A COMPLETE SYSTEM

OF

THEORETICAL AND MERCANTILE

ARITHMETIC.

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A COMPLETE SYSTEM

OF

THEORETICAL AND MERCANTILE

ARITHMETIC.

COMPREHENDING A

FULL VIEW OF THE VARIOUS RULES NECESSARY
IN CALCULATION.

WITH

PRACTICAL ILLUSTRATIONS

Of the most material Regulations and Transactions that occur in Commerce.

PARTICULARLY,

INTEREST, STOCKS, ANNUITIES,
MARINE INSURANCE,
EXCHANGE, &c. &c.

COMPILED
FOR THE USE OF THE STUDENTS AT THE
COMMERCIAL INSTITUTION,
WOODFORD.

By GEORGE G. CAREY,

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To JOHN JAY, Esq.

Principal of the Commercial Institution,

WOODFORD.

THERE is no person to whom I can with greater propriety inscribe this Work, than to you, Sir. The distinguished situation you have held in the commercial world, both in this Country and on the Continent, combined with the liberal course of mercantile instruction, to which you have since devoted your time and abilities, eminently qualify you to appreciate the claims of such a production to the patronage of the friends of the useful sciences, and to give a sanction to its object, that of contributing to the diffusion of a correct knowledge of that branch of science, to the cultivation and improvement of which your attention is at present so laudably and successfully devoted.

The utility of the institution established by you, and conducted under your auspices, in which a regular and enlightened system of edu-

cation is carried on, has been widely and deservedly acknowledged; indeed it is but justice to say, that the most solid advantages to the community may naturally be expected to arise from an Institution so conducted, where practice is combined with theory, and science with the principles of trade.

A treatise on the same extended plan, and embracing, in some degree, the same object, it is humbly deemed, cannot but be useful to the Public. That this Work, therefore, which has been compiled with this view and benefited by your suggestions, may be found deserving of your approbation, and that of the Public, is the earnest desire of

YOUR OBLIGED SERVANT,

G. G. CAREY.

PREFACE.

In a commercial country like this, it is needless to enlarge on the utility and necessity of arithmetical knowledge.—What has been found to be most requisite and desirable, is a *complete system* of that most useful branch of science; for, though there is no want of books on Arithmetic, the generality of these works are found extremely deficient in the course of instruction necessary for the commercial world.

The greater number of school books on this subject contain merely a few practical rules for performing some of the most common calculations that occur in conducting the affairs of a small business. Neither the theory of these rules, the contractions that may be introduced, nor a sufficient variety of exercises are given.

A few there are, that contain a great variety of rules, and even some of the contracted methods of performing the exercises; but they neither explain the rules, nor show how the contractions are obtained, and contain a set of exercises but little fitted for giving the student a knowledge of calculations connected with the counting-house.

Some of the best treatises on Arithmetic cannot be procured separately, being connected with other matter in the same volume, which renders them inconvenient for use, and too expensive for common text books; others, though very well calculated for teaching beginners the rudiments of the science, are so limited in size, that it is impossible

they can give either variety of rules or examples; and what may be strictly termed Commercial Arithmetic is scarcely introduced at all.

With the view of supplying the deficiencies so generally felt, in this respect, and of laying before the commercial student a comprehensive and systematic course of instruction, in all the probable cases of commercial calculation, and as they really take place in business, the present volume has been compiled.

In executing this work, it has appeared peculiarly necessary to elucidate the complex subjects of the Stocks, Marine Insurance, Exchanges, and Annuities, with other topics highly important to a great part of the population of this country, but scarcely noticed in, or entirely excluded from, works expressly intended for the instruction of those to whom such information is most necessary. The care taken to effect this, it is presumed, will be apparent from the manner in which the various subjects are treated.

As Logarithms are of so much use in abridging the labour of calculation, in many cases, particularly in Arbitration of Exchange, Compound Interest, and Annuities, an extensive and correct table of these is added to the volume.

It will be observed that the arrangement here adopted is different from that employed in any other work; the first part being entirely devoted to the consideration of Arithmetic simply as the science of *numbers*, and may be termed the theoretical part; the second, embracing a particular and ample view of mercantile transactions, and the business which naturally comes within the denomination of

the great branches of commerce herein considered. It is hoped this arrangement will be found convenient both for the student and teacher, as those students who wish only to acquire a general knowledge of calculation, will find sufficient rules and exercises in the first part for this purpose, whilst those who have already acquired a competent knowledge of the general principles of Arithmetic, may commence the study of the mercantile part without going through the whole.

The best works, on the subjects comprised in this volume, have been carefully consulted, and such articles as appeared necessary, have been included in this; but it would be superfluous to mention these. I cannot, however, but acknowledge the advantage I have derived from the excellent work of my esteemed friend, Mr. John Davidson, A.M. "the Practical Calculator," a production, perhaps, but too little known in this part of the kingdom.

It may be necessary to observe that, in the compilation of a work like this, and on its first appearance, some inaccuracies must be unavoidable, but it is hoped that such *errata* as may be discovered in this volume are but few, and such as ought not to weigh with the observer against the claim which the Author humbly puts in, on account of his labours, to public suffrage.

That it may be of use to those, for whom it is peculiarly intended, and contribute to those commercial advantages which form the basis of our national prosperity, is the sincere wish of

THE AUTHOR.

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ARITHMETIC.

1. Arithmetic is that science which treats of number.

2. The *theory of Arithmetic* treats of the properties of numbers in the most general and abstract manner.

3. *Practical Arithmetic* applies numbers to the performance of calculations in the various arts and common affairs of life.

DEFINITION OF NUMBERS, CHARACTERS, &c.

4. A *unit* is the number *one;* but is sometimes used to signify any of the nine digits.

5. A *whole number* consists of one or more units, unbroken, or not divided into parts; as 7, 24, 130, &c.

6. An *integer* is the whole of any thing; as a pound, a yard, &c. or, 1, 2, 7, 9, &c.

7. A *fraction* is a part of an integer or whole number; as $\frac{3}{4}$, $\frac{7}{8}$, $\frac{33}{142}$, &c.

8. A *mixed number* is, a whole number with a fraction annexed to it; as, $1\frac{1}{2}$, $4\frac{2}{3}$, $24\frac{7}{9}$, &c.

9. An *even number* is that which can be divided into two equal whole numbers; as, 8, 12, 64, &c.

10. An *odd number* is that which cannot be divided into two equal whole numbers; as, 9, 17, 29, &c.

11. A *prime number* is that which cannot be produced by the multiplication of two others, greater than unity; as 3, 5, 7, 11, 19, &c.

B

12. A *composite number* is that which can be produced by the multiplication of two or more numbers together; as, 6, 16, 24, 96, &c.

13. A *square number* is the product of a number multiplied into itself; as 4 is the square of 2; 49 of 7, &c.

14. A *cube number* is the product of a number multiplied twice into itself; as 8 is the cube of 2; 27 the cube of 3, &c.

15. A *perfect number* is that which is just equal to the sum of all its aliquot parts; as 6; for its aliquot parts are 1, 2, 3, which make 6, when added.

16. An *aliquot part* of a number, is that which is contained in another an exact number of times; as 2 is an aliquot part of 8; 3 an aliquot part of 15; and 7 of 28*.

17. *Ratio* is the relation that subsists between two numbers of the same kind, the comparison being made by considering what part, or parts, the one is of the other; as, when 6 is compared with 3, it is found to contain it twice, and the ratio of these two numbers is therefore said to be as two to one.

18. The *ratio* of *two numbers* is usually expressed by two points placed between them, thus, 8 : 4; and the former is called the *antecedent* of the ratio and the latter the *consequent*†.

19. *Analogy*, or *proportion*, is the similarity of ratios, when compared with one another; thus, 6 has to 3 the same ratio that 8 has to 4; therefore these numbers are proportional to each other; for 6 : 3 : : 8 : 4.

20. A *multiple* of a number is that which contains the number a certain number of times without a remainder; as 9 is a multiple of 3; 48 of 12; 72 of 8, &c.

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* For the different species and properties of numbers, see Hutton's Mathematical Recreations, Vol. I.

† Quantities, that are not of the same kind, cannot be compared together.—One line may have to another line the same ratio that one weight has to another weight, or that one portion of time has to another; but a line has no relation, in respect of magnitude, to a weight, or, in duration, to a portion of time.

21. That number which measures another, or is contained in it any number of times exactly, is called its *submultiple*.

22. The *greatest common measure* of two or more numbers, is the greatest number which will divide those numbers without a remainder; as 8 is the greatest common measure of 16, 24, and 40.

23. The *reciprocal* of a number is the quotient obtained by dividing unity, or 1, by that number; as $\frac{1}{8}$ is the reciprocal of of 8; $\frac{1}{28}$, of 28, &c.

24. *Homogenious quantities* are such as can be added together.

CHARACTERS.

21.	+ (Plus)	Is the sign of Addition.
	− (Minus)	——— of Subtraction.
	×	——— of Multiplication.
	÷	——— of Division.
	=	——— of Equality.
	: :: :	——— of Proportion.
	$\sqrt{}$, $\sqrt[2]{}$, or $^{\frac{1}{2}}$,	——— of the Square Root.
	$\sqrt[3]{}$, or $^{\frac{1}{3}}$	——— of the Cube Root.

25. A small figure, placed over any number, signifies that it is multiplied as many times into itself. Thus $8^2=64$, and $5^3=125$.

NOTATION.

1. The method of writing, or expressing numbers, by proper characters, is called NOTATION; and reckoning or reading their value, when written, is called NUMERATION.

2. Various contrivances have been used to accomplish the purposes of Notation and Numeration, at different periods, and by different nations of the world.

3. It is highly probable that short lines, or dots, were the first characters employed to denote or record numbers.

4. The letters of the alphabet appear also to have been used for this purpose, but each letter represented a particular number, which is evident in the divisions of Homer's Poems, and the 119th Psalm.

5. The Greeks used the initial letters of the names of the numbers, in their language, to express any number they wished to represent.

6. The Romans likewise employed a similar method, and, besides characters, for each rank of classes, they introduced others to represent five, fifty, and five hundred. The characters which they employed, and their values, are the following: I, for one; V, for five; X, ten; C, one hundred; D, five hundred; and M, one thousand.

7. By combining these characters according to the following rules, any number may be represented.

8. When the same letter is repeated twice, or oftener, its value is represented as often. Thus II signifies two; XXX, thirty; CC, two hundred.

9. When a numeral letter, of less value, is placed after one of greater, their values are added; thus, XI signifies eleven; LXV, sixty-five; MDCXXVIII, one thousand six hundred and twenty-eight.

10. When a numeral letter of less value is placed before one of greater, the value of the less is taken from that of the greater; thus, IV signifies four; XL, forty; XC, ninety; CD, four hundred. Sometimes IↃ is used instead of D, for five hundred; and the value is increased ten times by annexing Ↄ to the right hand.

Thus	IↃ	signifies 500	and	CIↃ,	1000.
	IↃↃ	5000		CCIↃↃ,	10,000.
	IↃↃↃ	50,000		CCCIↃↃↃ,	100,000.

11. Sometimes thousands are represented by drawing a line over the top of the numeral; $\bar{V}$ being used for five thousand; $\bar{L}$, for fifty thousand; $\bar{C}\bar{C}$, two hundred thousand.

12. The present mode of Notation, of giving each character a local value, that is, a value from the place it occupies, is found to be both easier and more extensive than any other method hitherto known, and is said to have been invented in India, and introduced into Europe by the Moors, who brought it from Arabia.

The figures or characters by which numbers are represented, at the present day, are also said to have been brought from Arabia.

13. These characters are ten in number, viz.

1	2	3	4	5	6	7	8	9	0
One	two	three	four	five	six	seven	eight	nine	cipher.

14. By these ten characters all numbers may be represented.

15. The nine first are called significant figures, or digits, and sometimes represent units, sometimes tens, hundreds, &c., according as they stand singly or joined to others.

16. When any significant figure stands alone, it denotes simply its own value; but it has besides a local value, which depends on the place it occupies, when connected with others: thus, a figure standing in the first or right hand place, denotes only its simple value, and is said to be so many units; but if another figure, or the cipher, be placed on the right of it, then its value is increased ten times what it was before; or it is shifted from the units place to the place of tens.

17. If two figures or ciphers be placed on the right of any figure, its value is increased one hundred times; if three, one thousand times, and so on, increasing in a tenfold proportion for every figure that is added.

18. The reason of this is obvious, for every figure or cipher that is placed on the right of any other figure, shifts the first figure, as it were, one place to the left of what it formerly held, and makes it occupy a higher or more valuable place, by which means its value is increased ten times what it was before, at every remove; for every place from the right is ten times more valuable than the one on the right of it.

NUMERATION.

1. The names of the places or classes may be learned from the following

TABLE.

2. 2.8 4 5,3 9 6.7 8 5,6 2 9.8 6 2,6 4 5

2	TRILLIONS.
8	Hundred thousand of billions
4	Ten thousand of billions....
5	Thousand billions ....
3	Hundred billions ....
9	Ten billions ....
6	BILLIONS.
7	Hundred thousand of millions
8	Ten thousand of millions ..
5	Thousand millions ....
6	Hundred millions ....
2	Ten millions ....
9	MILLIONS.
8	Hundred thousands...........
6	Ten thousands ....
2	Thousands ....
6	Hundreds ....
4	Tens ....
5	Units ....

3. The first six figures from the right hand are called the unit period; the next six the million period; after which the trillion, quadrillion, quintillion, sextillion, septillion, octillion, and nonillion periods, follow in their order.

4. The best method of reading any large number, is first, to divide it into periods and half periods, by different marks; then to begin at the left hand, and read the figures in their order, by naming them according to the places they hold, as in the preceding table.

5. In writing any number, care must be taken to give each figure its proper place, which may easily be done by means of the cipher, when significant figures are wanting, for this seems to be the only use of the cipher.

6. Write down in figures the following numbers.

1. Three hundred and fifty-four.
2. Five hundred and six.
3. Three thousand and eighty-six.
4. Nine thousand and seventy.
5. Sixteen thousand, seven hundred, and eighty-five.
6. Two hundred thousand, two hundred, and nine.
7. Forty-seven millions, eight hundred thousand.
8. Forty-five thousand, six hundred and seven millions, five hundred and sixty-four thousand, three hundred.

9. Six hundred and twenty-four thousand, three hundred and seventy-one billions, two hundred and sixty-four thousand, six hundred and thirty-five millions, nine hundred and ninety-five thousand, four hundred, and forty-two.

7. Write down, in words, the following numbers.

No.	1.	62	No.	6.	50,471
	2.	90		7.	307,605
	3.	630		8.	4,768,070
	4.	704		9.	25,024,738
	5.	6,789		10.	860,704,023

8. Numbers being susceptible of no other change than that of increase or diminution, it follows, that the whole of Arithmetic is comprehended in two operations, viz. *Addition* and *Subtraction*. However, as it is frequently required to add several *equal* numbers together, or to subtract several *equal* ones from a greater, till it be exhausted, proper methods have been invented to render these operations more easy, which are distinguished by the names of *Multiplication* and *Division*; therefore, these four rules are the basis of all practical Arithmetic.

ADDITION.

ADDITION is that operation by which we find the amount of two or more numbers.

RULE.

Place the numbers distinctly under each other; that is, units under units, tens under tens, and so on; and draw a line under the lowest number. Then reckon the amount of the right-hand column; and if it be under ten, mark it at the foot of the column. If it exceed ten, consider how many tens it contains, and carry as many units to the next column, and mark the excess of the sum above the tens, at the foot of the column. In like manner, carry the tens of each column to the next, and mark down the full sum of the left-hand column.

EXAMPLES.

1.	2.	3.	4.
276	3142	4135	5341
110	2314	5413	3415
473	4231	2541	4153
358	1423	1254	1536
589	3142	4125	5361
1806			

5.	6.	7.
47826	934826	217618
73840	751045	379824
21397	273876	218342
74186	482942	142729
21814	537268	374286

8.	9.	10.
36928	557896	5678964
58871	102357	7967343
36984	579982	1254710
72587	258989	6786786
97823	837254	434376
64834	919486	752698
75462	767358	1354968

11. Add together 308+5675+96398+5675+6987+7857+9375+8597+4984+6529.

12. Add together, 2976+5497+58384+97653+918+65837+97549+8968+56975+32817.

13. Add together, 98756+42918+36987+678+49186+38679+8196+76919+75679+89753.

14. Add together, 43724+3964+7982+53409+37654+38217+86214.

As it is of the utmost consequence in business, to perform addition readily and correctly, the student ought to practise this part of arithmetic till he can do it with facility.

In order to acquire this knowledge, the following remarks will be found useful:

1. If a person can readily add any two figures together, he will as easily add one figure to any sum.

2. For it is only to add the figure to the unit place of that sum, and if it exceed ten, it raises the ten place of the former sum, one. Thus, because seven and nine make sixteen, thirty-seven and nine make forty-six; forty-seven and nine, fifty-six, &c.

3. After a little practice in adding *two* figures together, or one figure to any sum, two or more figures may be considered as forming one sum, and added to any other sum at one glance. This is very easily done, when two or more figures amount to ten; as 6 and 4; 7 and 3; or 2, 3, and 5, stand in the same co-

lumn. To accomplish this with greater ease, it may sometimes be preferable to begin at the top of a column rather than at the bottom, and to add the carriage from a former line to the second or third figure from the top or bottom of the next column, instead of adding it to the first figure from the top or bottom, as is usually done.

4. In adding a number of columns, it is proper to write down the carriage from one column to the next, under its respective column, to prevent the trouble of summing the different columns over again.

METHODS OF PROOF.

1. Every operation should be examined or performed in a *different manner* from what it was done at first, in order to insure correctness; for, if the same mode be taken, the same error may be committed.

2. If the addition was *first* performed by beginning at the *bottom* of the columns, it ought to be performed the *second time* by beginning at the *top* of the columns; then, if the sum in both cases be the same, the *true* sum is obtained, otherwise not.

3. Divide the sums to be added into several divisions, add these separately, and then add their sums together. If the amount of these be the same as the amount obtained by adding at once, it may be presumed the true amount is obtained. This method should be adopted when the sums of the parts are required, as well as the whole.

4. Subtract each number, or term to be added from the amount, which will at last be wholly exhausted, without any remainder, if the addition has been rightly performed.

SUBTRACTION.

Subtraction is the operation by which we take a less number from a greater, in order to obtain their difference. The greater is called the *Minuend*, and the less, the *Subtrahend*.

RULE.

Place the subtrahend under the minuend, and subtract units from units, tens from tens, and so on. If any figure of the subtrahend be greater than the corresponding one of the minuend, borrow ten; that is, add ten to the upper figure, and then subtract the lower from the sum, put down the remainder, and carry one to the next figure of the subtrahend.

EXAMPLES.

	1.	2.	3.
Minuend	153876	7543879	17865432
Subtrahend	21754	4876294	2189345
Remainder	132122		

4.	5.	6.
894210	3812642	4218762
272849	1705897	4207579

7.	8.	9.
9386425	7214082	4218318
2098248	1880417	1688792

MISCELLANEOUS EXERCISES.

1. A person was born in 1780 and died in 1816, what was his age?

2. The art of printing being discovered in 1449, how many years is it since, this being 1816?

3. A merchant had two debtors, A, and B, who owed him, together, 1280; A owed him £879, what did B owe him?

4. If a person live till the year 1830, and is then fifty years old, in what year was he born?

The reason for borrowing or adding ten to the upper figure, when the under one is greater, will appear, if we consider that numbers increase in a tenfold ratio, and that, by increasing any two numbers equally, their difference is not altered. For by adding ten to the minuend, and one to the next higher place of the subtrahend, we add an equal sum to each.

METHODS OF PROOF.

1. Add the remainder to the subtrahend: if their sum be equal to the minuend, the operation is right.

2. Subtract the remainder from the minuend: if the difference be equal to the subtrahend, the operation is right.

MULTIPLICATION.

MULTIPLICATION is only a short method of performing Addition, when the same number is to be added any given number of times.

Thus, 4764 multiplied by 4, or 4764 added four times.

4764		4764
4		4764
		4764
19056		4764
		19056

From this it is evident, that, to multiply any number by another, is the same as to add one of the numbers to itself, as often as there are units contained in the other.

The numbers to be multiplied together are called *factors*; but the upper factor is also called the *multiplicand*, and the lower the *multiplier*. The number arising from the multiplication of the factors is called the *product*.

To enable the student to perform multiplication readily and correctly, it will be necessary to commit to memory the following

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

RULE I.

Place the multiplier under the multiplicand, and multiply the latter successively by the significant figures of the former; if the multiplier consist of more figures than one, place the right hand figure of each product under the figure of the multiplier from which it arises; then add the different products, which will be the whole product of both factors.

EXAMPLES.

1.	2.	3.
231037	7062526	3723104
6	7	8
1386222	49437682	29784832

4.	5.	6.
37846	312758	850467
235	789	234
189230	2814822	3401868
113538	2502064	2551401
75692	2189306	1700934
8893810	246766062	199009278

No. 7.	193524	by	456	No. 11.	56842	by	18642
8.	204916	by	678	12.	21593	by	91573
9.	83075	by	7684	13.	36898	by	98979
10.	84639	by	7695	14.	46897	by	57987

RULE II.

If the multiplier be a *composite* number, we may multiply by its component parts, instead of the number itself.

Example—4276 by 42, or by 7 times 6.

4276	4276
42	7
8552	29932
17104	6
179592	179592

EXERCISES.

No. 1.	Multiply	5492 by	72	No. 4.	Multiply	378914	by 54
2.	———	13759	56	5.	———	520813	by 63
3.	———	73048	48	6.	———	56417	by 144

If the multiplier be 5, which is the half of 10, we may annex a cipher, and then divide by 2. If it be 25, which is the fourth part of 100, we may annex two ciphers, and then divide by 4: if 125, which is the eighth part of 1000, three ciphers, and then divide by 8. Other contractions may be found in a similar manner.

RULE III.

1. To multiply any number by 9, which is one less than 10, annex a cipher to the number and then subtract the multiplicand from that number, increased by the cipher, and the remainder will be the product.

2. To multiply by 99, or any number of nines, annex as many ciphers as there are nines in the multiplier, and then subtract the multiplicand. A similar rule may be found for other figures as well as for 9.

EXAMPLE.

Multiply 74896 by 992

```
74896000 = × 1000
  599168 = ×    8 deducted
--------
74296832
```

EXERCISES.

No. 1. Multiply 4875967	by 99994	No. 3. 4759378	by 7997
2. 4169875	5995	4. 7093856	399958

It often happens that a line of the product is more easily found from a former line than from the multiplicand itself.

EXAMPLES.

```
1.   473526          2.   372568
         84                   36
     -------              -------
    1894104              2235408
   3788208              1117704
   --------             --------
   39776184             13412448
```

In the first example, instead of multiplying by 8, we may multiply 1894104 by 2, and in the second, instead of multiplying by 3, we may divide 2235408 by 2.

The product of two or more figures may sometimes be obtained at once, from the product of a figure already found.

EXAMPLES.

No. 1.

42397
546
254382
2289438
23148762

No. 2.

3522
54279
31698
95094
190188
191170638

In the *first* example, we first multiply by 6; then, because 6 times 9 is 54, we multiply that product by 9, instead of multiplying successively by 4 and 5.

In the second example, we first multiply by 9; then, because 3 times 9 is 27, we multiply the first line of the product by 3, instead of multiplying separately by 7 and 2; lastly, because twice 27 is 54, we multiply the second line of the product by 2, instead of multiplying separately by 4 and 5.

When this method is followed, care must be taken to place the right hand figure of each product under the right hand figure of that part of the multiplier from which it is derived.

Contractions may sometimes be obtained by beginning the work at the highest place of the multiplier, instead of the lowest.

METHODS OF PROOF.

1. Repeat the operation, using the multiplier as the multiplicand, and the multiplicand as the multiplier.

2. Cast the nines out of the multiplicand and also out of the multiplier, if above 9. Multiply the excesses together, and cast the 9's, if necessary, out of their product. Then cast the 9's out of the product of the factors, required to be multiplied together, and observe if this excess correspond with the former. The results are generally placed round a cross, thus:—

7898		3456	
48	6	76	0
	5×3		0×4
63184	6	20736	0
31592		24192	
379104		262656	

MISCELLANEOUS EXERCISES.

1. There are twelve signs in the Ecliptic, and every sign contains 30 degrees; how many degrees does the Ecliptic contain?

2. How many balls in 18 boxes, each containing 365?

3. How much hay will grow on 425 acres, supposing two crops, the first 212 loads, and the second 70 loads per acre, distinguishing the crops?

4. The sum of two numbers is 4584, and the less is 1876: what is their product?

5. What is the difference between twelve times sixty-seven, and twelve times seven and sixty?

6. How many miles will a person walk, in five years, supposing he travels twenty-three miles each day?

7. Suppose 200 men take a prize, and each man's share is £160, what is the value of the prize?

8. How many letters in a book of 7 vols., each vol. containing 328 pages, each page 34 lines, and each line 36 letters?

DIVISION.

In Division two numbers are given, and it is required to find how often the one is contained in the other.

Thus it may be required to find how many times 9 is contained in 58, and the answer is 6 times, with a remainder of 4. Here the 9, or number divided by, is called the Divisor; the (58) or number to be divided, the Dividend; and 6, or the number of times the divisor (9) is contained in the dividend, (58) is called the Quotient.

As the operation of Division would be tedious, when the divisor is contained a great many times in the dividend, it is proper to shorten the labour, as much as possible, by every convenient method that can be discovered.

RULE I.

When the divisor is less than 12, place the divisor on the left-hand of the dividend, separating them by a short line, then consider how often the divisor is contained in the first or left-hand figure of the dividend; if this figure shall be less than the divisor, take the first two, or even three, if necessary, and set down the quotient under the right-hand figure employed; if there shall be any remainder, carry it as so many tens to the next figure of the dividend, and continue this operation till all the dividend is exhausted.

EXAMPLES.

No. 1,	3)721472	No. 2,	7)6178298
	240490—2		882614

No. 3, 4)7210416
    1802604

No. 4, 9)4100742
    455638

No. 5, 5)7203287
    1440657—2

No. 6, 11)6745681
    613243—8

EXERCISES.

No. 7, 786425796 by 6.
8, 3148270546 by 11.
9, 1015304064 by 12.
No. 10, 541706652 by 9.
11, 384070019 by 12.
12, 701406214 by 11.

RULE II.

1. When the divisor is a composite number, and greater than 12, divide by the component parts of the divisor: that is, divide first by one of the component parts, and the quotient thence arising by the other component part, and so on; the last quotient will be the quotient required.

2. If the divisor be not a composite number, produced by the multiplication of two numbers, each under 12, this rule should not be applied, as the division will be more troublesome than by the next rule.

3. In using two divisors instead of one, if there should be any remainder in dividing by the second figure, multiply this remainder by the first divisor, and to the product add the first remainder (if any) and the sum will be the complete remainder*.

* If there be no remainder in dividing by the first divisor, it is not necessary to multiply the second remainder by the first divisor, the fraction being only in lower terms than when the whole divisor is its denominator.

EXAMPLES.

No. 1, 63 { 9)30114 by 63
7) 3346

Quotient 478

No. 2, 42 { 7)72408 by 42
6)10344

1724

No. 3, 28 { 7)76482 by 28
4)10926

2731—$\frac{1}{2}$

No. 4, 96 { 12)37984 by 96
8) 3165—4

395—$\frac{64}{96}$

EXERCISES.

No. 1,	824135 by 24.	No. 4,	576298 by 108.
2,	813718 by 72.	5,	904682 by 121.
3,	682762 by 81.	6,	372549 by 144.

When there are ciphers annexed to the divisor, cut them off, and cut off as many figures from the right-hand of the dividend; attach the figures cut off to the remainder, which will form the complete remainder.

RULE III.

1. When the divisor is not a composite number and greater than 12, assume as many figures on the left-hand of the multiplier as contain the divisor once, or oftener; find how many times they contain it; and place the answer as the highest figure of the quotient.

2. Multiply the divisor by the figure you have found, and place the product under the part of the dividend from which it is obtained.

3. Subtract the product from the figures above it.

4. Bring down the next figure of the dividend to the remainder, and divide the number it makes up, as before.

EXAMPLES

Divide 1760598 by 23

```
Divisor. Dividend. Quotient.
      23)1760598(76547
         161
         ---
          150
          138
          ---
           125
           115
           ---
            109
             92
            ---
             178
             161
             ---
              17 Remainder.
             ---
```

Divide 550914 by 234.

```
234)550914(2354
    468
    ---
     829
     702
     ----
     1271
     1170
     ----
      1014
       936
      ----
        78 Remainder.
      ----
```

It is usual to mark a point under the figures of the dividend, as they are brought down, to prevent mistakes.

EXERCISES.

No. 1,	378315 by 39.	No. 7,	975402 by 1338.
2,	832551 by 97.	8,	372109521 by 3071.
3,	589424 by 374.	9,	400716 by 6572.
4,	74879 by 563.	10,	832172 by 8708.
5,	382682 by 730.	11,	321047217 by 25204.
6,	718709 by 762.	12,	873467822 by 82873.

When the dividend is large, we may form a table of the products of the divisor by the nine digits, by which we may more readily perceive how often the divisor is contained in the various dividuals, or numbers to be divided.

EXAMPLE.

	73)5387269(73798
	511
73 by 2=146	277
3=219	219
4=292	
5=365	582
6=538	511
7=511	
8=584	716
9=657	657
	599
	484
	15 Remainder.

RULE IV.

To divide by 9, or any number of nines, write the dividend under itself, taking care to shift the highest place as many places to the right-hand as there are nines in the divisor.—Write it again, making the same change, as often as the length of the divideud admits; add these together, and then cut off as many figures from the right-hand of the sum as there are nines

in the divisor. The figures that remain on the left-hand compose the quotient, and those cut off, the remainder. If there be any carriage to the unit place of the quotient, add the same number to the remainder, and if the figures cut off be all nines, add 1 to the quotient, and there will be no remainder.

EXAMPLES.

No. 1, 99)432031
4320
43
Quotient 4363)94 Rem.

No. 2, 99)728862
7288
72
7362)22
2
Quotient 7362)24 Rem.

No. 3, 999)253746
253
253)999
1
Quotient 254

No. 4, 99)47235
472
4
477)11
1
Quotient 477)12 Rem.

EXERCISES.

No. 1, Divide 67403 by 99. No. 3, Divide 6748378 by 999.
2, —— 23548 by 990. 4, —— 5728376 by 9999.

The reason of this will be easily discovered, if we consider that any number of tens contains a like number of 9's, together with a like number of units; any number of hundreds a like number of 99's, together with an equal number of units; and the same thing is true for any multiple of ten, when divided by an equal number of 9's: hence the reason of the rule is obvious.

RULE V.

If the divisor can be multiplied, by a single figure, to make the product 100, or 1000, or any number of hundreds or thousands,

Multiply the dividend by the same figure that the divisor is multiplied by, and then divide it.

EXAMPLE I.

Divide 32875 by 25.

$$
\begin{array}{r}
32875 \\
4 \\
\hline
25\times4=1,00)1315,00 \\
\hline
1315 \quad \text{Quotient.}
\end{array}
$$

EXAMPLE II.

Divide 284175 by 75.

$$
\begin{array}{r}
284175 \\
4 \\
\hline
75\times4=3,00)11367,00 \\
\hline
3789 \quad \text{Quotient.}
\end{array}
$$

EXERCISES.

No. 1, Div.	59437 by 25.	No. 4, Div.	396765 by 225.
2, —	63728 by 125.	5, —	958452 by 275.
3, —	496475 by 175.	6, —	6880494 by 875.

RULE VI.

To divide by a number which becomes 10, 100, or 1000, &c. by adding one of its aliquot parts to it, or by subtracting one of its aliquot parts from it, add the same aliquot part of the dividend to itself, or deduct it from itself, which was added to, or subtracted from the divisor, and then divide the new dividend by the new divisor.

EXAMPLE I.

Divide 42750 by 75.

```
       75—42750
  ¼=25—14250
     ─────────
     1,00)570,00
          ──────
           570   Quotient.
```

EXAMPLE II.

Divide 3065125 by 875.

```
      875—3065125
 ⅐=125— 437875
   ─────────────
   1,000)3503,000
         ────────
          3503    Quotient.
```

EXERCISES.

No. 1, Div. 649378 by 875.		No. 4, Div.	64738 by 125.
2,	37968 by 12.	5,	8432076 by 1125.
3,	463798 by 15.	6.	579824 by 120.

The reason of these contractions are so apparent, that they require no explanation.

Those who are tolerably expert at performing division, by the foregoing methods, should endeavour to divide any number, by another, without putting down any more figures than the quotient and the several remainders, obtained by subtracting the different products of the divisor, by each of the quotient figures, as in the following example :

432)12345678(28577 Quotient.
3705
2496
3367
3438
414 Remainder.

This is sometimes called the *Italian* method, and differs only from the common method, in performing the necessary multiplications and subtractions *mentally*, instead of *visibly*.

Instead of bringing down each figure of the dividend, the arrangement would be neater to put down the figures only that remain, after subtracting the different products, and drawing a line through, or making a mark under each figure of the dividend, as it is used in the division.

The last example would then be arranged thus:

432)12345678(28577 Quotient.
3709604
24341
334

METHODS OF PROVING MULTIPLICATION AND DIVISION.

DIVISION, when compared with MULTIPLICATION, will be found to be exactly opposite in its tendency; hence these two rules mutually prove each other.

1. To prove Multiplication—Divide the product by either factor, the quotient will be the same as the other factor, without any remainder, when both operations are rightly performed.

2. To prove Division—Multiply the quotient and the divisor together; to their product add the remainder, if any; and, if

the operations have been accurately performed, it will make up the dividend.

3. Divide the dividend by the quotient, after having subtracted the remainder, when there is any, and the quotient, thus obtained, will be the same as the divisor, when the operations are accurately performed.

4. Add the remainder, and all the lower lines of figures together, and, if the work is right, the sum will be like the dividend.

This method of proof depends on this principle, that the product of the divisor and quotient, with the remainder added, equals the dividend; now the numbers directed to be added, are the products of the divisor by the several quotient figures, together with the remainder, which ought to make up the dividend, if the operation be rightly performed. This is, perhaps, the best method of proving Division.

EXAMPLE.

```
365)974932(2671 17/365      1st Product 730
     730                    2d          2190
     ----                   3d           2555
     2449                   4th           365
     2190                   Remainder      17
     ----                              ------
      2593                             974932
      2555                             ------
      ----
       382
       365
       ---
        17 Remainder.
       ---
```

3. Cast the 9's out of the divisor, and also out of the quotient, multiply the remainders together, and cast out the 9's from their product, if more than 9; cast the 9's also out of the remainder, and add these two last remainders together, and their sum, or the excess of their sum, above 9, will be equal to the remainder derived from the dividend, after casting out the 9's from it.

EXAMPLE.

```
          48)26680(555
Rem. 3       240     Rem. 6 × 3 = 18 ÷ 9 .. 0 Rem.
             ---
              268
              240
              ---
               280
               240
               ---
          Rem. 40 .................... 4 Rem.
               ---                     -
The remainder derived from the dividend is also   4 Sum.
```

The method of proving operations by casting out the 9's depends on the following principles:

If several numbers be separately divided by any divisor, and the respective remainders always added to the next number, the sum of the quotient figures and the last remainder will be the same as that obtained by dividing the sum of these numbers. Thus 14, 17, 25, contain as many 6's, as many 7's, &c. when divided separately, as their sum does, and the remainders are also the same: hence addition may be proved by division.

It is from the correspondence of the remainders, that the proof, by casting out the 9's, is inferred.

If any figure, with ciphers annexed to it, (except 9,) be divided by 9, the quotient will consist of that figure only, repeated as many times as there are ciphers attached to the significant figure, with the same figure as a remainder.

Thus, 50 divided by 9, quotes 5 with a remainder of 5; and 500 divided by 9, quotes 55, with a remainder of 5.

The reason of this will easily be perceived; for every figure, with a cipher annexed, contains exactly as many tens as there are ciphers; it must therefore contain an equal number of 9's and a remainder of an equal number of units.

If any number be divided by 9, the remainder is equal to the remainder of the sum of the figures, composing that number, when divided by 9.

For example, 4723, divided by 9, leaves a remainder of 7, and the sum of 4, 7, 2, and 3, which is 16, also leaves a remainder of 7, when divided by 9.

The reason of this will appear from the following illustration:

	4000	÷	9	=	444	and 4 of a	remainder.
	700	÷	9	=	77	7	remainder.
	20	÷	9	=	2	2	remainder.
	3	÷	9	=	0	3	remainder.
	4723				523	16	Sum remaining.
Again,	16	÷	9	=	1	7	remainder.
Wherefore	4723	÷	9	=	524	and 7	remainder,

for the reason stated above.

The number 9 has also this remarkable property, that the figures which compose its product, when multiplied by any of the nine digits, produce *nine* when added.

Thus,

$9 \times 2 = 18$ & $1 + 8 = 9$ $\quad 9 \times 6 = 54$ & $5 + 4 = 9$
$9 \times 3 = 27 \quad 2 + 7 = 9 \quad 9 \times 7 = 63 \quad 6 + 3 = 9$
$9 \times 4 = 36 \quad 3 + 6 = 9 \quad 9 \times 8 = 72 \quad 7 + 2 = 9$
$9 \times 5 = 45 \quad 4 + 5 = 9 \quad 9 \times 9 = 81 \quad 8 + 1 = 9$

It is from these principles that the rules, already given for proving any operation by casting out the 9's, are derived.

But this method of proving operations ought not to be recommended, as it cannot be depended upon; for, if an error of 9, or any multiple of 9, be committed, or if a figure be placed or reckoned in a wrong column, the results will nevertheless agree: and, therefore, the error will remain undetected.

If the method of proving operations, by casting out the 9's, be liable to the objection just stated, that of casting out the 3's is liable to the same objection, but in a three-fold degree; because every number contains three times as many 3's as it does 9's, and therefore there is three chances of committing an error by casting out the 3's, for one that there is by casting out the 9's.

MISCELLANEOUS EXERCISES.

1. Divide 4687 pounds equally among eleven men, required how much each man will receive?

2. There were planted 146835 trees in 585 rows. How many in each row?

3. The foundation of a wall measures 156 feet, and it contains 2,652 solid feet, what is its height?

4. If 76 equal pieces of linen contain 1767 yards, how many yards in each piece?

5. A garrison has 157½ sacks of corn. They consume daily 7 sacks and a half. How long will their corn last?

6. If the earth revolve round the sun in 365 days, 6 hours, 9 minutes, describing an orbit of 744457824 miles, how many miles does it move in a minute?

7. The circumference of the earth is 24,855 English miles, required how many English miles are in a degree?

PROPORTION.

PROPORTION, or *Analogy*, consists in the similitude of ratios.

Four numbers are said to be proportional, when the first contains the second, or any part of it, as often as the third contains the fourth, or the like part of it.*

* Or, which amounts to the same thing, four numbers are proportional, when the first, or any submultiple of it, is contained in the second as often as the third, or a like submultiple of it, is contained in the fourth.

Of four proportional numbers, the first of each pair is named the *antecedent*, and the second the *consequent*.

The first and fourth are called the *extreme* terms, and the second and third the *mean* terms.

When four numbers are proportional, the product of the extremes is equal to the product of the means.

Hence results the following rule for finding a fourth proportional to three given numbers.

RULE.

Multiply the second and third terms together, and divide the product by the first, and the quotient is the answer, or fourth proportional.

EXAMPLE I.

Required a fourth proportional to 3, 8, and 12.

$$\begin{array}{r} 3 : 8 :: 12 \\ 8 \\ \hline 3)96 \\ \hline 32 \text{ Answer.} \end{array}$$

EXAMPLE II.

Required a fourth proportional to 9, 6, and 24.

$$\begin{array}{r} 9 : 6 :: 24 \\ 6 \\ \hline 9)144 \\ \hline 16 \text{ Answer.} \end{array}$$

From the utility of this rule, and its extensive application, in the various species of calculation, it often happens, that the given terms consist of different denominations; and, as things can only be compared, or be proportional to each other, which are of the same kind or species, it becomes necessary to have some fixed principle or rule for arranging or stating the terms, when they are not abstract, previous to employing the above rule.

When a question is proposed to be solved by the rule of Proportion, the accountant must attend to the circumstance upon which the proportion depends; and common judgment will direct him to this, if the given terms be understood.

It is evident that the value, weight, and measure of any commodity, is proportional to its quantity; that the amount of work performed, or the quantity of provisions consumed, is proportioned to the time, or to the number of consumers, when the time does not vary; and that gain, loss, or interest, when the rate and time are fixed, is proportioned to the capital from which it arises, and that the effect, produced by any cause, is proportioned to the extent of the cause. In these and many other cases, the proportion is *direct*, or the number sought increases or diminishes along with the term of the same kind with itself, or with which it is to be compared.

In some questions, the number sought becomes less when the circumstance, from which it is derived, becomes greater. Thus when the price of goods increases, the quantity which can be bought for a given sum is less. When the number of men employed to perform any piece of work is increased, the time required to execute the same work will be shorter; and, when the energy of any cause is increased, the quantity necessary to produce any given effect is diminished. In these and similar cases, the proportion is said to be *inverse*.

Most of the writers on arithmetic divide Proportion into these two branches, *direct* and *inverse*, and give distinct rules for performing questions belonging to each branch; but, by attending to the above observations and the nature of the question, all questions, whether direct or inverse, may easily be stated by the following general rule:

RULE FOR STATING ALL QUESTIONS.

Place that number for the *third* term which signifies the same kind of thing with what is required, and then consider whether the answer to the question ought to be greater or less than that term. If *greater*, place the *least* of the other given numbers as the first term; but if *less*, place the *greatest* as the first term, and the remaining number as the second term.

The terms being thus stated, proceed to *multiply* the second and third terms together, and *divide* by the first, for the answer, as already directed.

EXAMPLE I.

If 6 men perform a piece of work in 8 days, how many men will do the same in 4 days?

```
   days  days   men.
If  4  :  8  ::  6
                 8
               ----
               4)48
               ----
                12 men.
```

EXAMPLE II.

If 16 men do a piece of work in 40 days, in how many days will 10 men do the same work?

```
men    men     days.
10  :  16  ::  40
       40
      ----
  1,0)64,0
      ----
       64  days.
```

In the first of these examples, *men* are required; therefore, the term, which is of the same kind with it, (6 men,) is made

the third term; and, because it will require more men to do a piece of work in 4 days than in 8 days, the least of the remaining terms is made the first term and the other the second term.

In the second example, *days* are required; therefore, 40 days are made the third term; and, because 10 men will require longer time to do the work than 16, the least of the remaining terms is made the first term and the other the second term.

The 1st of the above examples is *direct*, and the 2d *inverse*.

After a question is stated, the work may often be abridged by a person acquainted with the nature of proportion. To assist the student in accomplishing this important object, the following observations will very much contribute:

CONTRACTIONS.

1. When the second or third term can be divided by the first, or if any number occur to the memory, which will divide the first and second, or first and third term, without a remainder, it will shorten the work to divide these terms by that number, and use the quotients thence arising, instead of the original terms.

2. When the first term can be divided by the second, divide the third term by the quotient and the last quotient will be the answer.

3. When the first term can be divided by the third, divide the second by the quotient, and the last quotient will be the answer.

The three following examples will illustrate these different modes of contraction.

EXAMPLE I.

If 27 yards of cloth cost £54, what will 81 yards cost?

$$\begin{array}{ccccc} \text{Yds.} & & \text{Yds.} & & £ \\ 27 & : & 81 & :: & 54 \end{array}$$

$$\text{1st Contraction}\left\{\begin{array}{l} 81 \div 27 = 3 \times 54 = £162 \\ \text{or } 54 \div 27 = 2 \times 81 = £162 \end{array}\right\}\text{Ans.}$$

EXAMPLE II.

If 81 yards of cloth cost £162, what will 27 yards cost?

2d Contraction.

	Yds.		Yds.		£
81	:	27	: :		162
81	÷	27	=	3	)162

£54 Answer.

EXAMPLE III.

If £162 be paid for 81 yards of cloth, how many yards of the same may be bought for £54?

3d Contraction.

£		£			Yds.
162	:	54	: :		81
162	÷	81	=	2	)54

£ 27

MISCELLANEOUS EXERCISES.

1. If 16 men do a piece of work in 12 days, in how many days will 6 men do it?

2. If 8 men do a piece of work in 18 days, how many men will do it in 12 days?

3. If 12 men consume a certain quantity of provisions in 15 days, how long will the same quantity serve 20 men at that rate?

4. How many barrels, of 32 gallons each, will hold as much as 48 barrels of 42 gallons each?

5. A farmer had his crop cut down last year by 15 men in 24 days; how many men must he engage this year to cut it down in 18 days?

6. There are two numbers to one another, as 8 is to 11; the greater is 44, what is the less?

7. If 8 cwt. be carried 51 miles for a certain sum, how far ought 34 cwt. to be carried for the same money?

8. If the interest of £100 for a year be £4, what sum will produce £500 of interest in the same time?

9. If 6 horses eat 21 bushels of oats in a week, how many bushels will serve 30 horses the same time?

10. How many hours will a person require to count £245000, supposing him to count £250 in 3 minutes?

11. How many crowns are equal in value to 100 half-guineas?

/800

12. If there be ~~181~~ children born in a town, and 12 girls born for 13 boys, how many boys and how many girls are born?

13. A steeple projected a shadow to the distance of 35 yards, when a 4-foot staff, perpendicularly erected, cast a shadow 5 feet, required the height of the steeple?

14. How many yards of carpet (yard wide,) will cover a room, 25 feet long and 18 feet wide?

15. It is between 4 and 5 o'clock, and the hour and minute hands are exactly together; what is the precise time?

16. Suppose the arms of a deceitful balance to be to each other as 12 to $11\frac{1}{2}$, what weight will be required at the end of the shorter arm to counterbalance a weight of 46 lb. suspended from the end of the longer arm?

17. A traveller walks 24 miles a day, and after he had advanced 42 miles, another follows him, who walks 32 miles a day; in what time will he overtake him?

18. Janet can spin a certain quantity of yarn in 12 days, Margaret an equal quantity in 16 days; in what time will it be finished, if both work together?

19. A cistern, which would be filled in 3 hours, by 2 pipes running into it, would be filled in 12 hours by one of the pipes alone; in what time would it be filled by the other pipe alone?

20. If 21 oxen eat up 8 acres of grass in six weeks, and 18 oxen eat up the same in 9 weeks; how many oxen will it maintain for 18 weeks, if the grass grow uniformly during that time?

COMPOUND PROPORTION.

WHEN a proportion depends on several circumstances, connected with each other, it may be divided into as many simple proportions as these circumstances require, or it may be performed by one operation.

Thus, it may be asked, if 12 men consume 32 lbs. of bread in 18 days, how much will 24 men consume in 36 days?

Here the quantity of bread required depends, partly on the number of men and partly on the time; and, therefore, two distinct operations would be necessary to obtain the answer by simple proportion.

This, and all similar questions, forms what is called a Compound Proportion, which may be solved, by resolving it into as many simple proportions as there are particulars required in the question; or, what is much better, by the following rule:

RULE.

Place that number, which is of the same kind with the one required, as the third term; then state the several numbers, on which the question depends, as so many simple proportions, directly under each other, attending to the nature of the question, to discover whether the answer ought to be greater or less than the third term; then multiply all the numbers in the first row together, for a divisor, and all those in the second row together, and then proceed in every respect as directed in the simple rule of proportion.

```
men.      men.      lbs.
 12   :   24   : :  32
 18 days  36 days
 ---      ---
 216      144
 ---       72
          ---
          864
           32
         ----
         1728
        2592
        -----
   216)27648(128
        216
        ---
         604
         432
         ---
         1728
         1728
         ----
```

1. After stating the proportion, if the same number occur in both rows, it may be struck out of both; and if any number present itself to the mind, that will divide any number, in *both rows*, without a remainder, strike out the original numbers, and use the quotients in their stead.

2. Though it is in general better to multiply the second and third terms together, *before* dividing by the *first*; yet, if any of these terms can be divided by the first, without a remainder, this should be done before *multiplying* them together.

EXAMPLE.

If 10 horses consume £30 value of oats, in 8 weeks, when the oats cost 48s. per quarter, how many horses will consume £54 value, in 14 weeks, when the price is 27s. per quarter?

£30	: £54	: :	10 horses
14 months	: 7 months		
27 shillings	: 48 shillings		
Or £5	: 8 shillings		
1 shillings	: 2£		
2 months	: 1 month		
——	——		
10	: 16 horses the answer	: :	10.

Here 30 in the first row, and 48 in the second, are divided by 6, which is a submultiple of each, and their quotients, 5 and 8, used in their stead: 54, in the second row, being divisible by 27 in the first, and 14 in the first row by 7 in the second, these numbers are neglected, and their quotients used instead of them: and, lastly, the first and third terms being equal, they are also neglected; and, therefore 16, the second term, is the answer.

EXERCISES.

1. If 50 men cut down 30 acres of wheat in 6 days, how many men will cut down 400 acres in 20 days?

2. If 50 men cut down 30 acres of wheat in six days, how many acres will 10 men cut down in 40 days?

3. If 9 men build a wall, 120 feet long, $2\frac{1}{2}$ broad, and 16 high, in 12 days; how many will build a wall, 100 feet long, 2 broad, and 10 high, in 4 days?

4. If the interest of £100, for one year, be £5, what will be the interest of £75, for 9 months, at the same rate?

5. If 3 horses eat 12 bushels of oats, in 16 days, how much will 200 horses eat, in 24 days?

6. If £30 gain £4, in 5 months, what sum will gain £24, in 15 months?

7. Suppose the salary of 6 clerks, for 21 weeks, to be £120; what will be the sum necessary to pay 14 clerks, for 46 weeks, at the same rate?

8. A person put out a sum of money to interest, at 4 per cent, by which he cleared £35, in 21 months; required the sum?

9. If 4 oxen be maintained on 5 acres for 6 months, how many sheep may be kept on 56 acres, for 4 months, if 6 sheep eat as much as one ox?

10. A gentleman agreed with a labourer for 40 days, on condition that he should receive 20d. for every day he worked, and forfeit 10d. for every day he remained idle; at last he received 500d. for his labour; how many days did he work, and how many was he idle?

11. If 35 ells, at Vienna, make 24 at Lyons, and 3 ells at Lyons make 5 ells at Antwerp, and 100 ells at Antwerp make 125 at Frankfort, how many ells at Frankfort make 42 at Vienna?

12. If 78 lasts of Hamburgh make 70 of Amsterdam, and 30 of Amsterdam make 315 English quarters, and 100 English quarters make 198 bolls, how many bolls in 1000 lasts of Hamburgh, and how many lasts of Hamburgh in 1000 bolls?

13. A person contracted to make 240 yards of a road in 4 weeks, and for that purpose engaged 30 men; but, at the end of 8 days, found he had executed only 60 yards; how many additional men must he engage to finish the work in the stipulated time?

14. If 9 oxen eat up 3 acres of grass, in 5 weeks, and 20 oxen eat up 10 acres of the same in 10 weeks, how many oxen will eat up 30 acres of the same, in 25 weeks, if the grass grow uniformly during that time?

VULGAR FRACTIONS.

1. A Vulgar Fraction is one or more parts of unity, or one, and is represented by two numbers, one of which is placed above a small line, and the other under it: thus, $\frac{3}{4}$.

2. The upper figure, or number, is called the *numerator*, and the lower, the *denominator*.

3. The denominator shows into how many parts unity or one is divided; and the numerator shows how many parts of these are represented.

4. As the denominator may vary from one to any extent, while the numerator remains the same; and the numerator may vary in a similar manner, while the denominator remains the same, or both may vary; it follows that a vulgar fraction may express any part or parts of an integer, in terms of that integer.

5. From the nature of Vulgar Fractions it is evident that, when the numerator is increased and the denominator remains the same, the value of the fraction is increased; but, when the denominator is increased and the numerator remains the same, the value of the fraction is diminished.

6. If the numerator and denominator be both increased or diminished in the same proportion, that is, if they are both multiplied or divided by the same number, the value of the fraction is not altered; therefore every Vulgar Fraction may be expressed in a variety of forms, and yet retain the same value.

7. Vulgar Fractions are usually divided into proper and improper, simple and compound.

8. A *proper* fraction has its numerator less than its denominator; as, $\frac{4}{5}$, $\frac{8}{11}$, $\frac{7}{15}$, &c.—An *improper* fraction has its numerator greater than its denominator; as, $\frac{8}{3}$, $\frac{4}{3}$, $\frac{17}{9}$, &c.

9. A simple fraction consists but of one numerator, and one denominator, as $\frac{7}{8}$.

10. A compound fraction consists of two or more simple fractions, with the word *of* between them; and signifies a fraction of a fraction; as, $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$.

11. A mixed number is a whole number, with a fraction annexed to it; as, $3\frac{1}{4}$, $7\frac{3}{8}$, $17\frac{5}{6}$.

12. A complex fraction is a fraction having a mixed number,

either for its numerator or denominator; or a mixed number both for its numerator and denominator; as, $\frac{3\frac{1}{2}}{4}$ $\frac{5}{7\frac{3}{8}}$ $\frac{9\frac{3}{4}}{11\frac{3}{5}}$.

13. The common measure of a fraction is a number, which will divide both the numerator and denominator, without a remainder.

14. A common multiple, of two or more numbers, is a number which is divisible by the numbers of which it is a multiple, without a remainder.

15. The least common multiple of one or more numbers, is the smallest number that each of them will divide without a remainder.

CASE I.

To find the greatest common measure of two numbers.

RULE.

Divide the greater number by the less, and this divisor by the remainder. Proceed in this manner, always dividing the last divisor by the last remainder, till nothing remains; the last divisor is the greatest common measure.

EXAMPLE.

Required the greatest common measure of 84 and 144.

```
                         84)144(1
                             84
                            ----
                             60)84(1
                                60
                               ----
                                24)60(2
                                   48
                                  ----
Greatest common measure            12)24(2
                                      24
                                     ----
```

Required the greatest common measure of 536 and 792?

```
536)792(1
    536
    ---
    256)536(2
        512
        ---
         24)256(10
            24
            ---
             16)24(1
                16
                ---
Greatest common measure  8)16(2
                           16
                           ---
```

If the greatest common measure of 3 numbers be required, find the greatest common measure of the two first, and then the greatest common measure of that number and the third.— If there be more numbers, proceed in the same manner.

EXAMPLE.

Required the greatest common measure of 747, 945, and 1080.

```
747)945(1
    747
    ---
    198)747(3
        594
        ---
        153)198(1
            153
            ---
             45)153(3
                135
                ---
                 18)45(2
                    36
                    ---
Greatest common measure  9)18(2
                           18
                           ---
```

$$9)\underline{1080} \qquad 9)\underline{945} \qquad 9)\underline{747}$$
$$120 \qquad\quad 105 \qquad\quad 83$$

Here 9 is the greatest common measure of all the three numbers, and of course divides each of them without a remainder.

The reason of this operation is, that any number which measures two other numbers, will also measure both their sum and their difference, and likewise any *multiple* of either of them.—Thus, in Example I, 12 measures 24, it must measure $12+(24\times2)$ or 60; and, because it measures both 60 and 24, it must also measure $60+24$ or 84; and, again, because it measures 60 and 84, it must likewise measure $60+84$ or 144; therefore 12 is the greatest common measure of 84 and 144.

EXAMPLES.

What is the greatest common measure of the following numbers:

No. 1, 475 and 589.
2, 376 and 940.
3, 144 and 240.
No. 4, 32, 48, and 68.
5, 144. 216, and 324.
6, 376, 940, 1034, and 1081.

CASE II.

To find the least common multiple of two given numbers.

RULE.

Find the greatest common measure of the numbers, by last Case.

Multiply the two given numbers together, and divide their product by their greatest common measure, their quotient will be the least common multiple required.

Required the least common multiple of 12 and 18.

```
                          12)18(1
                             12
                             --
Greatest common measure    6)12(2
                             12
                             --
```

Then $12 \times 18 = 216 \div 6 = 36$ least common multiple.

If the least common multiple of three or more numbers be required.—Find the least common multiple of any two of them, and the least common multiple of this number and the last found multiple, which will be the answer for three numbers.

Continue this operation if more than three numbers are given.

RULE II.

Place the numbers in a line, and then divide as many of them as you can, by any small number, that will divide some of them without a remainder; place the quotients and the undivided numbers in another line under the former: divide the numbers in this line, in the same manner, by some other divisor, and write the quotients and the numbers, still undivided, in another line, which divide as before; and so on, till all the quotients become 1. Then multiply all the divisors together, and their *product* will be the least common multiple required.

EXAMPLE.

Required the least common multiple of 4, 9, 12, and 18.

2	4,	9,	12,	18
3	2	9	6	9
2	2	3	2	3
3	1	3	1,	3
	1	1	1	1

Then $2 \times 3 \times 2 \times 3 = 36$ the least common multiple.

If any of the given numbers be a submultiple of any of the other numbers, of which the least common multiple is required,

neglect such number and find the least common multiple of the remaining numbers.

Thus, in the last example, 4 is a submultiple of 12, and so is 9 of 18; therefore they may be neglected; because every multiple of 12 will be some multiple of 4, and every multiple of 18 will be some multiple of 9; and therefore it remains only to find the least common multiple of 12 and 18, which is 36, as in the example.

EXERCISES.

Required the least common multiple of the following numbers:

No. 1.	3, 4, and 8.	No. 4.	2, 3, 4, 5, and 6.
2.	3, 5, 8, and 10.	5.	10, 18, 30, and 45.
3.	4, 6, 8, and 12.	6.	24, 28, 32, and 56.

CASE III.

To reduce fractions to their lowest terms.

RULE.

Find the greatest common measure of the numerator and denominator by Case I.

Divide both terms of the fraction by their greatest common measure, and the quotients will be the numerator and denominator of the fraction in its lowest terms. If the greatest common measure be 1, the fraction is already in its lowest terms.

EXAMPLE.

Reduce $\frac{475}{589}$ to its lowest terms.

```
                            475)589(1
                                475
                                ---
                                114)475(4
                                    456
                                    ---
Greatest common measure              19)114(6
                                        114
                                        ---
```

Then $\frac{475}{589} \div 19 = \frac{25}{31}$ Answer.

EXERCISES.

No. 1.	$\frac{36}{48}$	No. 5.	$\frac{189}{279}$	No. 9.	$\frac{273}{429}$
2.	$\frac{45}{375}$	6.	$\frac{374}{418}$	10.	$\frac{64}{154}$
3.	$\frac{784}{1008}$	7.	$\frac{266}{456}$	11.	$\frac{732}{4408}$
4.	$\frac{306}{486}$	8.	$\frac{304}{840}$	12.	$\frac{1741}{1649}$

The above rule, for reducing fractions to their lowest terms, by first finding the greatest common measure and then dividing both parts of the fraction by it, is general; but it ought only to be had recourse to, when no number can be readily discovered that will divide both parts of the fraction without a remainder. For when any number can be found, by inspection, which will divide both the numerator and denominator without a remainder, employ this number as a common measure; that is, divide both parts of the fraction by it, and the resulting fraction again by any number that can be found to divide both parts of it without a remainder. Continue this process as long as any number can be found to divide the numerator and denominator without any remainder, and the original fraction will then be reduced to its lowest terms.

EXAMPLE.

Reduce $\frac{2592}{3456}$ to its lowest terms.

9)	4)	12)		2)	
2592	228	72	6	3	
3456	384	96	8	4	

In reducing fractions to their lowest terms, in this manner, the following remarks will be found useful:

If any number has a cipher in the unit's place, it is divisible by 10.

If any number has a 5 or 0, in the unit's place, it is divisible by 5.

If the two right-hand figures of any number be any multiple of 4, the whole number is divisible by 4.

If the three right-hand figures be any multiple of 8, the whole number is divisible by 8.

If the sum of the figures of any number be any multiple of 3 or 9, the whole number is divisible by 3 or 9.

If the right hand figure of any number be even, the whole number is divisible by 2.

If the right hand figure of any number be even, and the sum of all the figures any multiple of 6, the whole number is divisible by 6.

When the sum of the odd places, in any number, is equal to the sum of the even places of figures, that number is divisible by 11.

CASE IV.

To reduce whole or mixed numbers to improper fractions.

If the whole number has no given denominator, its denominator will be 1; if its denominator be given, multiply the whole number by that denominator, for a numerator to the given denominator; if the whole number has a fraction annexed to it, multiply the whole number by the denominator of the fraction, and add the numerator to the product, for a numerator to the given denominator.

EXAMPLE I.

Reduce 7 to an improper fraction.

$$7 = \frac{7}{1}$$

EXAMPLE II.

Reduce 9 to an improper fraction, whose denominator shall be 3.

$$9 = \frac{9 \times 3}{3} = \frac{27}{3}$$

EXAMPLE III.

Reduce $7\frac{2}{5}$ to an improper fraction.

$$7\tfrac{2}{5} = \frac{7 \times 5 + 2}{5} = \frac{37}{5}$$

EXERCISES.

1. Reduce 11 to an improper fraction.

2. Reduce 18 to an improper fraction.

3. Reduce 6 to an improper fraction, whose denominator shall be 7.

4. Reduce 19 to an improper fraction, whose denominator shall be 13.

5. Reduce $5\frac{3}{7}$ to an improper fraction.

6. Reduce $11\frac{7}{9}$ to an improper fraction.

7. Reduce $13\frac{11}{17}$ to an improper fraction.

8. Reduce $23\frac{3}{11}$ to an improper fraction.

9. Reduce $248\frac{23}{29}$ to an improper fraction.

10. Reduce $45\frac{1}{5}$. to an improper fraction.

CASE V.

To reduce improper fractions to whole or mixed numbers.

RULE.

Divide the numerator by the denominator, and the quotient will be the whole number, to which annex the remainder, (if any) with the denominator placed under it.

EXAMPLE I.

$$\frac{20}{4} = 20 \div 4 = 5$$

EXAMPLE II.

$$\frac{61}{8} = 61 \div 8 = 7\frac{5}{8}$$

EXERCISES.

1. Reduce $\frac{20}{6}$ to its equivalent whole or mixed number.
2. Reduce $\frac{41}{7}$ to its equivalent whole or mixed number.
3. Reduce $\frac{48}{16}$ to its equivalent whole or mixed number.
4. Reduce $\frac{234}{12}$ to its equivalent whole or mixed number.
5. Reduce $\frac{4362}{318}$ to its equivalent whole or mixed number.
6. Reduce $\frac{15764}{358}$ to its equivalent whole or mixed number.

CASE VI.

To reduce a compound fraction to an equivalent simple fraction.

RULE.

Multiply all the numerators together for the numerator of the simple fraction, and all the denominators together for its denominator, then abridge the fraction, when it can be done.

If any of the terms are whole or mixed numbers, first reduce them to improper fractions (by Case IV.) and then proceed as before.

EXAMPLE I.

Reduce $\frac{1}{2}$ of $\frac{3}{4}$ to a simple fraction.

$$\frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

EXAMPLE II.

Reduce $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{4}{9}$ to a simple fraction.

$$\frac{3 \times 5 \times 4}{5 \times 8 \times 9} = \frac{60}{360} = \frac{1}{6}$$

EXERCISES.

1. Reduce $\frac{4}{7}$ of $\frac{4}{6}$ of $\frac{7}{9}$ to a simple fraction.

2. Reduce $\frac{5}{8}$ of $\frac{3}{5}$ of $\frac{8}{9}$ to a ditto.

3. Reduce $\frac{3}{5}$ of $\frac{7}{16}$ of $\frac{9}{15}$ to a ditto.

4. Reduce $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $17\frac{3}{5}$ to a ditto.

5. Reduce $\frac{7}{9}$ of $\frac{4}{7}$ of $12\frac{1}{4}$ to a ditto.

6. Reduce $\frac{4}{5}$ of $\frac{3}{7}$ of $5\frac{1}{4}$ to a ditto.

CASE VII.

To reduce fractions of different denominators to equivalent fractions, having a common denominator.

RULE.

Multiply each numerator by all the denominators, except its own, for the new numerators; and multiply all the denominators together for a common denominator.

EXAMPLE.

Reduce $\frac{3}{8}$, $\frac{2}{5}$, $\frac{3}{4}$, to a common denominator.

$$\left.\begin{array}{l} 3 \times 5 \times 4 = 60 \\ 2 \times 8 \times 4 = 64 \\ 3 \times 8 \times 5 = 120 \end{array}\right\} \begin{array}{l} \frac{60}{160} \\ \frac{64}{160} \\ \frac{120}{160} \end{array}$$

$$8 \times 5 \times 4 = 160$$

EXERCISES.

1. Reduce $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{3}{4}$ to a common denominator.

2. Reduce $\frac{2}{3}$, $\frac{5}{8}$, and $\frac{7}{9}$ to a ditto.

3. Reduce $\frac{2}{7}$, $\frac{4}{5}$, and $\frac{1}{6}$ to a common denominator.

4. Reduce $\frac{2}{3}$, $\frac{4}{7}$, $\frac{3}{5}$, and $\frac{1}{2}$ to a ditto.

5. Reduce $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{5}$, and $8\frac{3}{7}$ to a ditto.

6. Reduce $\frac{2}{5}$ of $\frac{6}{7}$, $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{4}{5}$ of $8\frac{1}{8}$, to a ditto.

As the common denominator is a multiple of all the given denominators, we have only to find a number that is divisible by all the given denominators, for this number will be the common denominator of the respective fractions; but the common denominator found by the above rule, is not always the *least* number that can be divided by the given denominators, and as it is more convenient to work with small numbers than great, and also to have the fractions reduced to their *least* common denominator, the following rule should be employed for this purpose.

CASE VIII.

To reduce fractions to their least common denominator.*

RULE.

Find the least common multiple of all the denominators, (by Case 2, Rule 2,) for the new denominator; which divide by each of the given denominators; then multiply the quotients by the respective numerators, for the new numerators.

EXAMPLE.

Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$, to their least common denominator.

$$\begin{array}{r|l} 4 & 2,\ 4,\ 8,\ 12 \\ \hline 2 & 2,\ 1,\ 2,\ \ 3 \\ \hline 3 & 1,\ 1,\ 1,\ \ 3 \\ \hline & 1,\ 1,\ 1,\ \ 1 \end{array}$$

then, Num. Fractions.

$$\left.\begin{array}{l} \frac{24}{2} \times 1 = 12 \\ \frac{24}{4} \times 3 = 18 \\ \frac{24}{8} \times 5 = 15 \\ \frac{24}{12} \times 11 = 22 \end{array}\right\} \begin{array}{l} \frac{12}{24} = \frac{1}{2} \\ \frac{18}{24} = \frac{3}{4} \\ \frac{15}{24} = \frac{5}{8} \\ \frac{22}{24} = \frac{11}{12} \end{array}$$

$4 \times 2 \times 3 = 24$ common denominator.

* This is one of the most useful cases in reduction of fractions; for they can neither be added nor subtracted until they are first reduced to a common denominator; this case ought, therefore, to be well understood by the student, before he proceeds farther.

After arranging the numbers for multiplication, if the same number occur in every rank, they may be struck out and neglected; and if numbers occur in every rank which have a common measure, they may likewise be struck out and the quotients used in their stead.

A number may sometimes occur, on inspection, which is the least multiple of all the denominators, and the new numerators found, as directed in the rule.

EXERCISES.

1. Reduce $\frac{3}{4}$, $\frac{7}{10}$, and $\frac{11}{12}$ to their least common denominator.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{7}$, and $\frac{8}{9}$ to ditto.

3. Reduce $\frac{1}{2}$, $\frac{7}{8}$, $\frac{2}{3}$, and $\frac{11}{12}$ to ditto.

4. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{12}$, and $\frac{19}{36}$ to ditto.

5. Reduce $\frac{11}{24}$, $\frac{15}{28}$, $\frac{29}{32}$, and $\frac{31}{56}$ to ditto.

6. Reduce $\frac{7}{8}$, $\frac{11}{12}$, $\frac{31}{15}$, and $\frac{19}{24}$ to ditto.

CASE IX.

To reduce money, weights, and measures, to fractions.

RULE.

Reduce the given quantity to the lowest denomination mentioned in it, and make it the numerator; then reduce the proposed integer to the same denomination, and make it the denominator, and reduce the fraction to its lowest terms.*

EXAMPLE 1.

Reduce 4s. 2d. to the fraction of a £.

$$\frac{4s.\ 2d.}{£1} = \frac{50d.}{240d.} = \frac{5}{24} \text{ Answer.}$$

* This case differs little in its nature from the last case.

EXAMPLE II.

Reduce 1 cwt. 2 qrs. 7 lb. to the fraction of a ton.

$$\frac{1\text{cwt. }2\text{qr. }7\text{lb.} = 175\text{lb.}}{1\text{ ton} = 2240\text{lb.}} = \frac{5}{64} \text{ Answer.}$$

3. Reduce 5d. to the fraction of a shilling.
4. Reduce 9s. 7d. to the fraction of a £.
5. Reduce 6oz. 2dwt. $10\frac{1}{4}$ grains, to the fraction of a lb. troy.
6. Reduce 5 hours, 48 minutes, 48 sec. to the fraction of a day.
7. Reduce 1r. 10p. 12yds. to the fraction of an acre.
8. Reduce 1b. 3p. to the fraction of a quarter.

CASE X.

To find the value of a fraction in the known parts of the integer.

RULE.

Multiply the numerator by the number of parts contained in the next lower denomination; then divide the product by the denominator, and, if there be any remainder, multiply it by the parts in the next inferior denomination, and again divide by the denominator, and continue this mode of operation to the lowest denomination possible.

EXAMPLE I.

What is the value of $\frac{7}{8}$ of a £.?

```
   7
  20
 ----
8)140(17s. 6d. Answer.
  8
 ----
  60
  56
 ----
   4
  12
 ----
  48
  48
 ----
```

EXAMPLE II.

What is the value of $\frac{5}{9}$ of a ton?

```
   5
  20
 ----- cwt. qr. lb.  oz.
9)100( 11.  0.  12.  7 1/9. Answer.
   9
 ----
  10
   9
  ---
   1
   4
  ---
   4
  28
 ----
 112
   9
 ----
  22
  18
  ---
   4
  16
  ---
  64
  63
  ---
   1
```

EXERCISES.

3. What is the value of $\frac{17}{60}$ of a £.?

4. What is the value of $\frac{4}{67}$ of a moidore?

5. What is the value of $\frac{3}{7}$ of a lb. troy?

6. What is the value of $\frac{4}{5}$ of an acre?

7. What is the value of $\frac{1}{31}$ of a cwt.?

8, What is the value of $\frac{3}{16}$ of a cwt. ?

9, What is the value of $\frac{2}{15}$ of an oz. troy. ?

10. What is the value of $\frac{1}{4}$ of a quarter. ?

Although the value of the fraction should not amount to a unit of the lowest denomination, yet it may be reduced to a fraction of *that* or any *other* denomination, by multiplying the numerator according to the value of the denominations, as directed in the rule.

ADDITION OF VULGAR FRACTIONS.

RULE.

1. If the fractions to be added have a common denominator, add the numerators together and place the common denominator under the sum.

2. If the fractions have not a common denominator, reduce them to one, and then add the numerators.

3. When mixed numbers are to be added, find the sum of the integers and fractions separately, and add them together; or reduce them to improper fractions, and then add them.

4. If the given fractions are of different denominations, find the value of each, and add them, or reduce them to one denomination, and then add them.

EXAMPLE I.

Add $\frac{3}{8}$, $\frac{7}{8}$, and $\frac{5}{8}$ together.

$$\frac{3+7+5=15}{8} = 1\tfrac{7}{8} \text{ Sum.}$$

EXAMPLE II.

Add $\frac{5}{8}$, $\frac{2}{3}$, and $\frac{3}{4}$ together.

$$\begin{array}{rl} 5\times3\times4 = & 60 \\ 2\times8\times4 = & 64 \\ 3\times8\times3 = & 72 \\ \hline & 196 \\ 8\times3\times4 = & 96 \end{array} = 2\tfrac{1}{24} \qquad \text{or thus,} \quad \begin{array}{l} \frac{5}{8} = \frac{12}{24} \\ \frac{2}{3} = \frac{16}{24} \\ \frac{3}{4} = \frac{18}{24} \\ \hline \frac{49}{24} = 2\frac{1}{24} \end{array}$$

EXAMPLE III.

Add $14\frac{2}{5}$, $17\frac{3}{8}$, and $9\frac{7}{11}$ together.

$$\begin{array}{rrr} 2\times8\times11 = & 176 & 14 \\ 3\times5\times11 = & 165 & 17 \\ 7\times5\times\ 8 = & 280 & 9 \\ \hline & 621 & \\ 5\times8\times11 = & 440 & = 1\frac{181}{440} \\ & & \overline{41\frac{181}{440}} \text{ Sum.} \end{array}$$

EXAMPLE IV.

Add $\frac{2}{3}$ of a crown, $\frac{5}{6}$ of a guinea, and $\frac{3}{7}$ of a pound together.

$\frac{5}{6}$ of a guinea	£0	17	6
$\frac{3}{7}$ of a pound	0	8	$6\frac{3}{4}$—$\frac{3}{7}$
$\frac{2}{3}$ of a crown	0	3	4
	£1	9	$4\frac{3}{4}$—$\frac{3}{7}$ Answer.

Or thus,

$$\left.\begin{array}{r} £\frac{3}{7} = \frac{72}{168} \\ \frac{5}{6} \text{ guinea } £\frac{7}{8} = \frac{147}{168} \\ \frac{2}{3} \text{ crown } £\frac{1}{6} = \frac{28}{168} \end{array}\right\} = 1\frac{79}{168} = £1 \quad 9 \quad 4\frac{3}{4}-\frac{3}{7}.$$

The reason for reducing fractions to a common denominator, before adding them, is obvious; for, when the denominators are the same, the numerators signify like parts of unity, or the integer of which they are parts; and, therefore, the rule for adding them is similar, in effect, to that for adding pence, shillings, or any other inferior denominator of an integer.

EXERCISES.

5. Add $\frac{2}{3}$, $\frac{1}{3}$, and $\frac{5}{8}$ together.

6. Add $\frac{4}{9}$, $\frac{7}{9}$, and $\frac{8}{9}$ together.

7. Add $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{8}{9}$ together.

8. Add $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$ together.

9. Add $3\frac{5}{8}$, $9\frac{3}{7}$, and $2\frac{4}{5}$ together.

10. Add $47\frac{3}{4}$, $29\frac{7}{8}$, and $35\frac{9}{14}$ together.

11. Add $\frac{3}{11}$ of a guinea, $\frac{4}{7}$ of a moidore, and $\frac{3}{8}$ of a shilling.

12. Add $\frac{3}{8}$ of a lb. troy, $\frac{8}{9}$ of an oz., $\frac{7}{10}$ of a dwt., and $\frac{3}{8}$ of a grain.

13. Add $\frac{3}{14}$ of a cwt., $\frac{5}{7}$ of a qr., and $\frac{9}{11}$ of a lb.

14. Add $35\frac{1}{7}$ qr., $\frac{3}{4}$ of $\frac{5}{7}$ bushels, and $5\frac{4}{5}$ bushels.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions, when necessary, as directed in Addition, then subtract the one numerator from the other; place the common denominator under the difference, and this *new fraction* will be the difference between the given fractions*.

EXAMPLE.

Subtract $\frac{2}{7}$ from $\frac{4}{11}$.

$$
\begin{array}{l}
4 \times 7 = 28 \text{ num.} \\
2 \times 11 = 22 \text{ num.} \\
\hline
\qquad\qquad\quad 6 \\
7 \times 11 = \overline{77} \text{ diff.}
\end{array}
$$

When a proper fraction, or a mixed number, is to be subtracted from an integer, take the numerator of the fraction from the denominator, and place the remainder above the denominator, and diminish the minuend by *one;* or carry one to the right hand figure of the subtrahend, if it be a mixed number.

---

* The reason of the operations in this rule, and Addition, will be evident, if the numerators of the fractions be placed in a column, like the lower denomination of an integer, and then adding or subtracting them as integers, and carrying and borrowing, when necessary, according to the value of the higher denomination.

EXAMPLE I.

From 38 take $27\frac{7}{9}$.

$$\begin{array}{r} 38 \\ 27\frac{7}{9} \\ \hline 10\frac{2}{9} \end{array}$$

EXAMPLE II.

From $29\frac{1}{6}$, take $14\frac{3}{8}$.

$$\begin{array}{rcl} 29\frac{1}{6} & = & 29\frac{4}{24} \\ 14\frac{3}{8} & = & 14\frac{9}{24} \\ \hline & & 14\frac{19}{24} \end{array}$$

EXERCISES.

3. From $\frac{5}{6}$ take $\frac{3}{4}$.
4. From $\frac{2}{3}$ take $\frac{4}{7}$.
5. From $\frac{8}{11}$ take $\frac{3}{7}$.
6. From $3\frac{2}{5}$ take $1\frac{11}{12}$.
7. From 25 take $21\frac{1}{9}$.
8. From $36\frac{3}{11}$ take $16\frac{2}{3}$.
9. From 28 take $27\frac{23}{24}$.
10. From $19\frac{87}{140}$ take $9\frac{121}{375}$.
11. From $\frac{3}{4}$ of $\frac{4}{5}$, take $\frac{1}{8}$ of $\frac{7}{9}$.
12. From $\frac{3}{8}$ of $\frac{11}{12}$, take $\frac{1}{7}$ of $\frac{2}{3}$.
13. From $\frac{1}{8}$ of 4, take $\frac{1}{7}$ of 3.
14. From $\frac{1}{7}$ of $\frac{3}{4}$ of 86, take $\frac{1}{2}$ of $\frac{2}{3}$ of 2.
15. From £$\frac{3}{5}$, take $\frac{2}{9}$ of a shilling.
16. From $\frac{1}{7}$ of a guinea, take £$\frac{3}{16}$.
17. From $\frac{4}{7}$ of a lb. troy, take $3\frac{4}{7}$ dwt.
18. From $\frac{3}{8}$ of a ton, take $\frac{1}{5}$ cwt.

MULTIPLICATION OF VULGAR FRACTIONS.

RULE I.

Reduce mixed numbers to improper fractions, if there be any given; then multiply all the numerators together for the numerator of the product, and all the denominators together for the denominator of the product*.

EXAMPLE I.

Multiply $\frac{3}{8}$ by $\frac{7}{9}$.

$$\begin{array}{l} 3 \times 7 = 21 \\ 8 \times 9 = 72 \end{array} \quad \frac{21}{72} = \frac{7}{24} \text{ product.}$$

EXAMPLE II.

Multiply $9\frac{5}{8}$ by $12\frac{1}{3}$.

$$9\tfrac{5}{8} = \frac{77}{8} \quad \text{and} \quad 12\tfrac{1}{3} = \frac{37}{3}$$

$$\begin{array}{l} 77 \times 37 = 2849 \\ 8 \times 3 = 24 \end{array} \quad \frac{2849}{24} = 118\tfrac{17}{24} \text{ product.}$$

---

* When whole numbers are multiplied together, their value is increased, or the product is greater than either of the factors; but when proper fractions are multiplied together, their product is less than either factor; for fractions that are to be multiplied together, are, virtually, compound fractions; and are, therefore, fractions of fractions, and must, of necessity, be less than the fractions of which they are a part.

Thus, [illegible] multiplied by $\frac{3}{4}$, is the same as [illegible] of $\frac{3}{4}$, or [illegible], by Case 6th, which is, in effect, the same as the Rule.

RULE II.

1. To multiply a mixed number by a whole number, multiply the numerator and then the integer, carrying one to the integer for every time the denominator is contained in the product of the numerator by the multiplier.

2. To multiply a fraction by a whole number, divide the denominator of the fraction, if it can be done, which is the same, in effect, as multiplying the numerator.

EXAMPLE I.

Multiply $9\frac{11}{12}$ by 8.

$$9\tfrac{11}{12} \times 8 = 79\tfrac{1}{3}$$

EXAMPLE II.

Multiply $\frac{15}{32}$ by 8.

$$\frac{15}{32} \div 8 = \frac{15}{4} = \left(\frac{15}{32} \times 8\right) = \frac{120}{32} = 3\tfrac{3}{4}$$

EXERCISES.

3. Multiply $\frac{6}{7}$ by $\frac{3}{4}$.

4. Multiply $\frac{7}{8}$ by $\frac{5}{6}$.

5. Multiply $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$ together.

6. Multiply $\frac{3}{5}$ of $\frac{7}{9}$, by $\frac{2}{3}$ of $\frac{4}{7}$.

7. Multiply $\frac{4}{7}$ of $\frac{5}{6}$, by $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{8}$.

8. Multiply $\frac{11}{24}$ by 8.

9. Multiply $18\frac{3}{7}$ by 12.

10. Multiply $14\frac{3}{8}$ by $\frac{3}{7}$.

11. Multiply $85\frac{5}{8}$ by $9\frac{1}{4}$.

12. What cost $7\frac{3}{5}$ yards of cloth, at £$3\frac{1}{5}$ per yard.?

13. What cost $11\frac{4}{9}$ yards of ditto, at $2\frac{3}{7}$ guineas per yard.?

14. What cost $17\frac{3}{7}$cwt. at £$5\frac{2}{9}$ per cwt.?

15. What cost $4\frac{1}{3}$ oz. troy of gold, at £$3\frac{7}{8}$ per oz.?

DIVISION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers, and compound-fractions, to simple fractions, if there be any given; then invert the terms of the divisor, and multiply the dividend by it, as in Multiplication.

EXAMPLE I.

Divide $\frac{4}{5}$ by $\frac{2}{3}$.

$$\frac{4}{5} \times \frac{3}{2} = \frac{12}{10} = 1\frac{1}{5} \text{ Quotient.}$$

EXAMPLE II.

Divide $6\frac{3}{5}$ by $4\frac{7}{8}$.

$6\frac{3}{5} = \frac{33}{5}$ then, $\dfrac{33}{5} \times \dfrac{8}{39} = \dfrac{264}{195} = 1\frac{23}{65}$.

$4\frac{7}{8} = \frac{39}{8}$

In order to explain the reason of this rule, by the first example, suppose it had been required to divide $\frac{4}{5}$ by 2, or to take the half of that fraction, it is plain the quotient would have been $\frac{4}{10}$, by multiplying the denominator (5) by 2, for fractions are diminished by multiplying their denominators; but $\frac{2}{3}$ is three times less than 2; therefore, the quotient of any number divided by $\frac{2}{3}$ will be 3 times greater than the quotient arising from the same number divided by 2; consequently, it must be multiplied by 3, which is done by multiplying the numerator by 3; and, therefore, the true quotient is $1\frac{1}{5}$.

1. If the divisor and dividend have the same denominator, it is only necessary to divide the *numerator* of the dividend, by the *numerator* of the *divisor*.

2. When the dividend is a whole number, and the divisor a proper fraction, multiply the dividend by the denominator of the divisor, and divide the product by the numerator.

3. When the divisor is a whole number, and the dividend a fraction, multiply the denominator by the divisor, and place the numerator over the product*.

By observing the quotient arising from the division of any number, by a proper fraction, it will be found to be greater than the dividend. The reason of this is, that any number contains any fraction of a number oftener than the number of which it *is a fraction;* and, it is obvious, that the smaller the fraction is, the dividend will contain it the oftener, or the quotient will be the greater.

EXERCISES.

3. Divide $\frac{2}{3}$ by $\frac{8}{9}$.	6. Divide $\frac{9}{11}$ by $\frac{4}{7}$.
4. Divide $\frac{36}{32}$ by $\frac{5}{8}$.	7. Divide 4 by $\frac{8}{9}$.
5. Divide $\frac{5}{6}$ by $\frac{7}{12}$.	8. Divide 9 by $\frac{2}{3}$.

* In the multiplication and division of fractions, by whole numbers, and *vice versa*, 1 may be placed as the numerator or denominator of whole numbers, which will, perhaps, make the method of proceeding, with these numbers, plainer to the student.

9. Divide 6 by $\frac{13}{15}$.

10. Divide $\frac{13}{45}$ by 6.

11. Divide $\frac{11}{12}$ by 5.

12. Divide 97 by $5\frac{4}{7}$.

13. Divide $56\frac{4}{7}$ by $\frac{3}{4}$.

14. Divide $85\frac{9}{10}$ by $29\frac{7}{8}$.

15. Divide $\frac{2}{3}$ of $7\frac{3}{4}$ by $\frac{1}{5}$ of 4.

16. Divide £$1501\frac{2}{3}$, equally, among 54 men.

17. Divide $15\frac{4}{5}$ acres into $9\frac{1}{4}$ equal parts.

18. What number, multiplied by $\frac{4}{5}$, will amount to $17\frac{3}{7}$?

19. Divide £$160\frac{5}{6}$ among D, E, F, and G; so that D, E, and F, may have equal shares, and G $\frac{3}{5}$ of one of their shares.

PROPORTION OF VULGAR FRACTIONS.

RULE.

State the question, as in proportion of whole numbers, and reduce all the terms, if necessary, to simple fractions, and the first and second to the same denomination; then multiply the second and third terms together and divide by the first, as directed in the rules of Multiplication and Division*

* After any question in proportion is stated, the remaining part of the work is only an exercise in multiplication and division; and, as these operations have been fully explained already, it would be superfluous to give a number of examples in this place, as many opportunities will occur of applying fractions in the commercial part of the work.

EXAMPLE. I.

If $\frac{3}{4}$ of a yard cost £$\frac{5}{8}$, what will $\frac{7}{12}$ of a yard cost?

$$\begin{array}{ccccc} \text{yds.} & & \text{yds.} & & £ \\ \frac{3}{4} & : & \frac{7}{12} & :: & \frac{5}{8} \end{array}$$

then, $$\frac{7}{12} \times \frac{5}{8} \times \frac{4}{3} = \frac{140}{288} = £\frac{35}{72} = 9\text{s. } 8\tfrac{2}{3}\text{d.}$$

EXAMPLE II.

If $\frac{5}{8}$ cwt. cost £$4\frac{7}{9}$, what will $4\frac{1}{2}$ lb. cost?

$$\frac{5}{8} \text{ of } \frac{112}{1} = \frac{560}{8} = \overset{\text{lb.}}{70} : \overset{\text{lb.}}{4\tfrac{1}{2}} :: \overset{£}{4\tfrac{7}{9}}$$

then, $$\frac{9}{2} \times \frac{43}{9}$$

or, $$\frac{1}{2} \times \frac{43}{1} \times \frac{1}{70} = £\frac{43}{140} = 6\text{s. } 1\tfrac{1}{2}\text{d.}—\tfrac{6}{7}.$$

EXERCISES.

3. If $\frac{2}{3}$ lb. cost $8\frac{3}{4}$d., how many lbs. may be bought for £25?

4. If $2\frac{2}{5}$ yards cost £$3\frac{3}{4}$, what will $4\frac{4}{5}$ yards cost?

5. If the carriage of $2\frac{1}{2}$ tons of goods, $2\frac{9}{10}$ miles, be $\frac{1}{40}$ guinea, what is that per cwt. for one mile?

6. If $2\frac{1}{5}$ cwt. of sugar cost £$10\frac{1}{8}$, what cost $1\frac{3}{8}$ cwt.?

7. If 7 men cut down a field of grass in $4\frac{7}{8}$ days, how long would 4 men require to do the same?

8. If $\frac{2}{11}$ of a cwt. of sugar cost £$2\frac{3}{8}$, what sum will purchase the remainder of the cwt.?

9. If $\frac{1}{5}$ of $\frac{7}{9}$ of a yard of cloth cost $\frac{3}{4}$ of $\frac{5}{8}$ of a £, what will 180 yards cost?

10. If $3\frac{1}{4}$ cwt. be carried $15\frac{1}{2}$ miles for 4 guineas, how far ought $9\frac{1}{2}$ cwt. be carried for the same money?

Compound proportion being performed exactly like simple proportion, after the terms are stated and prepared, it is unnecessary to give any examples in this place, as a sufficient number and variety will be found in the commercial part of this work.

The Arithmetic of Vulgar Fractions is both tedious and intricate to those who are but little acquainted with the nature of numbers. This arises chiefly from the variety of *denominators* that are employed; for, when numbers are divided into different kinds, it becomes difficult to compare them with each other.—On this account the *Decimal* division of units and integers was introduced, which is much more simple; and calculations by them are often much shorter and easier performed than by Vulgar Fractions.

DECIMAL FRACTIONS.

In Decimal Fractions unity is supposed to be divided into 10 equal parts, or some multiple of 10; as, 100, 1000, 10000, &c.—The denominator of a decimal fraction is, therefore, 10, or some *multiple* of 10.

If the *decimal* consist of *one* figure, it is evident its denominator must be 10, or 1 with a *cipher* attached to it; for the division here employed is the same as in whole numbers, and as numbers increase in a tenfold proportion to the *left* hand, they decrease in the same ratio towards the *right* hand; therefore, the first place, to the *right* of *units*, must be the place of *tenth parts*; the *second*, that of *hundredth*; the *third*, that of *thousandth* parts, &c.; consequently, the denominator of a decimal fraction is 1, with as many ciphers annexed to the right of it as there are figures in the decimal.

The denominator of a decimal being thus always understood, it is not expressed, or put down, the value of the fraction being known by *inspection* of the *decimal.*

The figures of the decimal are *written* exactly as whole numbers, but are *distinguished* by having a *dot*, or point, placed on the left hand, which is called the *decimal point;* and which serves to separate them from whole numbers, when forming part of the same sum. Thus, the decimal expression for $\frac{4}{10}$ is .4; for $\frac{44}{100}$ is .44; for $9\frac{3}{10}$ is 9.3; for $\frac{7}{1000}$ is .007, &c.; in this last expression, two ciphers are prefixed before the numerator, that the decimal may contain as many places as there are ciphers in the denominator. This being the case, it is evident that ciphers on the *right* hand of a decimal neither *increase* nor *diminish* the *value* of the decimal; but, if ciphers be placed on the *left* hand of a *decimal,* it diminishes its value 10 times, for every cipher that is added; for, as already observed, the denominator of any decimal consists of 1, and as many *ciphers* annexed to it as there are figures in the decimal, whether they be ciphers or significant figures. The use of the cipher in decimals, as well as whole numbers, is merely to give significant figures their proper places, on which their value depends.

Decimals being only a continuation of the same scale as that of integers, the first place to the right of the decimal point, as just stated, signifies tenth parts; the second, hundredth parts; and so on, decreasing in a tenfold proportion, towards the right, as in the following table:

	Integers.							Decimals.						
&c.	3	4	8	7	5	2	4	.2	5	7	8	4	3,	&c.
	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units	Tenth parts	Hundred parts	Thousand parts	Ten thousand parts	Hundred thousand parts	Million parts*	

* The figure next the decimal point is sometimes called decimal *primes*, the next decimal *seconds*, the next *thirds*, and so on.

When a vulgar fraction is reduced to a decimal without any remainder, it is called a *finite* or terminate decimal; if the quotient consist of the same figure always returning, it is called a pure repeater or single *repetend;* if two or more figures always return in the same order, they are called a *circulate* or *compound repetend;* if there are figures before those that repeat, the decimal is called a *mixed repetend;* and the figures before those that repeat are called the *finite* part. A *single repetend* is usually marked with a point over it, and a compound repetend with a point over the *first* and *last* figure of it, which is the plan that will be adopted throughout this work, to distinguish these species of decimals from *finite* decimals.

REDUCTION OF DECIMALS.

CASE I.

To reduce a vulgar fraction to a decimal.

Annex as many ciphers to the numerator as may be deemed necessary, then divide it by the denominator, and the quotient is the *decimal;* which must consist of as many figures as there were ciphers added to the numerator; therefore, if there should not be so many, after performing the division, supply the defect by prefixing ciphers to the *left hand* of the quotient.

EXAMPLE I.

Reduce $\frac{5}{8}$ to a decimal.

8)5000

·625 Answer.

EXAMPLE II.

Reduce $\frac{1}{200}$ to a decimal.

$$200)1000$$

$$\cdot005 \text{ decimal.}$$

If the denominator be greater than 11, and the decimal is wanted to a great many places, divide by the denominatar till the remainder consist of a single figure, multiply this remainder by itself, and also the quotient, adding in the carriage from the remainder; place this product on the *right hand* of the quotient already found, instead of the remainder, and if there be still a remainder, repeat this operation till the decimal be deemed sufficiently exact. Each step of this operation will *double* the places in the preceding quotient.

EXAMPLE.

Reduce $\frac{1}{23}$ to a decimal.

$$\tfrac{1}{23} = .043\tfrac{11}{23}$$

$$.043\tfrac{11}{23} \times 11 = .478\tfrac{6}{23}$$

$$\text{therefore,} \qquad \tfrac{1}{23} = .043478\tfrac{6}{23}$$

$$\text{Again,} \qquad .043478\tfrac{6}{23} \times 6 = .260869\tfrac{13}{23}$$

$$\text{and } \tfrac{1}{23} = .043478260869\tfrac{13}{23}.$$

The reason of this method of extending the quotient is so obvious to those acquainted with *vulgar fractions*, that it requires no explanation.

EXERCISES.

3. Reduce $\frac{1}{2}$ to a decimal.
4. Reduce $\frac{3}{4}$ to ditto.
5. Reduce $\frac{1}{4}$ to ditto.
6. Reduce $\frac{7}{8}$ to ditto.
7. Reduce $\frac{9}{13}$ to ditto.
8. Reduce $\frac{1}{400}$ to ditto.

9. Reduce $\frac{16}{27}$ to a decimal.

10. Reduce $\frac{3}{32}$ to ditto.

11. Reduce $\frac{93}{178}$ to ditto.

12. Reduce $\frac{3}{335}$ to a decimal.

13. Reduce $\frac{17}{168}$ to ditto.

14. Reduce $\frac{1}{79}$ to ditto.

CASE II.

To reduce money, weights, and measures, to decimals.

RULE I.

Reduce the different denominations to a vulgar fraction of the integer, by Case 9th of Vulgar Fractions, and this fraction to a decimal by the last case.

EXAMPLE I.

Reduce 7s. 6d. to the decimal of a £.

$$\begin{array}{l} 7s.\ 6d. \\ \quad 2 \end{array}$$

$$20s. \times 2 = \frac{15}{40} = £\frac{3}{8} \qquad \text{then } 8)3.00 \qquad .375 \text{ decimal.}$$

EXAMPLE II.

Reduce 5 cwt. 2 qr. 14 lb. to the decimal of a ton.

$$\begin{array}{lcl} \text{cwt. qr. lb.} & & \\ (5.\ 2.\ 14.) & = & 630\text{lb.} \\ 1 \text{ ton is} & = & 2240\text{lb.} \end{array} = \frac{9}{32} \text{ of a ton.}$$

$$\text{then } 32 \begin{cases} 4)9.00 \\ 8)2.25 \end{cases}$$

$$.28125 \text{ decimal.}$$

RULE II.

Place all the different denominations under each other, the lowest being at the top of the column; on the left of each of these, place that number for a divisor, which would reduce it to the next superior denomination; then divide each of the denominations, together with the quotient arising from the denomination above it, by its proper divisor, and the last quotient will be the decimal required*.

EXAMPLE.

Reduce 12s. $7\frac{1}{2}$d. to the decimal of a £.

$$\begin{array}{r|l} 2 & 1 \\ \hline 12 & 7.5 \\ \hline 20 & 12.625 \\ \hline & £\ .63125 \text{ decimal.} \end{array}$$

EXAMPLE II.

Reduce 3 oz. 5 dwt. 15 gr. to the decimal of a lb. troy.

$$\begin{array}{r|l} 24 & 15. \\ \hline 20 & 5.625 \\ \hline 12 & 3.28125 \\ \hline & \text{lb. } .2734375 \text{ decimal.} \end{array}$$

---

* It often happens, in reducing vulgar fractions, or money, weights, measures, &c. to decimals, that the decimal will consist of *one* figure, repeating, *ad infinitum*; sometimes of *two*, or more figures, returning in the same order, and sometimes of one or more figures, and *then* the remaining part of the decimal will assume one of these forms. Thus, $\frac{1}{3}$, = .3 repeating; $\frac{1}{9}$ = .1 repeating; and 13s 4d. reduced to the decimal of a £. is = .6 repeating. Again, $\frac{10}{11}$ = $\dot{9}\dot{0}$ repeating; and 4d. reduced to the decimal of a £. is = $01\dot{6}$.

As many calculations cannot be performed decimally without introducing repeating decimals, it was deemed necessary to intersperse the rules for managing them with those of *finite* decimals.

For a further account of their nature, as well as of the vulgar fractions that produce the several kinds of decimals, See the end of Decimals.

EXERCISES.

3. Reduce 4d. to the decimal of a £.

4. Reduce 18s. 3d. to the decimal of a £.

5. Reduce 12s. 10¼d. to the decimal of a £.

6. Reduce 14s. 3d. to the decimal of a guinea.

7. Reduce 9 oz. 13 dwt. 6 gr. to the decimal of a lb.

8. Reduce 1 qr. 13 lb. to the decimal of a cwt.

9. Reduce 3 cwt. 17 lb. 9 oz. 15 dr. to the decimal of a cwt.

10. Reduce 1 rood, 30 poles, to the decimal of an acre.

11. Reduce 3 pecks, 5 pints, to the decimal of a quarter.

12. Reduce 121 days, 10 hours, 13 min. 15 sec. to the decimal of a common year.

Shillings, pence, and farthings, may be reduced, mentally, to the decimal of a pound, consisting of three places, thus: put half the number of shilings for the first decimal place, and the number of farthings, in the remainder, will form the other two places, only the number must be increased by 1, if above 24; by 2, if above 48; and by 3, if above 72*.

EXAMPLE.

Reduce 3s. 7½d., mentally, to the decimal of a £.

3s.	= .15		2s.	= .1
7½d.	= .031	or	1s. 7½d.	= .081
3s. 7½d.	= .181			.181

* The reason of this is, that 1 shilling being the 20th part of a £, 2 shillings are the 10th part; and, as the first decimal place signifies 10th parts, the number of shillings will be truly represented by half that number in the first decimal place; and as 960 (the number of farthings in a £) wants $\frac{1}{24}$th part of itself to make it 1000, if 1 be added to every 24 farthings, (which is to be reduced to a decimal) they will then be the 1000th parts of a £; and, consequently, truly represented by making them occupy the place of thousands in the decimal, which is the third place from the point.

EXERCISES.

Reduce, mentally, to the decimal of a £, the following sums:

2.	2s. 9¼d.	6.	2s. 4¾d.
3.	8s. 7¾d.	7.	1s. 11¾d.
4.	5s. 8¼d.	8.	14s. 3d.
5.	18s. 7½d.	9.	11s. 5¼d.

CASE III.

To find the value of a decimal.

RULE.

Multiply the given decimal by the number of parts contained in the next inferior denomination, and point off as many decimal places from the right hand of the product as there are in the multiplicand; then multiply the decimal places, thus pointed off, by the parts in the next inferior denomination, and, from this product, point off as before. Proceed, in this manner, through all the denominations, into which the integer is divided, and the figures on the left of the decimal point, in the different products, will be the value of the decimal.

EXAMPLE I.

Required the value of .228125 of a £.

$$
\begin{array}{r}
.228125 \\
20 \\
\hline
4.562500 \\
12 \\
\hline
6.750000 \\
4 \\
\hline
3.000000 \\
\hline
\end{array}
\quad = 4s.\ 6\tfrac{3}{4}d.\ \text{Answer.}
$$

EXAMPLE II.

Required the value of .47125 of a ton.

$$
\begin{array}{r l}
 & \text{cwt. qr. lb. oz. dr.} \\
.47125 & = 9.\ 1.\ 19.\ 9.\ 9\tfrac{3}{5}\ \text{Ans.} \\
20 & \\
\hline
9.42500 & \\
4 & \\
\hline
1.70000 & \\
28 & \\
\hline
19.60000 & \\
16 & \\
\hline
9.60000 & \\
16 & \\
\hline
9.60000 & \\
\hline
\end{array}
$$

EXERCISES.

3. Required the value of £.875.

4. Required the value of £.275.

5. Required the value of £.0375.

6. Required the value of .625 shilling.

7. Required the value of .3125 guinea.

8. Required the value of .6875 yard.

9. Required the value of .625 cwt.

10. Required the value of .8125 lb. troy.

11. Required the value of .584 quarter.

12. Required the value of .461 chaldron.

13. Required the value of .3375 acre.

14. Required the value of .42857 month.

CASE IV.

When the given decimal is a single repetend, or pure repeater.

RULE.

Proceed as already directed, only carry 1 for every 9, in multiplying the right hand figure; and if there should be a cipher on the right of the multiplier, annex the repeating figure (arising from the multiplication) instead of the cipher.

EXAMPLE.

Required the value of £$.08\dot{3}$

$$
\begin{array}{r l}
.08\dot{3} & = \text{1s. 8d.} \\
20 & \\
\hline
1.666 & \\
12 & \\
\hline
8.000 &
\end{array}
$$

EXERCISES.

2. Required the value of £$.36458\dot{3}$.

3. Required the value of £$.991\dot{6}$.

4. Required the value of £$.958\dot{3}$.

5. Required the value of $.\dot{8}$ guinea.

6. Required the value of $.41\dot{6}$ lb. troy.

7. Required the value of $.41\dot{6}$ ton.

8. Required the value of $.795\dot{5}$ ton.

9. Required the value of $.455569\dot{4}$ day.

CASE V.

When the decimal is a compound repetend.

RULE.

Suppose a few of the first places of the circulate to be annexed to the right hand figure, and, in multiplying, add the carriage arising from these additional figures to the product of the right hand figure of the circulate*.

EXAMPLE.

Required the value of $\dot{7}3\dot{0}$ of a £.

$$
\begin{array}{r}
\dot{7}3\dot{0} \\
20 \\
\hline
14.614 \\
12 \\
\hline
7.375 \\
4 \\
\hline
1.500
\end{array}
\quad = \quad 14s.\ 7\tfrac{1}{4}\text{—}\tfrac{1}{8}
$$

EXERCISES.

2. Required the value of £.$\dot{2}1\dot{6}$.

3. Required the value of £.$\dot{6}\dot{3}$.

4. Required the value of £.$852\dot{2}\dot{7}$.

5. Required the value of .$\dot{4}390\dot{2}$ acre.

6. Required the value of .$32\dot{1}4285\dot{7}$ guinea.

7. Required the value of .$6\dot{8}\dot{1}$ quarter.

---

* Or reduce it to a vulgar fraction, and then multiply by it.

8. Required the value of $.\dot{9}\dot{0}$ mile.

9. Required the value of $.\dot{8}5\dot{1}$ week.

Any decimal of a pound may be valued, mentally, to the nearest farthing, thus: double the first figure for shillings, to which add 1, when the *second* figure amounts to 5; then divide the *second* and *third* figures, when *under* 50, or their excess, if *above* 50, by 4, after deducting 1 for every 25 in the number; the quotient is pence, and the remainder farthings*.

EXERCISES.

1. Value £.95.	5. Value £.795.
2. Value .825.	6. Value .007.
3. Value .513.	7. Value .094.
4. Value .739.	8. Value .578.

CASE VI.

To reduce a decimal to a vulgar fraction.

RULE.

Under the given decimal place 1, with as many ciphers after it as there are figures in the decimal, for its *denominator;* and then reduce the fraction to its lowest terms.

EXAMPLE.

Reduce .625 to a vulgar fraction.

$$.625 = \frac{625}{1000} = \frac{125}{200} = \frac{25}{40} = \frac{5}{8} \text{ Ansr.}$$

---

* This is exactly the converse of the rule for reducing shillings, &c. to the decimal of a £; and the reason of the rule will be understood, if the note at page 74 be carefully considered.

EXERCISES.

2. Reduce .6 to a vulgar fraction.

3. Reduce .75 to a vulgar fraction.

4. Reduce .25 to a vulgar fraction.

5. Reduce .125 to a vulgar fraction.

6. Reduce .875 to a vulgar fraction.

7. Reduce .8125 to a vulgar fraction.

8. Reduce .80375 to a vulgar fraction.

9. Reduce .0025 to a vulgar fraction.

10. Reduce $.472\frac{1}{8}$ to a vulgar fraction.

11. Reduce .008125 to a vulgar fraction.

12. Reduce $.207\frac{1}{7}$ to a vulgar fraction.

CASE VII.

When the decimal is a pure repetend.

RULE.

Under the given repetend place as many *nines* as there are figures in the repetend, for the denominator; then reduce the fraction to its lowest terms.

EXAMPLE.

Reduce $.\dot{6}\dot{3}$ to a vulgar fraction.

$$.\dot{6}\dot{3} = \frac{63}{99} = \frac{7}{11} \text{ Answer.}$$

EXERCISES.

2. Reduce $.\dot{3}$ to a vulgar fraction.

3. Reduce $.\dot{1}4\dot{8}$ to a vulgar fraction.

4. Reduce $.\dot{0}06\dot{3}$ to a vulgar fraction.

5. Reduce $.\dot{1}463\dot{4}$ to a vulgar fraction.

6. Reduce $.\dot{0}\dot{6}$ to a vulgar fraction.

7. Reduce $.\dot{0}0\dot{3}$ to a vulgar fraction.

8. Reduce $.\dot{7}1428\dot{5}$ to a vulgar fraction.

9. Reduce $.\dot{6}1538\dot{4}$ to a vulgar fraction.

CASE VIII.

When the decimal is a mixed repetend.

RULE.

Subtract the finite part from the whole decimal, for the numerator, under which place as many nines as there are places in the *repetend*, and as many ciphers, on the right of these, as there are *finite* places in the decimal, for the denominator; then reduce the fraction to its lowest terms*.

EXAMPLE.

Reduce $.254\dot{6}2\dot{9}$ to a vulgar fraction.

$$\begin{array}{r} .254\dot{6}2\dot{9} \\ 254 \\ \hline \end{array}$$

$$5)\frac{254375}{999000} = 5)\frac{50875}{199800} = 5)\frac{10175}{39960} = 37)\frac{2035}{7992} = \frac{55}{216} \text{ Ansr.}$$

* This and the former case will be better understood, after perusing the observations at page 102.

EXERCISES.

2. Reduce $.0\dot{6}$ to a vulgar fraction.

3. Reduce $.00\dot{6}$ to a vulgar fraction.

4. Reduce $.07\dot{9}5\dot{4}$ to a vulgar fraction.

5. Reduce $.8\dot{3}$ to a vulgar fraction.

6. Reduce $.21\dot{3}$ to a vulgar fraction.

7. Reduce $.208\dot{3}$ to a vulgar fraction.

8. Reduce $.76\dot{2}195\dot{1}$ to a vulgar fraction.

CASE IX.

To make repetends similar and conterminous*.

RULE.

Extend the finite part of each, as far as the longest, and then extend all the circulates to as many places beyond that, as is expressed by the least common multiple of the number of places in each circulate.

EXAMPLE.

Make $14.\dot{3}$, $6.\dot{5}\dot{7}$, $.\dot{1}2\dot{3}$, and $\dot{3}2\dot{1}$, similar and conterminous.

$$
\begin{array}{r}
14.\dot{3}3333\dot{3} \\
6.\dot{5}7575\dot{7} \\
.\dot{1}2312\dot{3} \\
.\dot{3}2132\dot{1}
\end{array}
$$

As the repetends, in this example, all begin at the same distance from the decimal point, it is only necessary to extend each of them to six places, the least multiple of 1, 2, and 3, the number of figures they contain.

* That is, *begin* and *end* at the same distance from the decimal point.

EXERCISES.

2. Make $.41\dot{6}$, $.\dot{6}\dot{3}$, and $.\dot{3}9\dot{6}$, similar and conterminous.

3. Make $.23\dot{1}4\dot{8}$, $.\dot{9}\dot{0}$, and $.1041\dot{6}$, similar and conterminous.

4. Make $.\dot{2}\dot{7}$, $.\dot{0}975\dot{6}$, and $.\dot{6}$, similar and conterminous.

5. Make $.9527\dot{4}390\dot{2}$, $381\dot{0}975\dot{6}$, and $56\dot{8}\dot{1}$, similar and conterminous.

ADDITION OF DECIMALS.

RULE.

Arrange the decimal places under each other, according to their *value,* so that the decimal points may be directly under each other; then add the figures that stand in the same column, as in integers, and place the decimal point, in the *sum, directly* under the *other* points.

EXAMPLE.

Add together 65.24+1.397+563+.0046+1.525+29.076+.25.

```
 65.24
  1.397
563.
   .0046
  1.525
 29.076
   .25
---------
660.4926 Sum.
```

EXERCISES.

2. Add together 2.175 + 21.75 + .0625 + 810 + 51.5 + 400.125 + .00576.

3. Add together 376.25 + 86.125 + 637.4725 + 6.5 + 358.865 + 41.02.

4. Add together 3.5 + 47.25 + 927.01 + 2.0073 + 1.5.

5. Add together .01825 + 17.5 + .00375 + 199.25 + 144 + 14310.0125.

CASE II.

When single repetends are given.

RULE.

1. Extend the repeating figures one place beyond the farthest extended finite places, then add as before; only, in adding the right hand column, carry 1, for every 9 that it contains, to the next column.

2. The repeating figures may be converted into vulgar fractions, by Case VII, and then added, as directed in Addition of Vulgar Fractions*.

EXAMPLE.

Add together $.8\dot{3} + 7.41\dot{6} + 6.25 + 4.3\dot{8} + 8.\dot{6}$.

$$
\begin{array}{r}
.8\dot{3}\dot{3} \\
7.41\dot{6} \\
6.25 \\
4.38\dot{8} \\
8.66\dot{6} \\
\hline
27.54\dot{5}
\end{array}
\text{ Sum.}
$$

* This mode of proceeding is more laborious than by the first part of the rule.

EXERCISES.

2. Add together $45.\dot{3}+4.\dot{6}+148.8\dot{3}+.\dot{6}+4625+.\dot{3}$.

3. Add together $6.\dot{4}+23.0\dot{6}+.\dot{3}+.27545+.8\dot{3}+7.\dot{8}$.

4. Add together $12\frac{1}{3}+3\frac{5}{6}+41\frac{5}{11}+15\frac{1}{8}+7\frac{1}{2}+69\frac{3}{8}+5\frac{5}{24}$ after reducing the fractions to decimals.

5. Add together £6 10s. 8d.+ £1 3s. 6d.+£17 13s. 4d.+ £1 10s.+£3 11s. 5d.+£10 17s. $3\frac{1}{4}$d.+£33 12s. $2\frac{1}{4}$d. after reducing the inferior denominations to the decimal of a £.

6. Add together £7 6s. 8d.+ £3 3s. 4d.+£9 1s. 8d.+ £2 16s. 8d.+ £32 12s. $7\frac{1}{2}$d. decimally.

7. Add together 3 cwt. 1 qr. $9\frac{1}{3}$ lb.+5 cwt. $12\frac{4}{5}$ lb.+3 qrs. $9\frac{1}{3}$ lb.+7 cwt. 3 qrs. 7 lb. decimally.

8. Add together 3 oz. 5 dwt. 8 grs.+9 oz. 16 grs.+10 oz. 3 dwt.+7 oz. 8 dwt. 6 gr. troy, decimally.

CASE III.

When compound repetends are given.

RULE.

Make the circulates similar and conterminous, as directed at page 82, and, in adding the right hand column, include the carriage that would have arisen, had the repetends been extended farther.

EXAMPLE.

Add together $79.875+831.\dot{8}1\dot{3}+915.\dot{7}+16.525+79.\dot{5}\dot{4}+58.\dot{6}+399.\dot{1}\dot{5}+17.111\dot{5}+53.\dot{1}4\dot{8}+932.8125+87.\dot{1}9047\dot{6}$.

$$
\begin{array}{r}
97.875 \\
831.\dot{8}1\dot{3}8\dot{1}38138\dot{8} \\
915.7777\dot{7}77777\dot{7} \\
16.525 \\
79.5454\dot{5}45454\dot{4} \\
58.6666\dot{6}66666\dot{6} \\
399.1515\dot{1}51515\dot{5} \\
17.1115\dot{5}55555\dot{5} \\
53.1481\dot{4}81481\dot{1} \\
932.8125 \\
87.1904\dot{7}61904\dot{4} \\
\hline
3471.6179078494 \\
\hline
\end{array}
$$

EXERCISES.

2. Add together $89\frac{1}{8}+42\frac{3}{7}+56\frac{5}{33}+6\frac{10}{11}$ and $+15\frac{1}{4}$, after the fractions are reduced to decimals.

3. Add together $.\dot{5}+.\dot{2}\dot{5}+17.4\dot{7}+9.\dot{6}5\dot{1}$, and $67.\dot{3}4\dot{5}$.

4. Add together 2 cwt. 3 qrs. 7 lb. + 9 cwt. 2 qrs. 16 lb. + 15 cwt. 1 qr. + 2 qrs. 14 lb. + 3 cwt. 1 qr. 8 lb. + 2 cwt. 16 lb. + 7 cwt. 3 qrs. + 1 qr. 14 lb. + 5 cwt. 2 qrs. 7 lb. + 16 lb.

5. Add together 3 years, 123 days + 2 years, 97 days + 23 years, 283 days + 11 years, 33 days; and 1 year, 212 days, allowing 365 days to the year.

SUBTRACTION OF DECIMALS.

RULE.

Write the decimals under each other, according to the value of their places, as directed in Addition, and subtract as in integers; observing to place the decimal point, in the remainder, directly under the other points.

EXAMPLE I.

From 83.35 subtract 19.625.

```
83.35
19.625
------
63.725 Remainder.
```

EXAMPLE II.

From £2. 12s. subtract £1. 17s. 6d.

```
£2 12  0  =  2.6
  1 17  6  =  1.875
---------     -----
£0 14  6  =  0.725 Remainder.
```

EXERCISES.

3. From .45 subtract .0045.

4. From 12.3456 subtract .78095.

5. From 37.825 subtract .95.

6. From .081545 subtract .0787.

7. From £81 12s. 6d. subtract £37 9s. $1\frac{1}{4}$d.

8. From 3 cwt. 3 qrs. 7 lb. subtract 2 cwt. 3 qrs. 21 lb.

CASE II.

When repetends are given.

RULE.

Extend the repetends, as in Addition, and borrow from 9, at the right hand place, if the subtrahend exceed the minuend in that place*.

EXAMPLE I.

From $21.\dot{4}5\dot{3}$ subtract $13.\dot{7}\dot{2}$.

$$
\begin{array}{r}
21.\dot{4}53453\dot{3} \\
13.\dot{7}27272\dot{2} \\
\hline
7.\dot{7}26180\dot{0}
\end{array}
\text{ Remainder.}
$$

EXAMPLE II.

From 5 cwt. 2 qrs. 25 lb. subtract 3 cwt. 3 qrs. 17 lb.

$$
\begin{array}{lcl}
\text{cwt. qr. lb.} & & \\
5.\ \ 2.\ 25 & = & 5.7232\dot{1}4285\dot{7} \\
3.\ \ 3.\ 17 & = & 3.9017\dot{8}5714\dot{2} \\
\hline
 & & 1.82\dot{1}4285\dot{7} \text{ Remainder.}
\end{array}
$$

* Or convert the repetends into vulgar fractions, and then subtract them.

EXERCISES.

3. From $34.\dot{8}5\dot{1}$ subtract 5.47325.

4. From $39.2\dot{1}7\dot{8}$ subtract $17.\dot{6}\dot{8}$.

5. From 1.3 subtract $1.004\dot{7}$.

6. From $69.41\dot{6}$ subtract 25.375.

7. From $91\frac{3}{56}$ subtract $47\frac{4}{9}$ decimally.

8. From $76\frac{9}{41}$ subtract $29\frac{4}{7}$ decimally.

9. From £91 18s. $1\frac{1}{2}$d. subtract £55 4s. $11\frac{3}{4}$d. decimally.

10. From 9 cwt. 1 qr. 13 lb. subtract 7 cwt. 3 qrs. 12 lb.

MULTIPLICATION OF DECIMALS.

RULE.

Place the right hand figure of the multiplier under the right hand figure of the multiplicand, and multiply, as in whole numbers; then point off as many decimal places, from the right hand of the product, as there are in both *factors*: if there are *fewer* figures in the product than in *both factors*, supply the defect, by adding ciphers to the left hand.

EXAMPLE I.

Multiply 9.625 by 2.75.

```
     2.75
   ------
    48125
   67375
  19250
  -------
  26.46875
  -------
```

EXAMPLE II.

Multiply .125 by .025.

```
   .025
   ----
    625
   250
   ----
 .003125
   ----
```

In Example II. the number of decimal places, in both factors, is *six*, and in the product only *four*; therefore two ciphers must be added to the left hand.

To multiply any decimal by 10, remove the decimal *point* *one* place to the right; to multiply by 100, *two* places to the right; by 1000, *three*; and so on, removing it one place to the right, for every cipher in the multiplier.

EXAMPLE.

Multiply .4175 by 10, by 100, by 1000, and by 10000.

$.4175 \times 10 = 4.175.$
$.4175 \times 100 = 41.75.$
$.4175 \times 1000 = 417.5.$
$.4175 \times 10000 = 4175.$

EXERCISES.

2. Multiply 1.37 by 1.8.
3. ——— .1572 by .12.
4. ——— 12.34567 by 3.53.
5. ——— 4.82 by 3.5847.
6. ——— .0825 by .0625.
7. Multiply .0756 by 5.47.
8. ——— 47.55 by 2.43.
9. ——— 4.3125 by 10.
10. ——— 75.5 by 100.
11. ——— .0755 by 1000.

CASE II.

When the multiplicand is a repetend and the multiplier finite.

RULE.

To the product of the right hand figure of each line, add the carriage which would have been derived from the repetend, if it had been farther extended; and make the repetends of the several products begin and end together, before adding them up.

If the product repeat, its repetend will be similar to that of the multiplicand.

When there are ciphers on the right hand of the multiplier, put down a like number of the repeating figures instead of them.

EXAMPLE I.

$$
\begin{array}{rl}
\text{Multiply} \quad 61.47\dot{6} & \text{by } 42.5 \\
42.5 & \\
\hline
307383 & \\
1229533 & \\
24590666 & \\
\hline
2612.758\dot{3} & \text{Product.}
\end{array}
$$

EXAMPLE II.

$$
\begin{array}{rl}
\text{Multiply} \quad .92\dot{9}3\dot{7} & \text{by } 1500. \\
1500 & \\
\hline
4646896 & \\
9293793 & \\
\hline
1394.\dot{0}6\dot{9} & \\
\hline
\end{array}
$$

EXERCISES.

3. Multiply $5.68\dot{3}$ by 475.

4. Multiply $29.1\dot{6}$ by 26000.

5. Multiply $.4983\dot{8}$ by 12.64

6. Multiply $879.8\dot{3}$ by .75.

7. Multiply $8.\dot{7}6923\dot{0}$ by 786.4.

8. Multiply $365.4\dot{8}\dot{1}$ by .00325.

CASE III.

When the multiplier is a repetend,

RULE.

1. Find the product as in finite multipliers, and place under it the products which would have arisen from the repeating or circulating figures, if extended. Or,

2. Reduce the multiplier to a vulgar fraction; then multiply the multiplicand by the numerator and divide the product by the denominator.

3. When *both* factors repeat, the easiest method is to reduce *one* or *both* factors to vulgar fractions, and then find the product by the rules given for multiplying vulgar fractions; and as the denominator of the fraction (or the divisor) consists of 9's, the quotient may be obtained very easily by rule 4th of Division of Whole Numbers.

EXAMPLE I.

Multiply .958 by $.\dot{8}$

$$\begin{array}{r}
.958 \\
.\dot{8} \\
\hline
7664 \\
7664 \\
7664 \\
7664 \\
7664 \\
\hline
.851\dot{5}^{*}
\end{array}$$

or, $.\dot{8} = \frac{8}{9}$

therefore, $\frac{.958 \times 8}{9} = .851\dot{5}$

* It is evident that if a multiplier be a pure repeater, and extended

EXAMPLE II.

Multiply .735 by $.3\dot{2}\dot{6} = \frac{323}{990}$

$$
\begin{array}{r}
.735 \\
.323 \\
\hline
2205 \\
1470 \\
2205 \\
\hline
99.0)237405 \\
2374.05 \\
23.74 \\
.23 \\
\hline
.239803.03 \text{ Product.}
\end{array}
\qquad
\begin{array}{r}
.326 \\
3 \\
\hline
.323
\end{array}
$$

EXAMPLE III.

Multiply $12.\dot{3}$ by $.\dot{4}5\dot{6}$

$12.\dot{3} = 12\frac{1}{3}$

$$
\begin{array}{r}
12\frac{1}{3} \\
\hline
5477 \\
152 \\
\hline
5.\dot{6}2\dot{9}
\end{array}
$$

EXAMPLE IV.

Multiply $42.\dot{6}2\dot{5}$ by $.4\dot{6}2\dot{9}$

$$
\begin{array}{r}
.4\dot{6}2\dot{9} \\
4 \\
\hline
4625 \\
\hline
9990
\end{array}
\qquad\qquad
\begin{array}{r}
42,\dot{6}2\dot{5} \\
.4\dot{6}2\dot{5} \\
\hline
213128 \\
852512 \\
25575375 \\
170502502 \\
\hline
999)19.7143.518 \\
197143 \\
197 \\
\hline
19.7340859+
\end{array}
$$

---

to any number of places, the product arising from each figure will be the same as the first, and each line of the product will stand one place to the right hand of the former, as in Example I.

The mark + (plus) placed after any number, denotes that it requires something to be added to make it exact; and — (minus) that it requires something subtracted.

EXERCISES.

5. Multiply .12 by $.\dot{0}\dot{3}$
6. Multiply .843 by $.\dot{3}$
7. Multiply .8761 by $.\dot{6}$
8. Multiply $.37.2\dot{3}$ by $.\dot{2}6$
9. Multiply $234.\dot{6}$ by $1.\dot{3}$
10. Multiply $67.\dot{6}$ by $45.\dot{2}\dot{7}$
11. Multiply $.42\dot{8}\dot{1}$ by $.63\dot{8}5\dot{1}$
12. Multiply $23.\dot{1}4\dot{8}$ by $3.\dot{9}024\dot{3}$

CASE IV.

To limit the product to any number of decimal places.

RULE.

Invert the multiplier, and write its unit's place under that decimal place of the multiplicand, which is as many places to the right of the decimal point as are meant to be retained in the product. Then multiply, as in whole numbers, only begin with that figure which stands above the figure multiplied by, neglecting all the figures to the *right*, but adding in the carriage which would have arisen from them, had they been multiplied, and place the right hand figure of the products all under each other.

In order to have the last decimal place of the product exact, carry (to the right hand figure of each line) 1 for all numbers above 5 and under 14; 2 for all above 15 and under 24; 3 for all above 25 and under 34, &c. This will generally give the last place true to the nearest unit*.

---

* Or, instead of carrying in this manner, two figures more may be included than are necessary.

EXAMPLE I.

Multiply 6.827195 by 83.7945, and retain only 2 decimal places in the product.

6.827195		6.827195
5497.38	or thus,	5497.38
54617		5461756
2048		204815
478		47790
61		6144
4		273
		34
572.08		572.08

EXERCISES.

2. Multiply 6.827195 by 83.7945 and retain 2 decimal places.

3. Multiply 163.43785 by .0851795 and retain 3 decimal places.

4. Multiply 2.38645 by 8.2175 and retain 4 decimal places.

5. Multiply 128.678 by 38.24 and retain 1 decimal place.

6. Multiply .01546968 by 11.7145984 and retain 4 decimal places.

7. Multiply .58647473 by .0053948 and retain 8 decimal places.

DIVISION OF DECIMALS.

1. Divide, as in whole numbers, and point off from the right hand of the quotient as many figures for decimals as the decimal places in the dividend exceed those in the divisor; if there be not so many figures in the quotient, supply the defect by adding ciphers to the left hand*.

2. If there be any remainder, after bringing down all the figures in the dividend, or more decimal places in the divisor than there are in the dividend, ciphers may be added to the dividend, if it be finite, or the repeating figures, if it be a repetend, and the quotient carried to any degree of exactness required.

3. To divide by any number that has ciphers annexed, remove the decimal point in the dividend, as many places to the *left hand* as there are ciphers on the right of the divisor.

---

*.1. The number of decimal places in the divisor and quotient must always be equal in number to those in the dividend. Therefore, if there be the same number of decimal places in the divisor as in the dividend, there will be none in the quotient.

2. If there be more in the dividend, the quotient will have as many as the dividend exceeds the divisor

3. If there be more in the divisor than in the dividend, they must be made equal before dividing, and the quotient will then consist entirely of whole numbers.

4. The place of the decimal point may also be settled thus: the first figure of the quotient is of the same value with that figure in the dividend which stands over units, in the first product; or, which is the same, the first figure of the quotient is always at the *same distance* from the decimal point, and on the same side as the figure of the dividend, which stands above the unit place of the first product.

EXAMPLE I.

Divide 44.80515 by 3.45.

```
3.45)44.80515)12.987
     34.5
     ----
      1030
       690
      ----
       3405
       3105
       ----
        3001
        2760
        ----
         2415
         2415
         ----
```

EXAMPLE II.

Divide 78.$\dot{6}$ by 4.735.

```
4.735)78.$\dot{6}$66(16.6138
      4735
      -----
      31316
      28410
      -----
       29066
       28410
       -----
         6566
         4735
         -----
         18316
         14205
         -----
          41116
          37880
          -----
           3236
```

In Example I., there are 5 decimal places in the dividend, and only 2 in the divisor; therefore, the 3 right hand figures of the quotient must be pointed off for decimals.

In Example II., there are 2 more decimal places, in the divisor, than in the dividend; therefore 2 decimal places are added, to make them equal; and, as this is the case, there will be no decimal places in the quotient, when the dividend is exhausted, the first two figures (16) are, therefore, a whole number; but, to have the quotient more exact, other 4 places are added, and, consequently, the quotient will have as many decimal places.

The reason for adding 6's to the dividend is, because this figure is a repetend.

EXERCISES.

3. Divide 17.28 by .12.
4. Divide 1.728 by .12.
5. —— .1728 by .12.
6. —— 1728. by .12.

7. Divide 17.28 by 1.2.
8. —— .1728 by 12.
9. —— 37.25 by 281.5.
10. —— .5 by .00725.
11. —— .21975 by 124.
12. Divide .532$\dot{6}$ by 7.
13. —— .$\dot{5}$13$\dot{6}$ by 2.715.
14. —— .$\dot{2}\dot{7}$ by .4625.
15. —— 987.5 by 10.
16. —— 9.875 by 1000.

CASE II.

When the divisor is a repetend.

RULE.

Convert the divisor into a vulgar fraction, then multiply the given dividend by the denominator, and divide by the numerator of the fraction.

EXAMPLE.

Divide 211.9425 by 12.8$\dot{3}$.

```
12.83̇
 128     211.9425
 ----         900
1155 \  ----------
---- ) 1907.4825 (16.515*.
 900 / 1155
       ----
        7524
        6930
        ----
         5948
         5775
         ----
          1732
          1155
          ----
           5775
           5775
           ----
```

* The divisor, in this method, is converted into a vulgar fraction, by Case I. of Reduction; and in Method II, by Case IV. of Vulgar Fractions.

METHOD II.

$$12.8\dot{3} = 12\tfrac{5}{6} = \frac{77}{6}$$

then,

$$\begin{array}{rr|r}
 & & 211.9425 \\
 & & 6 \\ \hline
\multirow{2}{*}{$77\left\{\begin{array}{r} \\ \\ \end{array}\right.$} & 7 & 1271.6550 \\ \cline{2-3}
 & 11 & 181.6650 \\ \cline{3-3}
 & & 16.515 \\ \hline
\end{array}$$

When the multiplier is 9, 99, 999, &c. the easiest method of finding the product, is by Rule III, of Simple Multiplication.

EXERCISES.

2. Divide 4974 by $\dot{6}$.
3. Divide 315.625 by $11.5\dot{3}$.
4. Divide .6285 by $.\dot{1}4\dot{8}$.
5. Divide $.\dot{3}\dot{7}$ by $.\dot{2}3\dot{5}$.
6. Divide $.\dot{3}$ by $1.\dot{2}$.
7. Divide $234.\dot{6}$ by $1.\dot{3}$.
8. Divide $12.\dot{3}45\dot{6}$ by $.00\dot{8}\dot{1}$.
9. Divide $577.\dot{3}7\dot{5}$ by $23.\dot{8}5\dot{1}$.

CASE III.

To limit the quotient to any number of decimal places*.

RULE.

Find what place the first figure ought to occupy; that is, how many places from the decimal point it ought to stand: then consider how many figures the quotient ought to consist of, in order to have the required number of places, and point off as

* It often happens that there are many places in the divisor, and but few wanted in the quotient: in such cases, this rule is very convenient.

many from the left of the divisor, for a *new* divisor; and, in dividing by this new divisor, instead of annexing a figure to the remainder, omit one on the right hand of the divisor, but observe to take in the carriage arising from it, by the quotient figure multiplied by, as in Contracted Multiplication. Proceed in this manner, dropping a figure of the divisor, at each division, till it be exhausted.

EXAMPLE.

Divide 291.439765 by 27.65138725, retaining only 3 decimal places in the quotient.

```
27.651,38725)291.439765(10.539
              27651
              -----
               1492
               1382
               ----
                110
                 83
                ---
                 27
                 25
                 --
                  2
```

EXERCISES.

2. Divide 27457.55 by 32.1755, retaining only 4 decimal places in the quotient.

3. Divide 15.1275 by 9.813275, retaining only 2 decimal places in the quotient.

4. Divide 89.12543 by 12.34567, retaining only 4 decimal places in the quotient.

5. Divide 357.6543218 by 27.1234567, retaining only 5 decimal places in the quotient.

6. Divide $351.\dot{7}$ by $4125.65397\dot{2}\dot{5}$, retaining only 6 decimal places in the quotient.

It was observed, at page 65, that numbers were diminished when *multiplied* by proper fractions, but increased when *divided* by them. Hence *multiplication*, by *fractions*, has the same effect as *division*, by *integers*; and *division*, by *fractions*, as *multiplication* of *integers*; therefore, if any number be multiplied by $\frac{1}{2}$, or .5, the result will be the same, as when that number is divided by 2. Every integer has a decimal corresponding to it, which may be used in a similar manner: this *decimal* has received the name of the *reciprocal* of the number, and may often be employed with advantage, instead of the number itself, both in performing multiplication and division. (See page 8, Art. 23.)

To find the *reciprocal* of any number, divide 1, with ciphers annexed to it, by that number, and the quotient (after there is no remainder) is the *reciprocal*.

EXAMPLE.

Required the reciprocal of 25.

```
25)100(.04 Reciprocal.
   100
   ---
```

The product of any number, multiplied by .04, will be the same as the quotient of that number divided by 25.; and the quotient of any number, divided by .04, will be the same as the product of that number multiplied by 25.

EXAMPLE.

```
                    3875
                     .04
                  ------
25)3875(155   =   155.00
   25
   ---
   137
   125
   ---
    125
    125
    ---
```

When large numbers are to be multiplied, or divided by large numbers, and frequent use to be made of the *same* multiplier or divisor, it is preferable to use the reciprocal, instead of the number itself.

It is evident, from what has been exhibited of Decimals, that some are never complete, though extended to any number of places; others, that are finite, consist of so many places, that it would be extremely tedious and troublesome to apply them in calculations. In cases where the decimal extends to a great number of places, the three or four first places may be used and the others neglected, which will not materially affect the result, except the integer be very valuable; and, when this is the case, one or two more places may be retained But, for the purposes of business, three or four places are sufficiently exact.

Those Vulgar Fractions which have 2, 5, or any power* of these numbers, for their denominator, produce finite decimals, and the number of places is denoted by the exponent of the power. If the numerator be 1, the decimal, or reciprocal, of any power of 2, is the same power of 5; and the decimal, or reciprocal, of any power of 5, is the same power of 2: thus, the decimal for $\frac{1}{4}=.25=5^2$, and $\frac{1}{25}=.04=2^2$; this arises from the radix of the scale being 10, and the one of these figures being the reciprocal of the other, when the radix is divided by either of them: for $\frac{10}{5}=2$, and $\frac{10}{2}=5$; these are the only figures that will divide 10, (the radix of the decimal scale) without a remainder, and no numbers except these, or their powers, will measure 1, with any number of ciphers annexed; therefore no other denominators can produce *finite decimals.*

2. The denominators 3 and 9, produce *pure* repeaters; and, when the denominator is 9, the decimal consists of the *numerator* repeating. (See pages 29 and 30).

3. When the denominator is a power of 2 or 5, multiplied by 3 or 9, the decimal is a mixed repeater; and the number of finite places is denoted by the exponent of that power of 2 or 5, which is multiplied by 3 or 9. This will plainly appear, if the nume-

---

* Numbers produced by the successive multiplication of any figure, into itself, are called *powers* of that number; and the small figure, usually placed over any number, to show how many times it is multiplied into itself, is called the exponent of the power. (See page 3, Art. 25.)

rator be divided by the component parts of the denominator; for the first division, by 2 or 5, quotes a finite decimal, and the second, by 3 or 9, quotes a pure repeater, after the figures of the dividend, or first, quotient is exhausted. When the numerator is 1, the first quotient is either a power of 5 or 2; and, if the second divisor be 3, the remainder of the second division is the same as the result of that power, when the 3's are cast out.—Thus, if the 3's be cast out $2^2=4$, the result is 1; out of $2^3=8$, the result is 2; out of $2^4=16$, the result is 1; and, in general, the result of the even powers is 1, and of the odd powers 2: it is similar with the powers of 5 If the exponent of the power be *even*, and the division continued, the repeating figure will be $\dot{3}$; if odd, it will be $\dot{6}$. If the second divisor be 9, the repeating figure will be the same as the result of the first quotient, when the 9's are cast out.

4. All denominators, that are multiples of 2 or 5, and their powers, by any other numbers, except 3 and 9, produce mixed circulates; for, if the division be performed by the component parts of the denominator, the first divisor, being a power of 2 or 5, quotes a finite decimal, and the second quotes a circulate, after the figures of the first quotient are exhausted.

5. All denominators, of which 2, 5, and their powers, are not component parts, (except 3 and 9) quote pure circulates. The number of places in the circulate cannot exceed the denominator diminished by 1, but it often consists of much fewer places. If the denominator be any power of 3, the number of places is found, by dividing it by 9.

The form of all decimals depends upon the *denominator* of the vulgar fraction from which they are derived; and, therefore, the decimal remains the same, if the fraction be in its lowest terms, whatever the numerator be.

It would be a useful exercise for the student of this branch of Arithmetic, to find the reciprocal of every number, betwixt 1 and 30, dividing by the component parts of the number, when it can be done; for, by this means, he cannot fail to acquire a knowlege of the nature of decimals, especially if he keep in mind the foregoing observations.

The Arithmetic of Decimals is one of the most important branches of the science of numbers, being equally useful in philosophical and commercial calculations. To enable the student

to have a still clearer view of this subject, the following observations on Arithmetical Scales have been added*:

ARITHMETICAL SCALES.

To assist the mind in forming clear and accurate ideas of numbers, they have been arranged or formed into *classes*.

A certain number of units are conceived to form a class of the lowest kind, and an equal number of these classes, to form another of a higher kind; and superior ranks of classes are formed, in the same manner, as far as occasion requires.

This regular method renders our ideas of numbers exact, and our progress in their right application, in the various uses to which they are applied, comparatively easy.

The number of units, which constitutes a class of the lowest kind, is termed the *radix*, or root of the scale.

There does not seem to be any number *naturally* adapted to this purpose, to the exclusion of others. The number 10 has, however, been universally employed by all nations who have cultivated this science, and it is probably the most convenient for general use.

Additional figures might easily have been invented, so as to have carried the scale to 12 or 15, or any other number; but, by increasing the characters beyond certain limits, and applying to each of them an appropriate sign, we should fall into that very complexity which it is the object of a distribution of numbers, into limited periods, to avoid. On the other hand, though by employing fewer characters, we might render the operations

* It was thought superfluous to give examples of the application of decimals to the solution of questions in Proportion, as a sufficient variety will be found in the commercial part of this work.

of Arithmetic more simple, we should also make them proportionally tedious, because more figures would then be necessary for performing calculations. In our choice of a scale, we must therefore balance these advantages, and select the one which combines dispatch with simplicity.

One scale may likewise claim a preference to another, because its radix* has a greater number of small aliquot parts; this is of great importance in Fractional Arithmetic, as it diminishes the number of interminate radical fractions, or those fractions which cannot be reduced to a denominator that is some power of the radix.

When we have been long accustomed to represent any number by means of a particular sign, we are apt to imagine that there is a natural connection between the *number itself* and the *sign* by which it is expressed; and we find a good deal of difficulty in conceiving how it could be represented by any *other* character. Hence we think that *ten* is naturally represented by 10; twenty by 20; twenty-five by 25; and so on. This is owing partly to the influence of association, and partly to the strict analogy which subsists between the nomenclature and the notation of number. The names, however, by which numbers are expressed, as well as the signs by which they are represented, are altogether arbitrary, and 10 might have been used to denote twelve with perfect propriety, provided ten and eleven had been represented by appropriate characters, and the subsequent denominations of number made to correspond with this *new* arrangement.

Having made these general observations on Arithmetical Scales, I shall proceed to explain the method of transferring numbers from one radix to another.

According to the principles of the Common, or Arabian, Notation, the figures increase in a tenfold ratio, by each removal of one place to the left hand. Consequently, the value of any figure is equal to the product of its simple value and 10 raised to a power, whose index is denoted by the number of places which it occupies from the place of units: thus, $700=7\times10^2=7\times100$, because 7 is two places from the place of units; so also $8956=(8\times10^3)+(9\times10^2)+(5\times10)+6=(8\times1000)+(9\times100)+(5$

* Radical fractions are those whose denominators are some power of the radix of the scale.

$\times 10)+6$. It is easy to generalize this process; for we have only to substitute the radix of the scale for 10, and then find the local value of each figure accordingly. Suppose, for instance, I employ the *quaternary* scale, which has only four characters, viz. 1, 2, 3, 0, and that it were required to find the value of 3210 in terms of the decimal scale.—Since the radix is 4—

$$\begin{aligned} 3000 &= 3\times 4^3 = 192 \\ 200 &= 2\times 4^2 = 32 \\ 10 &= 1\times 4 = 4 \\ & \qquad\qquad\; \overline{228} \end{aligned}$$

By the quaternary scale then, 3210 represents two hundred and twenty-eight. In like manner, 8956, by the *duodecimal* scale, is fifteen thousand, one hundred, and eighty-six, according to the decimal scale.

$$\begin{aligned} \text{For } 8\times 12^3 &= 13824 \\ 9\times 12^2 &= 1296 \\ 5\times 12 &= 60 \\ 6 &= 6 \\ & \quad \overline{15186} \end{aligned}$$

And, in general, to find the value of an expression belonging to any scale whatever, in terms of the decimal scale, we must multiply each figure, respectively, by the radix of the given scale, raised to a power whose index is denoted by the number of places that the figure occupies from the place of units. It is evident, from what has been said, that conciseness of expression depends on the radix of the scale. On the other hand, to reduce numbers, expressed in the decimal form of notation, to any other scale, we must reverse the above operations, and begin with finding a power of the radix of the scale, either equal to the given number, or the next lowest. If the power which we have found be equal to the given number, its index will show how many places the highest figure (which, in this case, would be the character denoting unity) ought to be removed from the place of units; but, if it be less, we must divide the given number by the next lowest power of the radix, and the quotient will give the figure, and the index point out its local value.

If there be any remainder, we must divide it, in like manner, by the next lowest power of the radix; the quotient will give

the figure as before, and the index denote its local value. When, at last, we obtain no remainder, or a remainder less than the radix of the scale, we determine the places of the different quotients according to the indices, giving them a local value by means of the cipher, or a similar character, when it becomes necessary. The remainder, of course, belongs to the class of units.

This will be best illustrated by examples.

Required to express 156489 by the *senary* scale.

1.	2.	3.	4.	5.	6.	7.	
6,	36,	216,	1296,	7776,	46656,	279936,	powers of 6.

The 7th power is greater than the given number, and we therefore divide the given number by 46656, the next lowest power; the quotient is 3, and the remainder 16521, which we divide, in like manner, by the next lowest power of 6, viz. 7776, and the quotient is 2; and so on, till we get a remainder less than the radix of the scale. We then take the different quotients, and give them a local value, according to the indices of the powers by which they were obtained. They then stand thus, 3204253; and this expression, in the senary scale, corresponds to 156489 in the decimal scale. Thus,

```
46656)156489(3
      139968
      ------
   7776)16521(2
        15552
        -----
        216)969(4
            864
            ---
         36)105(2
             72
            ---
           6)33(5
             30
            ---
              3
```

Instead of finding the powers of the radix, as I have done in the above example, it will be more convenient, in practice, to divide the given number, and its successive products, by the ra-

dix, as in division, when the divisor is a composite number.—The last quotient and the remainders, taken in a reverse order, give the numerical expression sought.

By taking the same figures, as in the last example, we obtain the same result.

$$
\begin{array}{r}
6)156489 \\
\hline
6)26081-3 \\
\hline
6)4346-5 \\
\hline
6)724-2 \\
\hline
6)120-4 \\
\hline
6)20-0 \\
\hline
3-2
\end{array}
$$

Therefore 3204253 as before.

I shall take another example.—Required to express 15186, in the *duodecimal* scale.

$$
\begin{array}{r}
12)15186 \\
\hline
12)1265-6 \\
\hline
12)105-5 \\
\hline
8-9
\end{array}
$$

Therefore 8956 is the number.

The radix of this scale being 12, I divide the given number (15186) and the resulting quotients, successively by 12, till I obtain a quotient less than the radix. Then beginning with the last quotient, and taking the remainders in the inverse order in which they were obtained, the whole stands thus; (as above, 8956) which is, therefore, the expression in the duodecimal scale, for 15186 in the decimal.

Having shown the method of transferring numbers, from one scale to another, I shall now endeavour to show the relative advantages of different scales; but, before I enter upon this investigation, it is proper to observe, that, though some particular radix may be the most convenient, on the *whole*, yet each of

them may, in certain cases, be employed with advantage, and the principles of Arithmetic, will be best understood, when all of them can be used with equal facility.

Aristotle appears to have given some attention to this subject, and informs us, that a people of Thrace counted by periods of fours, as the Greeks did by periods of tens. The ancient Arithmetic, however, was so entirely different from the modern, that, comparatively, it was of very little consequence what scale it employed.

If we employed only one character to represent numbers, the operations of Arithmetic would be the simplest possible; but then they would derive no advantage whatever from the *notation*, for they would consist entirely in the gradual increase and decrease of numbers by unity.

Addition would be performed by writing, in succession, the numbers to be added together; Multiplication, by repeating, in like manner, the multiplicand as many times as was indicated by the multiplier; Subtraction, by writing the minuend, and then counting off the figures of the subtrahend; and Division, by dividing the dividend into periods, each of which was equal to the divisor. The simplicity of this species of Arithmetic would be more than counterbalanced by its extreme prolixity; and, while the operations with *large* numbers would always be liable to *error*, the numeration itself would be attended with insuperable difficulties. Accordingly, we find that an Arithmetic of this sort is only used by those who are little acquainted with numbers, and uninstructed in the more improved methods of calculation. The *Binary* Arithmetic makes use of two characters, and, on some occasions, this system may be employed with singular *advantage*.

Table of Numbers, according to the Binary Scale, as far as 41. See next page.

BINARY TABLE.

thirty twos.	six-teens.	eights	fours.	twos.	units.	Decimal Scale.
0	0	0	0	0	0	0
0	0	0	0	0	1	1
0	0	0	0	1	0	2
0	0	0	0	1	1	3
0	0	0	1	0	0	4
0	0	0	1	0	1	5
0	0	0	1	1	0	6
0	0	0	1	1	1	7
0	0	1	0	0	0	8
0	0	1	0	0	1	9
0	0	1	0	1	0	10
0	0	1	0	1	1	11
0	0	1	1	0	0	12
0	0	1	1	0	1	13
0	0	1	1	1	0	14
0	0	1	1	1	1	15
0	1	0	0	0	0	16
0	1	0	0	0	1	17
0	1	0	0	1	0	18
0	1	0	0	1	1	19
0	1	0	1	0	0	20
0	1	0	1	0	1	21
0	1	0	1	1	0	22
0	1	0	1	1	1	23
0	1	1	0	0	0	24
0	1	1	0	0	1	25
0	1	1	0	1	0	26
0	1	1	0	1	1	27
0	1	1	1	0	0	28
0	1	1	1	0	1	29
0	1	1	1	1	0	30
0	1	1	1	1	1	31
1	0	0	0	0	0	32
1	0	0	0	0	1	33
1	0	0	0	1	0	34
1	0	0	0	1	1	35
1	0	0	1	0	0	36
1	0	0	1	0	1	37
1	0	0	1	1	0	38
1	0	0	1	1	1	39
1	0	1	0	0	0	40
1	0	1	0	0	1	41

&c.

The ciphers on the left hand, are prefixed merely for the purpose of completing certain periods, and do not properly belong to the notation.

The great number of figures which are necessary, for expressing even small numbers, shows that this species of Arithmetic would be very unfit for common purposes. Thus, thirty-two is expressed by six characters, 100000. This inconvenience, however, is most observable in small numbers, and diminishes rapidly, as we advance in the numeration; for, while six figures are necessary for expressing thirty-two, only twice six are necessary for expressing a hundred and twenty-eight times thirty-two, or four thousand and ninety-six.

As the *binary* notation exhibits numbers under a very simple form of expression, the operations are abundantly easy, being all reducible to the addition and subtraction of unity; so that they are not more difficult than if we employed but one numerical character, and are much more expeditious.

The notation and numeration are all that are necessary to be committed to memory, and the difficulties they occasion, which are by no means great, are, in some degree, compensated by certain useful properties, which belong *peculiarly* to this scale.

I may mention, in the first place, the ready methods which the notation itself suggests, for the subdivision of weights, measures, &c.; and, by means of which, we are enabled to give several values with a few denominations. Let it be required, for example, to weigh 13: then all we have to do is to take 1101, the corresponding expression by the Binary Scale, and we obtain, at once, the inferior denominations of which it is composed; for $1101 = 1000 + 100 + 1$, that is, eight, four, and one. Another important advantage, resulting from this notation, consists in the easy method which it presents, for discovering the analogies of various classes of numbers. We observe, in each column of the table, as we proceed from right to left, a regular alternation of ciphers and units, increasing in a geometrical ratio of the radix. The square and cube numbers, and the other powers, as well as the triangular, pyramidal, and figurate numbers in general, have similar periods; so that tables of these numbers may be constructed, according to the binary notation, by simple inspection alone. No calculation is necessary; we have only to observe the nature of the returning period in a few examples, and then extend the table according to the particular law which we may have discovered.

M. de Lagny proposed to substitute the Binary Arithmetic for Logarithms, affirming that it was more simple and expeditious, and conducted, to the object in view, in a more direct manner. It was, however, thought that the advantages which he expected in *theory*, would not be realised in *practice*.

It is a most curious circumstance, that the Binary Scale seems to afford the only true explanation of the celebrated lines of *Fohi*, whom the Chinese regard as the founder of their empire, and author of their science: the whole of these lines are reducible to the notation of this Arithmetic. This will appear, from a representation of what is called,

THE FIGURE OF THE EIGHT COVA.

Eight Cova.								
Binary Sc.	000	100	010	110	001	101	011	111
Decimal Sc.	0	1	2	3	4	5	6	7

If we suppose that the broken line corresponds to the cipher, and the entire line to unity, the Cova presents a regular progression of numbers in the Binary Scale from 0 to 7. Father Bouset, who first suggested this explanation and communicated it to Leibnitz, afterwards procured, during his residence in China, the great figure of Fohi, which extends as far as 64. The exact coincidence between the combinations of these lines and the figures of the Binary Notation, left no doubt as to the justness of his conjecture, and it may be observed, that the restitution of the true sense of these characters, after so long an interval of time, is a very singular fact in the history of science.

On the whole, it may be concluded, that the Binary Scale, though possessing many valuable and important properties, is totally unfit for the more *common* purposes of calculation.

Its great defect consists in the *fewness* of the characters it employs, and the consequent prolixity of its operations; a circumstance the more to be regretted, because it constitutes its excellence in other respects. We must therefore regard it, rather as a curious instrument of research, than a useful means of promoting the practical operations of Arithmetic.

The notation of the *Ternary* Scale employs three characters. It circulates, like the binary, in periods.

TERNARY TABLE.

twenty sevens	nines	threes	units	Decimal Scale.
0	0	0	0	0
0	0	0	1	1
0	0	0	2	2
0	0	1	0	3
0	0	1	1	4
0	0	1	2	5
0	0	2	0	6
0	0	2	1	7
0	0	2	2	8
0	1	0	0	9
0	1	0	1	10
0	1	0	2	11
0	1	1	0	12
0	1	1	1	13
0	1	1	2	14
0	1	2	0	15
0	1	2	1	16
0	1	2	2	17
0	2	0	0	18
0	2	0	1	19
0	2	0	2	20
0	2	1	0	21
0	2	1	1	22
0	2	1	2	23
0	2	2	0	24
0	2	2	1	25
0	2	2	2	26
1	0	0	0	27
1	0	0	1	28
1	0	0	2	29
1	0	1	0	30
1	0	1	1	31
1	0	1	2	32
1	0	2	0	33

The circulates, in each column of the table, increase, as before, in the geometrical ratio of the radix. This circulating property of the characters of Notation, when they are placed in their numerical order, is not peculiar to the Binary and Ternary Scales, but belongs to Notation in general. In the higher scales, however, it is less perceptible, on account of the length of the periods.

The *Ternary* Scale is more convenient than the Binary, for the practical operations of Arithmetic; but, as its circulating periods return at longer intervals, it is not so well calculated for detecting the nature of *figurate* numbers and the laws of progressions.

The Quaternary has all the defects of the Binary Scale, without any of its advantages; it, therefore, deserves but little consideration. I may apply the same observation to the Octonary Scale; only the latter is more convenient, in point of *extent*, than any of those I have yet mentioned.

I may class together the *Quintuple*, or *Quirnary*, and the *Septenary* Scales, as they are equally unfit for constituting the bases of an arithmetical system. Their radius, besides, being too small, would render the greater number of radical fractions interminate.

With respect to the latter circumstance, the Senary Scale is less exceptionable than the Quintuple and Septenary Scales; but it is not more convenient, in point of extent, and would render the operations of arithmetic too diffuse.

The Nonary Scale is liable to all the objections which I have stated against the Ternary Notation; for though the operations are performed in a more concise manner, yet the same difficulties present themselves in the fractional arithmetic.

The number *ten* has been adopted, by every civilised nation, for the radix of the Numerical Scale. It has no peculiar advantages, however, to recommend it, and seems to have been selected, for that important purpose, merely because it expresses the number of the human fingers. It is to be regretted, that a circumstance so totally unconnected with every scientific consideration, should have determined an elemental principle, of the last importance, to one of the most abstract, as well as to one of the most useful of all the sciences; and that the Decimal Nota-

tion should still be retained, notwithstanding its evident imperfections, and the superior claims of the other scales.

The number *ten* has only two aliquot parts; (exclusive of unity) and, therefore, it is not so convenient, for the radix of a system, as the number *six;* for though the latter has, likewise, but two aliquot parts, yet, since one of these is a common multiple of both radices, and the powers of the remaining aliquot part of six include a greater proportion of number than those of the remaining aliquot part of ten, it will be found that the Senary Scale also includes a greater proportion of finite radical fractions than the Decimal Scale. Ten, however, is preferable to six, both with regard to conciseness of numerical expression and despatch in calculation, and these properties, in some measure, make up for its defects in the fractional arithmetic.

The Duodecimal Scale combines all the advantages of the *Senary* and *Decimal* Scales; it is no less convenient than the one, with respect to its aliquot parts, and still more so, than the other, with respect to the brevity of its operations. Nor is the number twelve so great, as to render computation, by the Duodecimal Scale, at all difficult; on the contrary, it seems to have been resorted to, in every age, as the most convenient number for the divisions of weights and measures. I have, therefore, no hesitation in giving it a decided preference to the decimal system. The Duodecimal Scale (says an able writer) would nowhere have been found of greater use, than when applied to the circle, the case in which the decimal division is liable to the strongest objections.

The only scale which, in my opinion, can at all be compared with the Duodecimal Scale, is the *Trigesimal;* for the number thirty has the aliquot parts of *ten* and *twelve,* at least the prime aliquot parts; and, in the present inquiry, we must consider only the prime aliquot parts of the radices, since it is the number of these alone that constitutes the value of a particular scale, in the Fractional Arithmetic. Thus, though *four* is an aliquot part of twelve, and not of thirty, the Duodecimal Scale has no advantage, on that account, over the Trigesimal; for, four being a power of two, and two an aliquot part of some power of thirty, four must also be an aliquot part of some power of thirty; and, consequently, those fractions, which have four for their denominator, must be finite radical fractions, by the Trigesimal Scale.

TABLE OF FRACTIONS,

having all the denominators, from 2 to 20, inclusive; exhibiting the relative advantages of the different systems, by a comparison of their number of radical fractions.

Scales.	Radices	FRACTIONS.																				
Binary	Two	($\frac{1}{2}$)	$\frac{1}{3}$	($\frac{1}{4}$)	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	($\frac{1}{8}$)	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	4	$\frac{14}{190}$
Ternary	Three	$\frac{1}{2}$	($\frac{1}{3}$)	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	($\frac{1}{9}$)	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	2	$\frac{8}{180}$
Quaternary	Four	($\frac{1}{2}$)	$\frac{1}{3}$	($\frac{1}{4}$)	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	($\frac{1}{8}$)	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	4	$\frac{14}{120}$
Quinary	Five	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	($\frac{1}{5}$)	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	($\frac{1}{18}$)	$\frac{1}{19}$	$\frac{1}{20}$	1	$\frac{4}{130}$
Senary	Six	($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{4}$)	$\frac{1}{5}$	($\frac{1}{6}$)	$\frac{1}{7}$	($\frac{1}{8}$)	($\frac{1}{9}$)	$\frac{1}{10}$	$\frac{1}{11}$	($\frac{1}{12}$)	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	9	$\frac{36}{99}$
Septenary	Seven	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	($\frac{1}{7}$)	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	1	$\frac{6}{180}$
Octonary	Eight	($\frac{1}{2}$)	$\frac{1}{3}$	($\frac{1}{4}$)	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	($\frac{1}{8}$)	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	4	$\frac{15}{119}$
Nonary	Nine	$\frac{1}{2}$	($\frac{1}{3}$)	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	($\frac{1}{9}$)	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	2	$\frac{8}{186}$
Decimal	Ten	($\frac{1}{2}$)	$\frac{1}{3}$	($\frac{1}{4}$)	($\frac{1}{5}$)	$\frac{1}{6}$	$\frac{1}{7}$	($\frac{1}{8}$)	$\frac{1}{9}$	($\frac{1}{10}$)	$\frac{1}{11}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	($\frac{1}{20}$)	7	$\frac{31}{105}$
Undecimal	Eleven	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	($\frac{1}{11}$)	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	$\frac{1}{16}$	$\frac{1}{17}$	$\frac{1}{18}$	$\frac{1}{19}$	$\frac{1}{20}$	1	$\frac{10}{190}$
Duodecimal	Twelve	($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{4}$)	$\frac{1}{5}$	($\frac{1}{6}$)	$\frac{1}{7}$	($\frac{1}{8}$)	($\frac{1}{9}$)	$\frac{1}{10}$	$\frac{1}{11}$	($\frac{1}{12}$)	$\frac{1}{13}$	$\frac{1}{14}$	$\frac{1}{15}$	($\frac{1}{16}$)	$\frac{1}{17}$	($\frac{1}{18}$)	$\frac{1}{19}$	$\frac{1}{20}$	9	$\frac{34}{99}$
Trigesimal	Thirty	($\frac{1}{2}$)	($\frac{1}{3}$)	($\frac{1}{4}$)	($\frac{1}{5}$)	($\frac{1}{6}$)	$\frac{1}{7}$	($\frac{1}{8}$)	($\frac{1}{9}$)	($\frac{1}{10}$)	$\frac{1}{11}$	($\frac{1}{12}$)	$\frac{1}{13}$	$\frac{1}{14}$	($\frac{1}{15}$)	($\frac{1}{16}$)	$\frac{1}{17}$	($\frac{1}{18}$)	$\frac{1}{19}$	($\frac{1}{20}$)	13	$\frac{66}{88}$

The fractions contained in the parentheses are the fractions that are finite, by the scale, to which they belong, when they are reduced to equivalent fractions, whose denominators are some power of the radix, and the column of integers, on the right hand, expresses their number. The column of fractions, on the right hand, points out the *number* of finite and interminate radical fractions, with every possible numerator, excluding those fractions which occur, under different forms, of the same value; as, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, &c.; $\frac{1}{3}$, $\frac{3}{9}$, $\frac{6}{18}$, &c.; the *numerators* express the finite, and the *denominators* the interminate radical fractions. It is not to be supposed, however, that the column of fractions expresses the real proportions of the finite and interminate radical fractions of each scale; for, in order to obtain these proportions accurately, it would be necessary to extend the denominators of the fractions, in the Table, from two to some number, which is a common multiple of all the radices.

It has been already remarked, that the value of a scale, in the Fractional Arithmetic, depends entirely on the number of its prime aliquot parts; and, therefore, no new scale, after thirty, would give any advantage, till we come to two hundred and ten, the product of the prime numbers, 2, 3, 5, 7. The number two hundred and ten is evidently by far too great for forming the radix of an arithmetical system; and, accordingly, we must restrict our choice to the Duodecimal and Trigesimal Scales. Each has its advantages, but the *Duodecimal* Scale is *preferable*, in this respect, that the transition to it, from the *Decimal* Scale, would be more practicable, because it would be attended with less violence and difficulty. At any rate, if no change should ever take place, it is a fortunate circumstance, that the *Decimal* Scale is the most valuable, after the *Duodecimal* and *Trigesimal* Scales; and that the present system of Arithmetic possesses so nearly the most perfect kind of notation that numbers can admit.

POSITION.

POSITION is a rule by which the *true* answer to a question is discovered by means of *supposed* numbers.

When the answer is found by *one* supposed number, it is called Single Position; and when *two* supposed numbers are employed, it is called Double Position.

SINGLE POSITION.

RULE.

Take any number, and try if it answer the conditions of the question; if it *do*, it is the answer; if not, say, as the result derived from this number is to the true result, stated in the question, so is the number supposed to the answer*.

EXAMPLE.

What number is that, which being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ of itself, the sum will be 155?

Suppose 24
$\frac{1}{2} = 12$
$\frac{1}{3} = 8$
$\frac{3}{4} = 18$

62 : 155 : : 24 : 60. Answer.

EXERCISES.

2. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, had £60 left, what had he at first?

* This rule is founded on the principle, that the results are proportional to the suppositions; which is so obvious, as to require no demonstration.

3. What sum, lent out at 5 per cent., will amount to £850, in 8 years?

4. The joint stock of 2 partners, A and B, was £1250, of which B advanced £160 less than A; required the stock of each?

5. Divide 1142 guineas among A, G, and D, in such a manner, that G may have 106 more than A, and D 58 more than A and G together?

6. A gentleman bought a chaise, horse, and harness, for £60; the price of the horse was double the harness, and the chaise double that of the horse and harness, what was the price of each?

7. Divide 436 guineas among A, B, and C, in such a manner, that B may have 18 more than $\frac{3}{5}$ of A's share, and C 16 more than $\frac{2}{3}$ of B's.

DOUBLE POSITION.

RULE.

Assume any two numbers, and work with each of them, as directed in Single Position, and mark the error of each result with the sign +, if in *excess*, and with —, if an error of defect, then multiply the *first position* by the *second error*, and the *second position* by the *first error*, and divide the *difference* of their *products* by the *difference* of the errors, when the signs are alike; but the *sum* of the products by the *sum* of the errors, when the signs are unlike, which will quote the answer.

EXAMPLE.

A farmer kept a servant for every 60 acres he possessed, and

on taking a lease of 40 acres more, he engaged 2 additional servants; after which, he had a servant for every 50 acres. How many acres and servants had he at first?

	Acres.	Serv.		Acres.	Serv.
Suppose he had	180	3	Suppose again	120	2
	40	2		40	2
	220	5		160	4
		50			50
		250			200
		220			160
	1st Error	30—		2d Error	40—

	Errors.	Products.
1st Position	180 × 40	= 7200
2d Position	120 × 30	= 3600
	10	) 3600
		360 Acres: and, therefore, 6 Servts.

Proof $360 \div 60 = 6$ Serv.
40 2 Serv.
$400 \div 50 = 8$ Servts.

After the errors are found, the answer may be obtained more easily, (on some occasions,) by the following proportion: As the *sum* of the errors, when they are of *different* kinds, or the *difference* of the errors, when they are of the *same* kind, is to the difference of the suppositions, so is the least *error* to a fourth number, which is to be *added* to the supposition by which that error is produced, if the errors be of the *same* kind, and this supposition, *greater* than the other; or, if the errors be of *different* kinds, and this supposition *less* than the other, in every other case it is *subtractive**.

Thus 30 and 40 were the errors in the last Example, which are of the same kind.

* Such questions as form *Simple* Equations, in *Algebra*, can only be performed by Position: but it is, nevertheless, very useful, and often saves much trouble, in approximating to the roots of the higher equations, &c. &c.

40 greater Error				
30 least Error				
—		diff. sup.		least Error
10 diff.	:	60	:	30
		30		
		10)1800		
		180 4th Number		
		180 Supposition		
		360 Acres, as before.		

Here the 4th number is *added* to 180, the supposition, from which 30, the least error, is produced; because the errors are of the *same* kind, and this supposition greater than the other.

EXERCISES.

2. What number is that, which being multiplied by 3, the product increased by 4, and that sum divided by 8, quotes 32?

3. A person had 2 horses and a *saddle*, worth £50; when the saddle was placed on the first horse, it made his value double the second; but, when placed on the second horse, made his value triple the first. Required the value of each horse?

4. A gentleman hired a labourer for 40 days, and agreed to give him 8d. for every day he worked, but he was to return 4d. for every day he was idle: at the end of the 40 days the labourer received 10s. 5d. How many days did he work?

5. A vessel, which could just contain 63 gallons, was filled with wine of two sorts; the one at 8s. per gallon, and the other at 10s. The mixture sold at £28 16s. per hhd. How much was there of each sort?

6. A merchant allows £100 *per annum* for the expenses of his family, and augments, yearly, that part of his stock which remains, by a third part of itself; at the end of 3 years his original stock was doubled. With what sum did he begin trade?

7. Required a number, consisting of 2 digits, which is equal to 4 times the sum of its digits; and, if to that number 27 were added, the digits would be *inverted*?

INVOLUTION.

Involution is the operation of raising powers, and is formed by successive multiplication. Any number, multiplied into itself, produces the square or second power of that number, and that product, multiplied into the original number, produces the cube or third power of that number, &c. See page 2, Art. 13 and 14. The number given to be involved is called the *root* or *first* power. The number expressing the power is called the *index* or *exponent* of that power. See page 3, Art. 25 and the note at page 102.

TO INVOLVE ANY NUMBER TO ANY POWER.

RULE.

Multiply the number as many times into itself as is denoted by the exponent of the power.

EXAMPLE.

Required to involve 4 to the 4th power*.

$$4\times4\times4\times4=4^4=256$$

EXERCISES.

2. Involve 9 to the 3d power.
3. Involve 234 to the square.

---

* The fourth power is called the biquadratic; the fifth power, the sursolid power, &c., but these names are obsolete.

4. Involve 54 to the 3d power.

5. Involve 35 to the 4th power.

6. Involve 55 to the 3d power.

TO INVOLVE A SIMPLE FRACTION TO ANY POWER.

RULE.

Involve the numerator and denominator separately to the proposed power, and the results will be the respective terms of the fraction raised to the power required*.

EXAMPLE.

Involve $\frac{2}{3}$ to the 3d power.

Numerator	$2\times2\times2=8$	third power.
Denominator	$3\times3\times3=\overline{27}$	

EXERCISES.

2. Involve $\frac{4}{5}$ to the second power.
3. Involve $\frac{3}{4}$ to the third power.
4. Involve $\frac{3}{5}$ to the fourth power.
5. Involve $3\frac{1}{5}$ to the third power.
6. Involve $7\frac{1}{3}$ to the fourth power.

---

* Decimals are involved like whole numbers, and mixed numbers may be reduced to improper fractions, and then involved as directed in the rule.

EVOLUTION.

EVOLUTION is the reverse of Involution, and is that operation by which any proposed root of any given number is discovered.

Although every number may be *involved* or raised to any power, yet there are many numbers of which the first power, or exact root, cannot be found.

The roots which are exactly found, are termed *rational* roots; and those which cannot be accurately found, are termed irrational, or *surd* roots

Roots are often denoted by placing the character $\surd$ before the power, with the index or exponent of the root over it; but *now*, more frequently, by placing the index of the required root above the number whose root is required, in the form of a vulgar fraction: the numerator denotes the power the given number is to be *raised* to, and the denominator the *root* required. Thus $\surd 12$, $\sqrt[2]{12}$, or $\overline{12}|^{\frac{1}{2}}$ denotes the square root of 12; $\sqrt[3]{2+6}$, or $\overline{2+6}|^{\frac{1}{3}}$ the cube root of $2+6$; and $\sqrt[\frac{m}{n}]{24}$ or $24^{\frac{m}{n}}$ that 24 is raised to the *m*th power, and then the *n*th root extracted.

TO EXTRACT THE SQUARE ROOT.

RULE.

Point off the given number into periods of two figures each, beginning at the place of units, and pointing off to the *left hand* in integers, and to the *right* in decimals.

Then find a number whose square is either equal to, or the next less than, the figure or figures in the left hand period: place this figure in the quotient, as in Division, and place its square *under*, and subtract it *from*, the above-mentioned period, and to the remainder annex the next period, for a dividend.

Double the part of the root already found, and place it on the left of the dividend for a *trial* divisor, find how often it is contained in the dividend, excluding the right hand figure, and put the figure thus found in the quotient, as the next figure of the root, and also on the right of the divisor; then multiply the divisor (thus increased) by the figure last put in the quotient, subtract the product from the dividend, and to the remainder, bring down the next period for a new dividend.

Find the next figure of the root, as before, by doubling that part of the root already found; or, what is the same thing, doubling the last figure that was put into the divisor, and find how often the trial divisor, thus formed, is contained in the dividend. Proceed in this manner till all the periods be used. If there be a remainder after the periods are brought down, periods of *ciphers* may be added, and the root obtained, in decimals, to any degree of exactness.

EXAMPLE.

What is the square root of 28742.25?

	28742.25(169.53539 Root.
	1
26	187
6	156
329	3142
9	2961
3385	18125
5	25169
33903	120000
3	101709
339065	1829100
5	1695325
3390703	13377500
3	10172100
33907069	320539100
9	305163621
33907078	15375479

The roots of *vulgar* fractions are found by extracting the root of the numerator for a *new* numerator, and the root of the denominator for a new denominator; thus the square root of $\frac{16}{36} = \frac{4}{6} = \frac{2}{3}$: when this cannot be done exactly, it may be reduced to a decimal, and then the root extracted.

The root of a *decimal* is extracted in the same manner as that of a whole number. If the decimal repeats or circulates, periods of the repeating or circulating figures must be annexed to the remainder.

To obtain the square root of a mixed number, it may be first reduced to an improper fraction, or a decimal, and then its root extracted*.

EXAMPLE.

What is the square root of $2\frac{1}{4}$?

$$\sqrt{2\tfrac{1}{4}} = \sqrt{\tfrac{9}{4}} = \tfrac{3}{2} = 1\tfrac{1}{2} \text{ or } \sqrt{2.25} = 1.5 = 1\tfrac{1}{2}\dagger$$

The number of figures in the *root* of any number is always equal to the number of *periods* into which the number can be divided: consequently the number of figures in the root is known as soon as the number is pointed off into periods.

This arises from the nature of multiplication; for *one* figure multiplied by another, can never produce more than two figures, and as *two* is the greatest number of figures contained in a period of which the *square* root is wanted, it is evident the root can never consist of more figures than there are *periods* in the number of which the root is to be extracted; and there cannot be *fewer*, for every *single* figure has *one* figure for its root; be-

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* The roots of numbers are most *easily* discovered by Logarithms. See page 145.

N B. After the root has been found to 5 or 6 figures, two or three more may be found by common division.

† There are some numbers whose square root can never be determined exactly, as 2, 3, 5, 6, 7, 8, 10, &c. and others whose cube root cannot be obtained, as 2, 3, 4, 5, 6, 7, 9, &c.

cause, when a single figure is multiplied by another single figure, (or by *itself*, which produces its *square*) it either produces one figure or *two*, but never more*.

EXERCISES.

2. What is the square root of 611524?
3. What is the square root of 5499025?
4. What is the square root of 451584?
5. What is the square root of 1227756;
6. What is the square root of 2?
7. What is the square root of 157.375?
8. What is the square root of .001225?
9. What is the square root of 412?
10. What is the square root of $\frac{3}{4}$?
11. What is the square root of $2\frac{1}{3}$?
12. What is the square root of $9\frac{43}{48}$?

MISCELLANEOUS EXERCISES.

1. The side of a square garden is 63 yards; how many square yards does it contain?

2. What will the inclosing of a square field, containing 2209 square yards, cost, at 3s. 6d. per yard?

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* The same holds good with respect to every *other* root as well as the *square*, for a similar reason; for any single figure, multiplied twice by a single figure (or by *itself*, which produces its *cube*) can only produce *one*, *two*, or *three* figures, but never *more*. Now the number of figures that can be in a period, of which the cube root is required, can never exceed three; therefore there will still be one figure in the root for every period, and no more.

As the demonstration of the rule itself can only be understood by those who have made some progress in Algebra, it was thought superfluous to insert it in this work.

3. Required a mean proportional between 36 and 2401*?

4. Required the side of a square, equal in area to a circle containing 3 English acres?

5. A cable, which is 3 feet long, and 9 inches in compass, weighs 22 lb.; what will a fathom of that cable weigh, which measures a foot in circumference?

6. The diameter of a standard bushel is $18\frac{1}{2}$ inches, and its depth 8 inches, what must the diameter of that bushel be, whose depth is $7\frac{1}{4}$ inches?

7. If 20 feet of iron railing weigh half a ton, when the bars are $1\frac{1}{4}$ inch square, what will 50 feet come to, at $3\frac{1}{4}$d. per lb., when the bars are $\frac{7}{8}$ inch square?

8. A pipe of 6 inches bore will be 3 hours in running off a certain quantity of water; in what time will 4 pipes, each 3 inches bore, be in discharging double the quantity?

9. There are two circular ponds in a gentleman's pleasure ground, the diameter of the less is 30 yards, and the other is twice as large. What is its diameter?

10. Glasgow is 44 miles west from Edinburgh; Peebles is exactly south from Edinburgh, and 49 miles in a straight line from Glasgow. What is the distance between Edinburgh and Peebles?

11. A line stretched from the top of a spire, to a point 50 feet from its bottom, measured 50 yards. Required the height of the spire?

---

* The geometrical mean, commonly called a mean proportional, is the square root of the product of the two extremes.

2. The square root of the area of any surface, is the side of a square, equal in content to that surface.

3. The square of the hypothenuse of a right angled triangle is equal to the sum of the squares of the two sides.

4. Similar surfaces are to each other as the squares of their like sides.

5. Circles are to one another as the squares of their diameters, or circumferences.

TO EXTRACT THE CUBE ROOT.

RULE.

Divide the given number into periods of three figures each, beginning with units, and pointing to the left in integers, and to the right in decimals. Then find a number whose cube is either equal to, or the next less than, the first, or left hand period, which place in the quotient, and subtract its cube from the said period, annexing to the remainder the next period for a dividend; to which find a divisor as follows:

Multiply the square of the quotient by 3, and to the product annex 2 ciphers for the first and greatest part of the divisor, find how often it is contained in the dividend, and put the answer in the quotient; then multiply the former quotient by this new figure, and by 3, annexing a cipher to the product for the second part of the divisor, which place under the first part.

Again, square the last figure put in the quotient for the third and last part of the divisor, which place under the other parts. Multiply the sum of these three parts by the figure last put in the quotient, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend, to which find a divisor, by multiplying the square of the whole root alread found by 3, and annexing two ciphers to the product, &c. as before; from these find the next figure of the root, and so on. If there be any remainder, after all the periods are used, the work may be continued by annexing periods of ciphers.

If any divisor, when multiplied by the last quotient figure, exceed the dividend, diminish it by one, at each trial, till it answer.

EXAMPLE.

What is the cube root of 1141166.125?

See the work, next page.

$$
\begin{array}{lrlr}
 & & & \dot{1}141\dot{1}66.12\dot{5}(104.5 \text{ Ans.} \\
 & & & 1 \\
\hline
1^2 \times 3 & = 30000 & & 141166 \\
1 \times 4 \times 3 & = 1200 & & \\
4^2 & = 16 & & \\
\hline
 & 31216 & \times 4 = & 124864 \\
\hline
104^2 \times 3 & = 3244800 & & 16302125 \\
104 \times 5 \times 3 & = 15600 & & \\
5^2 & = 25 & & \\
\hline
 & 3260425 & \times 5 = & 16302125
\end{array}
$$

TO EXTRACT THE CUBE ROOT OF A VULGAR FRACTION.

RULE.

Extract the root of its numerator for a *new* numerator, and the root of its denominator for a new denominator; if this cannot be done exactly, reduce the fraction to a *decimal*, and then extract the root.

In extracting the cube root of a decimal, care must be taken that the decimal places be *three*, or some multiple of three, before the operation is *begun*; because there are three times as many *decimal places* in the *cube*, as there are in the *root*. **See note, page 127.**

EXERCISES.

2. What is the cube root of 373248?
3. What is the cube root of 54872?
4. What is the cube root of 970299?
5. What is the cube root of 64964808?
6. What is the cube root of 2?
7. What is the cube root of 9?
8. What is the cube root of 67?
9. What is the cube root of 500?

10. What is the cube root of .0278180875 ?

11. What is the cube root of 35764.3715 ?

12. What is the cube root of $\frac{5}{9}$?

13. What is the cube root of $\frac{3}{8}$?

14. What is the cube root of $\frac{128}{378}$?

15. What is the cube root of 30[illegible] ?

16. What is the cube root of $13\frac{3}{7}$?

MISCELLANEOUS EXERCISES.

1. Required the side of a cubical box, which will contain 2774 cubical inches of brandy ?

2. Required two mean proportionals between 2 and 54*?

3. Required two mean proportionals between 6 and 1296 ?

4. There is a stone, of a cubic form, containing 21952 solid feet; what is the area of one of its sides ?

5. An iron ball of 4 inches diameter weighs 9lbs. what is the weight of an iron ball of 8 inches diameter ?

6. What is the diameter of a 42 lb. iron ball ?

7. The side of a cubical altar being 1 cubit; what is the side of one of the same form of three times that size ?

8. If 20 grains of gold gild a wooden ball which weighs 512

* Two mean proportionals between two extremes may be found thus: multiply each extreme by the square of the other, and then extract the cube root of each product for the mean proportionals sought.

2. Similar solids are to one another as the cubes of their like linear sides.

3. Spheres are to one another as the cubes of their diameters; and their surfaces as the squares of their diameters.

4. The squares of the periodic times of the planets are to each other as the cubes of their mean distances from the sun.

ounces, how many grains will gild a ball of the same kind that weighs 1331 ounces?

9. The solid content of a globe is 15625 cubic inches; required the side of a cube of equal solidity?

10. The length of a ship's keel is 72 feet, the breadth of the midship beam 25 feet, and the depth of the hold 13 feet; required the dimensions of two other ships of the same form, the one to carry twice as much, and the other only half as much?

11. If a ship of 250 tons be 72 feet long in the keel, required the tonnage of another ship of the same form, whose keel is 81 feet long?

12. The proportion between Jupiter's mean distance from the sun and the earth's is, according to Dr. Maskelyne, as 5.20279 to 1; and a tropical year is 365 d. 5 h. 48 m. 48 sec. How many days, &c. are there in Jupiter's year?

13. The Georgium Sidus being 19.08352 times more remote from the sun than the earth is; required the year of the Georgium Sidus?

14. If the earth be 95 millions of miles from the sun, how many miles are the planets Ceres and Saturn from the sun, their years being 1681 days, 12 hours, 9 min. and 10746 days, 19 hours, 16 min. 15 sec. respectively?

15. The first of Jupiter's Satellites is at the distance of $2\frac{5}{6}$ diameters of Jupiter from his centre, and revolves around that centre in 42 hrs. 27 m. 34 sec., and the fourth of Jupiter's Satellites revolves in 16 days, 16 hrs. 32 min. 9 sec. required the distance of the outermost Satellite from the centre of Jupiter, in diameters of Jupiter, and in English miles. Jupiter's mean diameter being 89170 English miles?

The operations, both in the square and cube root, may be proved various ways.

1. By involving the root to the given power, and adding in the remainder, if any; then, if the work be right, the result will be equal to the given power.

2. By casting out the nines, as follows; cast the nines out of the root, and multiply the remainder by itself; cast the nines

out of the product, reserving the *excess;* cast the nines also out of the remainder, subtract the excess from the dividend, then cast the nines out of what remains: if this excess be equal to the *former*, it may be presumed the work is right.

3. By adding the remainder, and all the lower lines, as directed for proving division. See page 28, Art. 4.

DUODECIMALS.

DUODECIMALS have received their name from the division of unity into 12 equal parts.

By this species of arithmetic, calculations are performed as if the arithmetical scale were regulated by 12, instead of 10, and characters for 10 and 11 added to the number of the digits.

Duodecimals, or *cross multiplication*, is a method of finding the content of any rectangular surface, the length and breadth being given in feet inches, and duodecimal parts. It is a rule used by workmen and artificers in calculating the content of their work.

As several kinds of artificers work are computed by different measures, and this subject more properly belonging to practical mathematics, than arithmetic, it will be sufficient, in this place, barely to state the general rule for computing the content of a rectangle, and give one or two examples.

RULE.

Multiply each denomination of the length, by the *feet* in breadth, beginning at the lowest place, and setting each product under that denomination of the multiplicand from which it arises, observing to carry 1, for every 12, to the next higher place.

Multiply, in the same manner, by the *inches* in breadth, if

any, setting each product one place to the right hand, and always carrying by 12 when the product exceeds that number.

Multiply, in a similar manner, by the next lower parts, if any, setting down each product one place farther to the right, than those of the next higher, and so on; the sum of the different products is the answer*.

When the number of feet in the multiplicand is great, multiply by the feet of the multiplier, and then perform the rest of the work, by taking parts of the multiplicand for the other parts of the multiplier: or questions of this kind may be often more easily performed, decimally.

If the measure be required in yards, or any other denomination greater than feet, we may first find it in feet and then bring it to the denomination required.

EXAMPLE I.

Multiply 10 ft. 4 in. 5″, by 7 ft. 8 in. 6″.

ft.	in.	″		
10	4	5		
7	8	6		
72	6	11		
6	10	11	4	
	5	2	2	6
79 . 11′	0″	6‴	6⁗	

or thus,

		ft.	in.	″		
		10	4	5		
		7	8	6		
4	$\frac{1}{3}$	70				
4	$\frac{1}{3}$	3	5	5	8	
6	$\frac{1}{6}$	3	5	5	8	
4	$\frac{1}{3}$	0	5	2	2	6
4	$\frac{1}{12}$	2	4			
1	$\frac{1}{4}$		2	4		
				7		
		ft. 79	11′	0″	6‴	6⁗

* Inches are sometimes called *primes*; the next lower denomination, parts or *seconds*; the next, *thirds*, &c; and marked thus: 4 ft. 7′ 9″ 8‴.

N.B. Feet, multiplied by feet, produce *feet*.
Feet, multiplied into inches, produce *inches*.
Feet, multiplied into seconds, produce *seconds*, &c.

Inches, multiplied into inches, produce *seconds*.
Inches, multiplied into seconds, produce *thirds*, &c.

Seconds, multiplied into seconds, produce *fourths*.
Seconds, multiplied into thirds, produce *fifths*, &c.

EXAMPLE II.

Multiply 9 ft. 6 in. by 3 ft. 9 in.

ft. in.		ft. in.	
9 . 6	or thus,	3 . 9 =	3.75
3 . 9		9 . 6 =	9.5
28 . 6			1875
7 . 1 . 6			3375
ft. 35 . 7′ . 6″		ft.	35.625
			12
		in.	7′.500
			12
			6″.000

The solid content of any rectangular, solid or parallelopiped, may be obtained by multiplying the length, breadth, and thickness, all into each other.

EXAMPLE.

Required the solid content of a wall, 13 feet, 6 inches long, 5 feet, 8 inches high, and 2 feet, 6 inches broad?

13 . 6	or decimally,		13.5 length
5 . 8			$5\frac{2}{3}$
67 . 6			675
9 . 0		$\frac{2}{3}$ =	90
76 . 6			76.5
2 . 6			2.5
153 . 0			3825
38 . 3			1530
ft. 191 . 3		ft.	191.25
			12
		in.	3.00

EXERCISES.

2. Multiply 5 ft. 8 in. by 2 ft. 5 in.

3. Multiply 3 ft. 6 in. by 7 ft. 9 in.

4. Multiply 14 ft. 9 in. by 5 ft. 6 in.

5. Multiply 13 ft. 7 in. by 9 ft. 11 in.

6. Multiply 25 ft. 3 in. by 10 ft. 10 in.

7. What is the measure of a court, 12 yards, 1 foot long, and 7 yards, 2 feet broad?

8. What is the measure of a carpet, 7 yards, 1 foot, 4 inches long, and 5 yards, 2 feet, 3 inches broad?

9. What is the measure of a floor 13 yards, 2 feet, 9 inches long, and 5 yards, 1 foot, 7 inches broad?

10. What is the solid content of a rampart, 926 feet long, 40 feet broad, and 23 feet high?

11. What is the solid content of a block of marble, 4 feet, 3 inches long, 2 feet, 6 inches broad, and 2 feet, 1 inch, 5 pts. high?

12. The canal, which joins the Forth and Clyde, is 27 miles long, 36 feet broad, at a mean rate, and 7 feet deep; required the number of cubical yards of excavation* ?

LOGARITHMS.

LOGARITHMS are the indices or exponents of a series of numbers in geometrical progression.

---

* 27 solid feet=1 cubical, or solid yard. See table of solid measure.

Thus, $\left\{\begin{array}{lllllll} 0. & 1. & 2. & 3. & 4. & 5. & \text{ind. or log.} \\ 1, & 10, & 100, & 1000, & 10000, & 100000, & \text{geo. pro.} \end{array}\right.$

These are the logarithms in common use, and are called *common logarithms*, in order to distinguish them from other kinds of logarithms*; as hyperbolical logarithms, &c.

Hence, in this kind of logarithms, the logarithm of 10 is 1; the log. of 100 is 2; the log. of 1000 is 3, &c. Therefore, the log. of any number between 1 and 10, being greater than 0, and less than 1, must be a fraction of 1, which is expressed decimally.

The logarithm of all numbers between 10 and 100, is greater than 1 and less than 2, hence the log. of any number betwixt 10 and 100 is 1, and a decimal fraction annexed; the log. of all numbers betwixt 100 and 1000 is 2, with a decimal fraction annexed, and so on.

The integral part of the logarithm is called the *index* or *characteristic*, and the other the *decimal* part. It is evident, from the above progression, that the *index* is one *less* than the number of figures in the integer number, of which it is the logarithm.

The index of the logarithm of a proper fraction is *negative*, and is usually marked thus (—); if there be no ciphers on the right of the decimal point, the index is —1; if there be a cipher in the place of tenth parts, the index is —2; and so on.

Instead of negative indices, their arithmetical complements are often used; because, by this means, the computations are rendered easier, especially to those unacquainted with the first principles of Algebra†.

The decimal parts of the logarithms of numbers, which consist of the same figures, are the same, whether the number be *integral*, *mixed*, or *fractional*. This may be illustrated as follows:

* As the limits of this volume will not permit of treating this subject fully, those who wish for farther information on the subject, may consult the introduction to Sherwin's or Hutton's Logarithms.

† The arithmetical complement of any logarithm is found by subtracting the given logarithm from 10; and the arithmetical complement of an *index* by subtracting it from 19.

Numbers.	Logarithms.
189600	5.2778
18960	4.2778
1896	3.2778
189.6	2.2778
18.96	1.2778
1.896	0.2778
.1896	—1.2778
.01896	—2.2778
.001896	—3.2778
.0001896	—4.2778
.00001896	—5.2778

The logarithm of any number under 100,000 may be found from the table of logarithms annexed to this work*.

TO FIND THE LOGARITHM OF ANY GIVEN NUMBER.

RULE.

1. If the number be under 100, its logarithm is found in the first page of the table, directly opposite to it, in the column immediately on the right of that containing the number†.

2. If the number consist of three figures, find it in the first or left hand column of the following pages of the table, opposite to which, in the column immediately on the right of it, and marked 0 at the top, is its logarithm.

3. If the given number consist of four figures, the three first are to be found in the left hand column, as already directed, and under the fourth at the top of the table, is the logarithm required, to which 3 is to be prefixed for its index, if the given number be an integer.

* The logarithms in the table consist of *six* places, which is sufficiently accurate for most purposes in arithmetic or practical mathematics.

† In the first page of the table, the columns containing numbers are marked Num., and those containing their logarithms are marked Log. In the other pages of the table, the left hand column of each page contains numbers and the other columns their logarithms.

4. If the given number exceed four figures, find the logarithm of the *first four* figures as before, to which prefix the index according to the number of figures in the given number; and then take the difference between the logarithm answering to the first four figures, and the next greater in the table*: multiply this difference by the remaining figure or figures in the given number, and strike off as many figures to the right hand as there are in the multiplier, and the remaining part of the product, added to the logarithm, answering to the first four figures, will be the required logarithm, nearly†.

EXAMPLE I.

Required the logarithm of 24?

Num.		Log.
24	=	1.380211

The number in this example being under 100, it is found in the first page of the table, opposite to which is its logarithm.

	Num.		Log.
Example 2.	364	=	2.561101
3.	795	=	2.900367
4.	4977	=	3.696968
5.	6748	=	3.829175

EXAMPLE VI.

Required the logarithm of 25768?

The logarithm of 25760 is 4.410946
Diff. log. 169 × 8 is 135,2

The logarithm of 25768 is 4.411081

* To facilitate this operation, the differences are usually placed opposite to the logarithms from which they are obtained, in the last column of each right hand page, which is marked Diff. as in the table annexed to this work.

† This mode of proceeding is founded on the supposition that the differences are alike between the logarithms of the intermediate numbers, which is not true; but the logarithm of a number consisting of 6 or 7 places, may be obtained sufficiently accurate, in this manner, for any practical purpose, especially if the table extend to 7 places.

In a similar manner, the logarithm of

	498797	is found to be	5.697923
and of	5678979	————	6.754270

TO FIND THE LOGARITHM OF A PROPER FRACTION.

RULE.

Subtract the logarithm of the denominator from the logarithm of the numerator, the remainder is the logarithm of the fraction.

EXAMPLE.

Required the logarithm of $\frac{9}{11}$?

Logarithm of 9 0.954242
Logarithm of 11 1.041393

Logarithm of $\frac{9}{11}$ = 9.912849 or —1.912849

TO FIND THE LOGARITHM OF A MIXED NUMBER.

RULE.

Reduce the mixed number to an improper fraction, and subtract the logarithm of the denominator from the logarithm of the numerator, the remainder is the logarithm required.

EXAMPLE.

Required the logarithm of $2\frac{1}{4}$?

Num.			Log.
$2\frac{1}{4}$	=	9 =	0.954242
		4 =	0.602060
		$2\frac{1}{4}$ =	0.352182

The logarithm of a decimal fraction is found exactly in the same manner as a whole number, therefore the fractional part

of a mixed number may be reduced to a decimal, and the logarithm of the integer and fractional part taken out at once; but the index, in this case, is to be regulated by the number of figures in the integral part.

EXAMPLE I.

Required the logarithm of .125?

Num. Log.
$.125 = -1.096910$ or 9.096910

EXAMPLE II.

Required the logarithm of $18\frac{1}{4}$?

Num. Log.
$18\frac{1}{4} = 18.25 = 1.261263$

TO FIND THE NUMBER ANSWERING TO ANY GIVEN LOGARITHM.

RULE.

Look for the given logarithm in the different columns of the table, (neglecting the index) until it be found exactly, or the next less: then the three first figures of the corresponding natural number will be found opposite to it, in the left hand column, marked Num. and the fourth figure, immediately above it, at the top of the column, which contains the logarithm. If the index of the *given logarithm* be 3, the *four* figures thus found are integers: if the index be 2, the *three* first figures are integers, and the fourth is a decimal: if the index be 1, the *two* first figures are integers, and the other two are decimals, and so on*.

If the logarithm cannot be found exactly in the table, and more than four figures are required in the natural number, subtract the next less logarithm in the table from the given loga-

* The index of the logarithm of a number being a unit *less* than the number of figures in the number, the number of figures must be one more than the index of the log. corresponding to it.

rithm, to the remainder annex as many ciphers as there are figures required above *four* in the natural number; which divide by the difference between the next *less* and the next *greater* logarithms, (which will be found in the column, marked Diff.,) and the quotient annexed to the four figures ,formerly found, will form the natural number required.

EXAMPLE I.

Required the natural number corresponding to the logarithm, 2.734240?

This logarithm is found opposite to 542 and under 3, and the index being 2, the fourth figure is a decimal, therefore, the corresponding number is 542.3.

EXAMPLE II.

Required the natural number corresponding to the logarithm, 5.132780.

The next less logarithm in the table is .132580, answering to the number 1357; the difference between which and the given logarithm is 200, to which two ciphers being annexed, makes 20000; which divided by 320, the difference between the next less and the next greater logarithm, the quotient is 62, which annexed to 1357, makes 135762, the number required.

The use of logarithms is to shorten laborious calculations, such as multiplying and dividing large numbers, raising powers, extracting roots, &c. This valuable property arises from their nature or connection with the decimal scale, which is such, that adding the *logarithms* corresponding to any two numbers, produces the logarithm of their product; and subtracting their logarithms, leaves the logarithm of their quotient. Hence, *addition* of logarithms serves the end of *multiplication*, and *subtraction* of logarithms, of division, of their corresponding numbers, &c.

TO PERFORM MULTIPLICATION BY LOGARITHMS.

RULE.

Add the logarithms of the factors, and the sum is the logarithm of their product.

If there be negative and affirmative indices, their difference is to be taken; or rather use the arithmetical complement of the negative indices, and then add them; only, in this case, *tens* must be rejected from the sum of the indices.

EXAMPLE I.

Multiply 278 by 16.

Factors	278	log.	2.444045
	16	log.	1.204120
Product	4448	log.	3.648165

EXAMPLE II.

Multiply 17.8, 0.4, and 0.0065 together.

Factors	17.8	log.	1.250420	or	1.250420
	0.4	log.	—1.602060		9.602060
	0.0065	log.	—3.812913		7.812913
Product	0.04628	log.	—2.665393		8.665393

TO PERFORM DIVISION BY LOGARITHMS.

RULE.

Subtract the logarithm of the divisor from the logarithm of the dividend, the remainder is the logarithm of the quotient.

EXAMPLE I.

Divide 25768 by 364.

Dividend	25768	log.	4.411081
Divisor	364	log.	2.561101
Quotient	70.7912	log.	1.849980

EXAMPLE II.

Divide 8.42 by .96.

Dividend	8.42	log.	0.925312
Divisor	.96	log.	—1.982271
Quotient	8.771	log.	0.943041

TO PERFORM PROPORTION BY LOGARITHMS.

RULE.

Add the logarithm of the second and third terms together, and subtract the logarithm of the first term from the sum; the remainder is the logarithm of the fourth term, or answer.

EXAMPLE.

If 497 yards of cloth cost £287; what will 389 yards cost?

As 497	log.	2.696356
is to 389	—	2.589949
so is 287	—	2.457882
		5.047831
		2.696356
To £224.633		2.351475

TO PERFORM INVOLUTION BY LOGARITHMS; THAT IS, TO RAISE ANY NUMBER TO ANY POWER.

RULE.

Multiply the logarithm of the given number by the index, or exponent of the power; the product is the logarithm of the power required.

EXAMPLE I.

What is the square, or second power, of 39?

Given number	39,	log.	1.591065
Expon. of power			2
Square	1521		3.182130

EXAMPLE II.

What is the hundredth power of 1.05.

Given number	1.05,	log.	0.0211893
Expon. power			100
Hundredth power	131.513		2.1189300

TO PERFORM EVOLUTION, THAT IS, TO EXTRACT ANY PROPOSED ROOT BY LOGARITHMS.

RULE.

Divide the logarithm of the given number by the exponent of the power, the quotient is the logarithm of the root.

If the given number be a decimal, and the arithmetical complement of the negative index used, prefix 1 to that index, for the square root; 2 for the cube root, &c.

EXAMPLE I.

Required the square root of 2?

Given number	2,	log.	0.301030
Square root	1.41421	$\frac{1}{2} =$	0.150515

EXAMPLE II.

Required the cube root of 6751269?

Given number 6751269, log. 6.829386
which, divided by 3, quotes 2.276462
the logarithm of 189, the root required.

PROGRESSION.

WHEN a rank or series of numbers *increases* or *decreases* by a common difference, it forms an arithmetical progression; thus, 1, 2, 3, 4, 5, 6, and 1, 3, 5, 7, 9, &c. are arithmetical progressions. In the *first*, the common difference is 1; and, in the second, it is 2*.

In every arithmetical progression there are five different particulars concerned; viz.

1. The least term.
2. The greatest term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

Any *three* of these being given, the other two may be found.

CASE I.

THE LEAST TERM, THE GREATEST TERM, AND THE NUMBER OF TERMS BEING GIVEN, TO FIND THE SUM OF ALL THE TERMS.

RULE.

Multiply the sum of the greatest and least terms by half the number of terms, or by the whole number of terms, and divide the product by 2, which will give the sum required.

* When the progression consists only of *three*, or *four terms*, it is commonly called an arithmetical proportion.

EXAMPLE.

The first term of an arithmetical series is 2, the last 16, and the number of terms 8: required the sum of the series?

Least term,		*Greatest term,*		*Sum,*		*Half numb. terms,*		*Sum series.*
2	+	16	=	18	×	4	=	72

2. The least term is 5, the greatest 205, and the number of terms 11; required the sum of all the terms?

3. The first term is 4, the last 800, and the number of terms 40; required the sum of all the terms.

CASE II.

THE LEAST TERM, THE GREATEST TERM, AND THE NUMBER OF TERMS BEING GIVEN, TO FIND THE COMMON DIFFERENCE.

RULE.

Subtract the least term from the greatest, then divide the remainder by 1 less than the number of terms: the quotient will be the common difference.

EXAMPLE.

The least term is 5, the greatest term 27, and the number of terms 12: required the common difference?

Greatest term,		*least term,*		*diff.*		*numb. terms less one,*		*com. diff.*
27	−	5	=	22	÷	11	=	2

2. The least term is 3, the greatest term 17, and the number of terms 8; required the common difference?

CASE III.

THE GREATEST TERM, THE LEAST TERM, AND THE COMMON DIFFERENCE BEING GIVEN, TO FIND THE NUMBER OF TERMS.

RULE.

Divide the difference between the greatest and least terms by the common difference; add 1 to the quotient, and the sum will be the number of terms.

EXAMPLE.

The least term is 4, the greatest 39, and the common difference 5; required the number of terms?

Greatest term,		*least term,*		*diff.*		*com. diff.*		*quotient,*				*num. terms.*
39	$-$	4	$=$	35	$\div$	5	$=$	7	$+$	1	$=$	8

2. The least term 6, the greatest 60, and the common difference 3: required the number of terms?

CASE IV.

THE GREATEST TERM, THE COMMON DIFFERENCE, AND THE NUMBER OF TERMS BEING GIVEN, TO FIND THE FIRST OR LEAST TERM.

RULE.

Multiply the number of terms, *minus* 1, by the common difference, the product, subtracted from the greatest term, leaves the least.

EXAMPLE.

The greatest term is 60, the common difference 3, and the number of terms 19; required the least term?

$(19 - 1) \times 3 = 54$; then $60 - 54 = 6$ least term.

CASE V.

THE NUMBER OF TERMS, THE COMMON DIFFERERCE, AND THE SUM OF ALL THE TERMS BEING GIVEN, TO FIND THE FIRST OR LEAST TERM.

RULE.

Divide the sum of all the terms by the number of terms, and from the quotient subtract half the product of the common difference, multiplied by the number of terms, *minus* 1, the remainder is the first term.

EXAMPLE.

The number of terms is 11, their sum is 154, and the common difference is 2: required the least term?

$$154 \div 11 = 14$$
$$\frac{(11 - 1) \times 2}{2} = 10$$
$$\text{diff. } 4 \text{ least term.}$$

CASE VI.

THE LEAST TERM, THE NUMBER OF TERMS, AND THE COMMON DIFFERENCE BEING GIVEN, TO FIND THE GREATEST TERM.

RULE.

Subtract the common difference from the product of the number of terms, multiplied by the common difference; the remainder, added to the first term, gives the greatest term.

EXAMPLE.

The least term is 6, the number of terms 21, and the common difference 3: required the greatest term?

$$21 \times 3 = 63 - 3 = 60 + 6 = 66 \text{ greatest term.}$$

As the foregoing cases comprehend the questions that *most commonly* occur in geometrical progression, it was thought unnecessary to insert the other cases in *words*; they are, therefore, given in the form of Algebraic Theorems, which may be

understood (or translated into words) by those who are acquainted with the common signs that are employed in Algebra*.

Let a represent the first term.
x ——— the last term.
d ——— the common difference.
n ——— the number of terms.
s ——— the sum of the series.

7. Given n, s, and x, to find a.

$$a = \frac{2s}{n} - x$$

8. Given d, s, and x, to find a.

$$a = \sqrt{(\tfrac{1}{2}d + x)^2 - 2ds}$$

9. Given a, n, and s, to find x.

$$x = \frac{2s}{n} - a$$

10. Given d, n, and s, to find x.

$$x = \frac{s}{n} + \tfrac{1}{2}d \times (n-1)$$

11. Given a, d, and s, to find x.

$$x = \sqrt{(\tfrac{1}{2}d - a)^2 + 2ds}$$

12. Given a, n, and s, to find d.

$$d = \frac{s - an}{n - 1} \times \frac{2}{n}$$

---

* To explain the methods of obtaining the following Theorems, as well as those in geometrical progression, belongs rather to Algebra than to Arithmetic; they are, therefore, purposely omitted in this work, which only embraces subjects immediately connected with *Arithmetic*.

13. Given n, s, and x, to find d.

$$d = \frac{xn - s}{n - 1} \times \frac{2}{n}$$

14. Given a, s, and x, to find d.

$$d = \frac{(x + a) \times (x - a)}{2s - (a + x)}$$

15. Given a, s, and x, to find n.

$$n = \frac{2s}{a + x}$$

16. Given a, d, and s, to find n.

$$n = \frac{\frac{1}{2}d - a + \sqrt{(\frac{1}{2}d - a)^2 + 2ds}}{d}$$

17. Given d, s, and x, to find n.

$$n = \frac{\frac{1}{2}d + x - \sqrt{(\frac{1}{2}d + x)^2 - 2ds}}{d}$$

18. Given a, d, and x, to find s.

$$s = \frac{a + x}{2} \times \frac{(x - a) + d}{d}$$

19. Given a, d, and n, to find s.

$$s = [2a + (n - 1) \times d] \times \frac{n}{2}$$

20. Given d, n, and x, to find s.

$$s = [2x - (n - 1) \times d] \times \frac{n}{2}$$

EXERCISES.

1. Required the sum of an arithmetical series, consisting of 100 terms; the first term being 3, and the common difference 4?

2. The first term of a decreasing arithmetical series is 420, the common difference 3, and the number of terms 50; required the sum of the series?

3. The greatest term of a decreasing arithmetical series is 23, the common difference $\frac{1}{2}$, and the number of terms 39; required the sum of the series?

4. The sum of an arithmetical series is 14820, the first term 0, and the last 297; required the number of terms, and common difference?

5. The last term of an arithmetical series is 505, the first 5, and common difference 5; required the sum of the series, and number of terms?

6. A person agrees to pay a debt in a year, by weekly payments; to pay 1s. the first week, 2s. the next, 3s. the next, &c. required the debt, and last payment?

7. How many strokes do the clocks of Venice, which go on to 24 o'clock, strike in a day?

8. A trader, while he continued in business, spent £4680. The first year he had spent £30, and he had increased his expenses yearly by £12. How long was he in trade?

9. A trader spent £60, the ninth year he was in business; his expenses had yearly increased, during that period, by an equal sum, and were in all £432. What did he spend the first year, and what was the yearly increase?

GEOMETRICAL PROGRESSION.

When a rank, or series of numbers, increase by the same *multiplier*, or decrease by the same divisor, they form a geometrical progression*. Thus, 2, 4, 8, 16, 32, 64, &c. is an increasing geometrical progression, in which the common multiplier is 2. The first and last terms are usually called the *extremes*, and the constant multiplier the ratio.

In every geometrical progression there are five different particulars concerned: viz.

1. The least term.
2. The greatest term.
3. The number of terms.
4. The common ratio.
5. The sum of all the terms.

Any three of these being given, the other two may be found.

As questions in geometrical progression are most easily performed by Logarithms, and can only be performed by those who have acquired a knowledge of the first principles of Algebra, the theorems for solving most of the cases are here inserted, in

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* 1. If the progression consist of *three* terms, the *product* of the two *extremes* is equal to the *square* of the *mean* term. Example; 4, 8, 16, is a geometrical progression; therefore, $16\times4=8^2$.

2. If it consist of *four* terms, the product of the two extremes is equal to the product of the two means. Example: 4, 8, 16, 32, is a geometrical progression; therefore, $32\times4=16\times8$.

3. In a geometrical progression consisting of any number of terms, the product of the two extremes is equal to the product of any two terms that are equally distant from them, or to the square of the mean term, when the number of terms are odd. Example: let the prog. be 2, 4, 8, 16, 32, 64, 128; then $128\times2=16^2=256=32\times8$.

as convenient a manner as possible, for the logarithmical process, when they are very laborious to solve by any other means.

Let a	represent	the least term.
x	———	the greatest term.
r	———	the common ratio.
n	———	the number of terms.
s	———	the sum of the series.
Log.	———	the logarithm of any letter.

THEOREMS.

1. Given a, x, and r, to find s.

$$s = \frac{xr-a}{r-1} \text{ or } \frac{x-a}{r-1} + x$$

2. Given a, r, and n, to find s.

$$s = a \times \frac{r^n - 1}{r - 1}$$

3. Given n, r, and x, to find s.

$$s = \frac{r^n - 1}{r - 1} \times \frac{x}{r^n - 1}$$

4. Given a, x, and n, to find s.

$$s = \frac{x-a}{\sqrt[n-1]{\frac{x}{a}} - 1} + x$$

5. Given n, r, and x, to find a.

$$a = \frac{x}{r^n - 1} \text{ or Log. } a = \text{log. } x. - \text{log. } r. \times (n-1)$$

6. Given r, s, and x, to find a.

$$a = xr - [s \times (r-1)]$$

7. Given r, s, and n, to find a.

$$a = s \times \frac{r-1}{r^n - 1}$$

8. Given a, r, and n, to find x.

$$x = ar^{n-1} \text{, or log. } x = \text{log. } r \times (n-1) + \text{log. } a.$$

9. Given a, r, and s, to find x.

$$x = \frac{s \times (r-1) + a}{r}$$

10. Given r, s, and n, to find x.

$$x = \frac{r-1}{r^n - 1} sr^{n-1}$$

11. Given a, x, and s, to find r.

$$r = \frac{s-a}{s-x}$$

12. Given a, x, and n, to find r.

$$r = \sqrt[n-1]{\frac{x}{a}}, \text{ or log. } r = \frac{\text{log. } x - \text{log. } a}{n-1}$$

13. Given a, n, and s, to find r.

$$r = \sqrt[n]{\frac{s \times (r-1) + a}{a}} \text{ or log. } r = \text{log. } \frac{[s \times (r-1) + a] - \text{log.} a}{n}$$

14. Given a, r, and x, to find n.

$$n = \frac{\log. x - \log. a}{\log. r} + 1$$

15. Given a, r, and s, to find n.

$$n = \log. \frac{[s \times (r-1) + a] - \log a}{\log. r}$$

16. Given a, s, and x, to find n.

$$n = \frac{\log. x - \log. a}{\log. (s-a) - (\log. s - x} + 1$$

17. Given r, s, and x, to find n.

$$n = \frac{\log. x - \log. [(xr-s) \times (r-1)]}{\log. r} + 1$$

The above theorems comprehend the greatest number, and the most useful cases, in geometrical progression.

EXAMPLE I.

The least term 2, the greatest 256, and the ratio 2, of a geometrical progression, required the sum of the series?

Here $a=2$, $x=256$, and $r=2$

By Theorem 1.

$$\begin{array}{rr} x = & 256 \\ r = & 2 \\ \hline xr = & 512 \\ a = & 2 \\ \hline xr - a = & 510 \\ \hline r-1 = & 1 \end{array} \qquad \text{or } \frac{x-a}{r-1} = \frac{254}{1} = 254$$

$$x = +256$$

$$= \text{Sum of Series} \ldots \; 510$$

EXAMPLE II.

Required the last term of a geometrical series, consisting of 14 terms; the first term being 2 and the common ratio 2?

Here $a = 2$, $r = 2$, and $n = 14$, to find x.

By Theorem 8. $x = \text{Log. } r \times (n-1) + \text{log. } a$.

log. of r, or $2 =$	0.301030	or $2^{13} =$	8192
$(n-1)$ $13 =$	13		2
	3.913390	Ans.	16384
log. a, or $2 =$	0.301030		
Answer $x = 16384 =$	4.214420		

EXERCISES.

1. The least term of a geometrical series is 10, the greatest term 10000, and the ratio 6; required the sum of the series?

2. A fir plant, which weighed 4 oz. avoirdupois, increased to triple the weight each year, how much did it weigh at the end of the 7th year?

3. Required the last term and the sum of a geometrical series, whose first term is 1, common ratio 2, and number of terms 16?

4. Required the sum of a geometrical series, consisting of 20 terms; the first term being 1, and the last 524288?

5. The sum of a geometrical series is 1048575, the common ratio 2, and the number of terms 20; required the last term?

6. Required the sum of a geometrical series, decreasing to infinity, whose greatest term is 1 and common ratio 2?

7. Suppose 8 hogs, left on a certain island, increase fourfold each year; in how many years will their number amount to 131072?

8. Suppose 6 hogs, left on a certain island, to multiply so as to amount to 3188646 at the end of 12 years; at what rate did they multiply?

9. If the posterity of Noah, which consisted of 6 persons at the flood, increased so as to double their number in 20 years; how many inhabitants would there be in the world 2 years before the death of Shem, who lived 502 years after the flood?

PERMUTATION AND COMBINATION*.

THE different orders in which quantities can be arranged, are called their *permutations*. Thus, the permutations which the three letters, *a*, *b*, *c*, admit of, when taken two and two together, are six, viz. *ab*, *ba*, *ac*, *ca*, *bc*, *cb*.

The *combinations* of quantities are the different collections that can be formed out of these quantities, without regarding the order in which the quantities are placed. Thus, *ab*, *ac*, *bc*, are the combinations which can be formed of the letters *a*, *b*, *c*, when taken two and two; for, though *ab* and *ba* are different *permutations*, they form the same *combination*.

Let n = any given number of quantities, the number of *permutations* which can be formed out of these is $=1\times2\times3$, &c. continued to n terms, when the articles are all different.

EXAMPLE.

How many permutations does the letters *a*, *b*, *c*, *d*, *e*, *f*, admit of?

$$1\times2\times3\times4\times5\times6=720 \text{ Answer.}$$

* Besides arithmetical and geometrical progression, there are many other kinds of series. If the arithmetical series, 1, 2, 3, 4, 5, 6, &c. be successively added together, another series will arise, which is called a series of *triangular* numbers. If the series, 1, 3, 6, &c. of triangular

The number of permutations that can be formed out of n quantities, taken two and two together, is $n \times \overline{n-1}$; taken three and three together, is $n \times \overline{n-1} \times \overline{n-2}$.

The number of *combinations* that can be formed out of n things, taken two and two together, is $n \times \frac{n-1}{2}$; taken three and three together, is $n \times \frac{n-1}{2} \times \frac{n-2}{3}$.

It appears, from the above theorems, that there are *twice* as many *permutations* in any number of quantities, as there are *combinations*, when taken *two* and *two*; six times as many, when taken three and three, &c.

EXAMPLE.

How many ways may a party of 3 men be drawn out from a company of 50 men?

Here $n = 50$

Therefore $50 \times \frac{49}{2} \times \frac{48}{3} = 19600$ ways.

The varieties of one or more articles, not restricted to any particular number that may be chosen from a given quantity, is sometimes called *Election*.

The number of elections is $= 2^n - 1$, if n represent the whole number of articles.

EXAMPLE.

How many quantities is it possible to weigh, at one operation, with 5 weights, without using any of them as back weights.

Here $2^5 - 1 = 31$, the answer.

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numbers be added, the sums 1, 4, 10, &c. are called *pyramidal numbers*. If the series of pyramidal numbers be added in a similar manner, another series is obtained, and so on, without limitation.—These series are denominated figurate numbers.

EXERCISES.

2. How many combinations of 5 letters in 10?

3. How many elections may be made from 24 letters of the alphabet?

4. How many combinations can be made with 5 letters out of the 26 of the alphabet?

5. How many changes can be rung with 4 bells out of 8?

6. How many varieties are there in the stops of an organ that has 8 sets of pipes?

7. In how many different orders may a party of 12 men be placed?

8. In how many different orders may a gardener plant 3 elms, 4 firs, and 5 oaks?

Those who wish to see the doctrine of combinations, permutations, and figurate numbers, &c. fully explained, may consult Simpson's Algebra, Emerson's Treatise on these subjects, or Dodson's Mathematical Repository.

COMMERCIAL ARITHMETIC.

Before the student commences the study of Arithmetic, he should be well acquainted with most of the following Tables; but particularly before he enters on the study of Commercial Arithmetic.

TABLES OF MONEY, WEIGHTS, AND MEASURES.

Sterling Money.

Farthings.	Penny.	Shilling.	Pound.
4 =	1		
48	12 =	1	
960	240	20 =	1

Note 1. Scots money is divided in the same manner as Sterling, and is one twelfth of its value.

2. A joannes = 36s.; a moidore = 27s.; a jacobus = 25s.; a carolus=23s.; a guinea=21s.; a pistole=17s. 6d.; a mark=13s. 4d.; an angel = 10s.; a noble=6s. 8d.; a crown=5s.; a dollar=4s. 6d.; a tester=6d.; a groat=4d.

Troy Weight. For Gold, Silver, Jewels, and Liquors.

Grains.	dwt. Pennyweight.	oz. Ounce.	lb. Pound.
24 =	1		
480	20 =	1	
5760	240	12 =	1

In Scotland, gold and silver are weighed by the Troy ounce and pound; but the ounce is divided into 16 drops, and the drop into 30 grains, A lb. Troy = .822622 lb. Avoirdupois = .756302 lb. Scots Troye.

Apothecaries Weight. For compounding Medicines.

Grains.	scr. Scruple.	dr. Dram.	oz. Ounce.	lb. Pound.
20 =	1			
60	3 =	1		
480	24	8 =	1	
5760	288	96	12 =	1

This weight is the same as Troy, though differently divided.

Avoirdupois Weight. Used, in England, for every sort of Merchandise, except the above.

Troy Grains.	Dram.	Ounce.	Pound.	Qurter.	cwt.	Ton.
$27\frac{45}{128}$	1					
$437\frac{5}{8}$	16 =	1				
7002	256	16 =	1			
196056	7168	448	28 =	1		
784224	28672	1792	112	4 =	1	
15684480	573440	35840	2240	80	20 =	1

NOTE. A stone=14 lb., a sack of flour=280 lb., a barrel of flour =196 lb., a quartern loaf=$69\frac{1}{4}$ oz., a barrel of candles=10 dozen, a gallon of oil=$7\frac{1}{2}$ lb., a fother of lead=8 pigs of $21\frac{1}{4}$ stones each; at London, it is $19\frac{1}{2}$ cwt., and at the mines $22\frac{1}{2}$ cwt.; a load of hay at London=36 trusses of 56 lb. each of old hay, or 60 lb. each of new hay; a load of straw at London=36 trusses of 36 lb. each; a stone of glass=5 lb.; a seam of glass=120 lb.; a firkin of soap=64 lb.; a firkin of butter=56 lb.; a barrel of gunpowder=1 cwt.; a bushel of salt=56 lb.; a bushel of foreign salt=84 lb.; a bushel of rock salt=65 lb.

A lb. Avoirdupois=1.215625 lb. Troy=91938 lb. Scots Troye.

Wool Weight.

Avoir. Pounds	Clove.	Stone.	Todd.	Wey.	Sack.	Last.
7 =	1					
14	2 =	1				
28	4	2 =	1			
182	26	13	$6\frac{1}{2}$ =	1		
364	52	26	13	2 =	1	
4368	624	312	156	24	12	1

A pack of Wool=240 lb.

English Measures of Length.

Inches.	Gunter's link.	Foot	yard	Fathom	Pole or Rod.	Gunter's chain.	furlong	Mile
$7\frac{23}{25}$=	1							
12	$1\frac{17}{33}$	1						
36	$4\frac{6}{11}$	3=	1					
72	$9\frac{1}{11}$	6	2=	1				
198	25	$16\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{3}{4}$=	1			
792	100	66	22	11	4=	1		
7920	1000	660	220	110	40	10 =	1	
63360	8000	5280	1760	880	320	80	8 =	1

A palm=3 inches; a span=9 inches; a cubit=18 inches; a pace =5 feet. In Devon, and part of Somerset, a pole=5 yards; in Cornwall, a pole=6 yards; in Lancashire, a pole=7 yards; in Cheshire and Staffordshire, a pole=8 yards; in the Isle of Purbeck, and some other parts of Dorsetshire, a pole=15 feet, 1 inch. In Ireland, a pole=7 yards; and a mile=2240 yards.

Cloth Measure.

2¼ inches = 1 nail; 4 nails = 1 quarter; 4 quarters = 1 yard; 5 quarters=1 English ell; 3 quarters=1 Flemish ell.

English Square Measure.

Inches.	Gunter's link.	Foot.	Yard.	Pole, or Perch.	Gunter's chain.	Rood.	Acre.	Mile.
62[illegible] =	1							
144	2[illegible]	1						
1296	20[illegible]	9 =	1					
39204	625	272¼	30¼ =	1				
627264	10000	4356	484	16 =	1			
1568160	25000	10890	1210	40	2½ =	1		
6272640	100000	43560	4840	160	10	4	1	
4014489600	6400000	27878400	3097600	102400	6400	2560	640	1

The Irish acre is to the English as 49 to 30¼. An English acre =.7869407 Scots acre. The French arpent is 48400 French square feet=54974.4093 English square feet. The Roman acre was 28800 Roman square feet.

Wine Measure. For Wine, Spirits, Oil, Cider, Perry, Mead, Metheglin, Honey, &c.

Solid inches.	Pint.	Quart.	Gallon.	Runlet	Barrel.	Tierce.	Hogshead.	Puncheon.	Pipe or Butt.	Tun.
28 7/8 =	1									
57 3/4	2 =	1								
231	8	4 =	1							
4158	144	72	18 =	1						
7276 1/2	252	126	31 1/2	1 3/4 =	1					
9702	336	168	42	2 1/3	1 1/3 =	1				
14553	504	252	63	3 1/2	2	1 1/2 =	1			
19279	672	336	84	4 2/3	2 2/3	2	1 1/3 =	1		
29106	1008	504	126	7	4	3	2	1 1/2 =	1	
58212	2016	1008	252	14	8	6	4	3	2 =	1

An anker = 10 gallons; a wine gallon = .2792445 Scots gallon =.819148936 ale gallon; a barrel of herrings=32 wine gallons; a cran of herrings=34 wine gallons; a barrel of salmon=42 wine gallons; a barrel of pitch or tar=31½ gallons. Olive oil is sold by the tun of 236 gallons; port wine, by the pipe of 138 gallons; Lisbon, by the pipe of 140 gallons; Madeira, by the pipe of 110 gallons; Sherry. by the pipe of 130 gallons; Vidonia, by the pipe of 120 gallons.

Ale and Beer Measure. For Malt Liquors, Vinegar, &c.

Solid inches.	Pint.	Quart.	Gallon.	Firkin.	Kilderkin.	Barrel.	Hogshead.	Butt.
35¼=	1							
70½	2=	1						
282	8	4=	1					
2397	68	34	8½=	1				
4794	136	68	17	2=	1			
9588	272	136	34	4	2 =	1		
14382	408	204	51	6	3	1½=	1	
28764	816	408	102	12	6	3	2 =	1

The beer measure in London was 9 gallons to the firkin, 36 gallons to the barrel, and 54 gallons to the hogshead; and the ale measure, 8 gallons to the firkin, 32 gallons to the barrel, and 48 gallons to the hogshead. But by Act 43. Geo. 3. cap. 69. the barrel of beer, or ale, sold by the common brewers of Great Britain, must contain 36 gallons; but if brewed by any victualler or retailer, it contains only 34 gallons, as in the table.

An ale gallon=.340896 Scotch gallon=1.220779 wine gallon.

English Dry Measure.

Solid inches.	Pint.	Quart.	Gallon.	Peck	Bushel.	Qurter.
33.600315 + =	1					
67.20063 +	2=	1				
268.80252 +	8	4=	1			
537.60504 +	16	8=	2=	1		
2150.42017 +	64	32	8	4=	1	
17203.36137 +	512	256	64	32	8 =	1

A pottle or quartern=4 pints; a strike=2 bushels; a coomb=4 bushels; a chaldron of corn=32 bushels; a wey=40 bushels; a

last=80 bushels. A London chaldron of coals=36 bushels heaped up (each bushel containing a Winchester bushel and a quart) = 28 cwt. or 3136 lb. The Newcastle chaldron of coals=52¼ cwt.; 15 London chaldrons = 8 Newcastle chaldrons. A sack of coals = 3 bushels; a vat or quarter=9 bushels.

A quarter=1.3416964 barley boll=1 b. 1 f. 1 p. 1.868 lip.
A quarter=1.9572983 wheat boll=1 b. 3 f. 3 p. 1.267 lip.

Solid Measure.

Solid inches.	Solid foot.	Solid yard.	Solid mile.
1728	1		
46656	27 =	1	
254,358,061,056,000	147197952,000	5,451,776,000	1

A load of hewn timber	=	50 solid feet
A load of unhewn timber	=	40 ditto
A stack of wood	=	108 ditto
A cord of wood	=	128 ditto
A ton of bale goods	=	40 ditto
A wheat boll	=	5.0664 ditto
A barley boll	=	7.4202 ditto
A quarter	=	9.9556 ditto
A wine hogshead	=	8.4219 ditto
An ale hogshead	=	8.3229 ditto

Linen yarn.—The reel is 90 inches in circumference: 120 threads =1 cut; 12 cuts=1 hank; 4 hanks=1 spindle.

Cotton yarn.—The reel is 54 inches in circumference: 80 threads =1 skein; 7 skeins=1 hank; 18 hanks=1 spindle.

Paper.—24 sheets=1 quire; 20 quires=1 ream; 2 reams=1 bundle; 10 reams=1 bale.

Time.

Seconds.	Minute.	Hour	Day.	Common civil year.
60=	1			
3600	60=	1		
86400	1440	24=	1	
31,536,000	525600	8760	365=	1

Also—a week=7 days; a month=4 weeks; a year=13 months and 1 day, or 52 weeks 1 day.

Leap year=366 days; mean Julian year=365 days, 6 hours; solar year=365 days, 5 hours, 48′ 48″; sidereal year=365 d. 6h. 9′ 10″; a lunation, or lunar synodical month=29 d. 12 h. 44′ 3″. The civil year is divided into the following 12 civil, or calendar months:

	days.		days.
January	31	July	31
February { in common years	28	August	31
February { in leap years	29	September	30
March	31	October	31
April	30	November	30
May	31	December	31
June	30		

Thirty days have September,
April, June, and November:
February twenty-eight alone,
And all the rest have thirty-one.

SCOTS WEIGHTS, MEASURES, &c.

Scots Troye Weight. For Meal, Butcher-Meat, Hemp, Iron, &c.

Eng. Troy Grains.	Drop.	Ounce.	Pound.	Stone.
29¾ =	1			
476	16=	1		
7616	256	16=	1	
121656	4096	256	16=	1

200 lb. Scots Troye is reckoned = a barrel of bacon, beef, butter, honey, and ashes; a last=12 barrels.

Trone weight is much used for home productions; as butter, cheese, &c. but is different in almost every town. At Edinburgh, it stands thus: 29¾ English Troy Grains = 1 drop, 16 drops = 1 ounce, 20 ounces=1 lb., 16 lb=1 stone.

A lb. Scots Troye = 1.322222 lb. English Troy, = 1.087689 lb. Avoirdupois.

Scots Measures of Length.

A foot=1.344086 link.

Eng. inches.	Gunter's link.	Eng. foot.	Ell.	Fall or Rood.	Gunter's chain.	furlong	Mile
$8\frac{116}{125}$=	1						
12	$1\frac{32}{93}$=	1					
$37\frac{1}{5}$	$4\frac{1}{6}$	$3\frac{1}{10}$=	1				
$223\frac{1}{5}$	25	$18\frac{3}{5}$	6=	1			
$892\frac{4}{5}$	100	$74\frac{2}{5}$	24	4=	1		
8928	1000	744	240	40	10 =	1	
71424	8000	5952	1920	320	80	8 =	1

A Scots inch=$1\frac{1}{185}$ English inch: if we do not regard this difference, but reckon the chain=74 English feet, then a link=$8\frac{22}{25}$ inches; a foot=$1\frac{13}{37}$ link; an ell=$3\frac{1}{12}$ feet=37 inches; a rood=$18\frac{1}{2}$ feet=222 inches; a furlong=740 feet=8880 inches; a mile=5920 feet=71040 inches.

Scots Square Measure.

A Scots acre=1.2707438 English acre,
A square foot=1.80656723 square link.

Eng. inches.	Gunter's link.	English foot.	Ell.	Fall.	Gunter's chain.	Rood.	Acre
$79\frac{11081}{15625}$=	1						
144	$1\frac{6976}{8649}$=	1					
$1383\frac{21}{25}$	$17\frac{13}{36}$	$9\frac{61}{100}$=	1				
$49818\frac{6}{25}$	625	$345\frac{24}{25}$	36=	1			
$797091\frac{21}{25}$	10000	$5535\frac{9}{25}$	576	16=	1		
$1992729\frac{3}{5}$	25000	$13838\frac{2}{5}$	1440	40	$2\frac{1}{2}$=	1	
$7970918\frac{2}{5}$	100000	$55353\frac{3}{5}$	5760	160	10	4=	1

When the Scots chain is reckoned only 74 English feet; then a square link $=78\frac{534}{625}$ inches; a foot $=1\frac{1131}{1369}$ link; an ell $=9\frac{73}{144}$ feet $=1369$ inches; a fall $=342\frac{1}{4}$ feet $=49284$ inches; a chain $=5476$ feet $=788544$ inches; a rood $=13690$ feet $=1971360$ inches; an acre $=54760$ feet $=7885440$ inches. The acre, found by using a chain of 74 feet, is $593\frac{3}{5}$ square feet $=1$ fall, $25\frac{739}{961}$ ells less than the true Scots acre; or, if a field containing a true Scots acre be measured by a chain of 74 feet, it will amount to $101084\frac{4}{1369}$ square links, $=1$ acre, 1 fall $26\frac{2923}{6845}$ ells, which is $\frac{1}{92\frac{93}{371}}$ of the whole, above the true content.

Scots Liquid Measure.

Solid inches.	Gill.	Mutchkin.	Chopin.	Pint.	Quart.	Gallon.
$6\frac{1851}{4000}=$	1					
$25\frac{851}{1000}$	4=	1				
$51\frac{351}{500}$	8	2=	1			
$103\frac{101}{250}$	16	4	2=	1		
$206\frac{101}{125}$	32	8	4	2=	1	
$827\frac{29}{125}$	128	32	16	8	4 =	1

The standard pint contains 55 Scots Troye ounces of river water. The ale pint, in common use, and other ale measures, generally hold $\frac{1}{14}$ part above standard. In charging the duties on the species called twopenny ale, 36 English gallons are, in practice, allowed for the barrel, and 12 Scots gallons are reckoned equal thereto; which makes the pint$=105\frac{3}{4}$ solid inches. There is no Scots hogshead.

A Scots gallon=3.581091 wine gallons, =2.933447 ale gallons.

A Paris pint=48 Paris cubic inches. See page 177.

Scots Dry Measure. For Wheat, Beans, Pease, Rye, Salt, and Grass-seeds.

Solid inches.	Pint.	Lippie.	Peck.	Firlot.	Boll.	Chalder.
$103\frac{101}{250}$=	1					
$137\frac{1067}{3200}$	$1\frac{21}{64}$=	1				
$549\frac{267}{800}$	$5\frac{5}{16}$	4=	1			
$2197\frac{67}{200}$	$21\frac{1}{4}$	16	4=	1		
$8789\frac{17}{50}$	85	64	16	4=	1	
$140629\frac{11}{25}$	1360	1024	256	64	16=	1

A wheat boll=.5109083 quarter=4 bush. 0 pecks, 5.585 pints.

For Oats, Barley, and Malt.

Solid inches.	Pint.	Lippie.	Peck.	Firlot.	Boll.	Chalder.
$103\frac{101}{250}$=	1					
$200\frac{1381}{4000}$	$1\frac{15}{16}$=	1				
$801\frac{381}{1000}$	$7\frac{3}{4}$	4=	1			
$3205\frac{131}{250}$	31	16	4=	1		
$12822\frac{12}{125}$	124	64	16	4=	1	
$205153\frac{67}{125}$	1984	1024	256	64	16=	1

The barrel for apples, beef, and pork, was 8 gallons, the herring barrel 8½ gallons, the salmon barrel=10 gallons.

An oat boll=.7453251 quarter=5 bush. 3 p. 13.606 pints.

Orkney Weights. For Barley, Oats, Meal, Malt, Butter, and Oil.

Scots Troye. st. lb. oz.	Avoirdupois. lb.	Mark.	Setteen or Lyspund.	Meil on the malt pundlar	Chalder or last.
0 : 1 : 4=	1.3596=	1			
1 : 14 : 0	32.6306	24=	1		
11 : 4 : 0	195.7836	144	6 =	1	
270 : 0 : 0	4698.8165	3456	144	24 =	1

The chalder of Barley, in Orkney, measures at Leith from 18 to 20 bolls. Barley is sometimes reckoned by the barrel, which is no certain measure; but, in general, the chalder of barley is reckoned 29 or 30 barrels.

Butter and oil are generally reckoned by the barrel, and should weigh 6 lyspunds and 16 marks, or 12½ stones Scots Troye. The butter is packed up in half-barrels, which hold 32 Scots pints.

Shetland Weights. For the same Commodities as in Orkney.

Scots Troye. st. lb. oz. dr.	Avoirdupois. lb.	Mark.	Setteen or Lyspund.	Meil on the malt pundlar	Chalder or last.
0 : 1 : 2 : 10$\frac{2}{3}$	1.2689 =	1			
1 : 12 : 0 : 0	30.4553	24=	1		
10 : 8 : 0 : 0	182.7318	144	6 =	1	
252 : 0 : 0 : 0	4382.5632	3456	144	24 =	1

The Shetland chalder of barley should be between 17 and 18 bolls.

FOREIGN WEIGHTS, &c.

Paris Weight.

Eng. Tr. gr.	Grain.					
$\frac{105}{128}$ =	1	Denier.				
$19\frac{11}{16}$	24 =	1	Gros, or drachme.			
$59\frac{1}{16}$	72	3 =	1	Once.		
$472\frac{1}{2}$	576	24	8 =	1	Marc.	
3780	4608	192	64	8 =	1	Livre, or pound.
7560	9216	384	128	16	2 =	1

A Paris lb.=1.079691516 lb. avoirdupois=1.3125 lb. Troy.

The Scots Troye lb. was originally intended to be the same as the Paris lb. though it now exceeds it, (probably from the inaccuracy of the Scots standards,) by 56 English Troy grains.

Amsterdam Weight.

Amsterdam weight is commonly reckoned the same with Scots Troye; but it seems to be a little heavier, for a Dutch lb. belonging to the city of Glasgow, weighs 7628 English Troy grains; and a standard Dutch lb. in the possession of a gentleman at Edinburgh, weighs $7633\frac{1}{2}$ English Troy grains.

There is another lb. used by the Dutch, in retailing, called the *house pound,* which weighs about 16 oz. 9 dr. Avoirdupois.

Dutch weights. 24 grains = 1 dram; 3 drams = 1 gross; $4\frac{1}{16}$ gross, or 300 grains = 1 loot; 2 loots = 1 ounce; 8 ounces = 1 mark; 2 marks = 1 pound; 8 lb. = 1 stone; 15 lb. = 1 lispound; 20 lispounds = 1 shippound; 165 lb. = 1 wage; 400 lb. = 1 load.

Hamburgh Weight.

2 loots = 1 ounce; 16 ounces = 1 pound; 14 lbs. = 1 lispound; 8 lispounds = 1 centner; 20 lispounds = 1 shippound; 10 lb. = 1 stone of wool or feathers; 16 lb. = 1 lispound of ditto; 20 lispounds

= 1 shippound of ditto; 20 lb. = 1 stone of flax; 16 lispounds = 1 ton of butter or tallow.

In Russia, 40 lb. = 1 pood; 10 poods = 1 bercovitz; 63 poods nearly = 1 English ton of tallow, hemp, flax, iron, potashes, soap, lintseed oil, &c. A pood is generally reckoned = 36 lb. avoirdupois.

In Denmark, 16 lb. = 1 lispound; 20 lispounds = 1 shippound.

In Sweden, a shippound for flax, hemp, and fish = 400 lb.; for iron = 320 lb.

At Riga, 20 lb. = 1 lispound; 20 lispounds = 1 shippound; $6\frac{1}{16}$ shippounds are generally reckoned an English ton.

In Poland, 16 lb. = 1 stone; 34 lb. = 1 spruce stone; 120 lb. = 1 centner; 320 lb. = 1 shippound; 2560 lb. = 1 last; 3840 lb. = 1 great last.

At Konigsberg, Memel, &c. 33 lb. = 1 stone; 10 stones = 1 shippound; 6 shippounds = 1 last of hemp, flax, bees-wax, and tallow; 12 shlb. = 1 last of ashes; 18 cwt. English = 1 last of yarn. An English ton = $66\frac{3}{4}$ stones nearly.

In Portugal, 32 lb. = 1 arab; 4 arabs = quintal.

In Spain, 26 lb. = 1 arab; 4 arabs = quintal.

Foreign Corn Measures.

At Amsterdam, a corn last = 27 muids = 108 schepels = 36 sacks = about 82 bushels, or $80\frac{1}{4}$ wheat firlots, or 55 barley firlots.

At Hamburgh, a corn last = 90 schepels = about 90 bushels, or 88 wheat firlots, or $60\frac{3}{5}$ barley firlots.

At Konigsberg, a corn last = $56\frac{1}{8}$ schepels = about 84 bushels, or $82\frac{1}{5}$ wheat firlots, or $56\frac{7}{10}$ barley firlots; a barrel of lintseed = about $3\frac{1}{3}$ bushels.

At Riga, a last of wheat or barley = 48 loops = about 89 bushels, or 87 wheat firlots, or $59\frac{7}{10}$ barley firlots; a last of oats = 60 loops = about $111\frac{1}{4}$ bushels, or $74\frac{4}{5}$ barley firlots; a last of rye = about $83\frac{3}{5}$ bushels, or $81\frac{1}{5}$ wheat firlots; a barrel of lintseed = $3\frac{4}{5}$ bushels.

At Petersburgh, Archangel, &c. a chetvert = about $5\frac{3}{10}$ bushels, or $5\frac{1}{25}$ wheat firlots, or $3\frac{9}{10}$ barley firlots.

At Leghorn, a sack = $2\frac{1}{10}$ bushels, or 159 sacks = 40 quarters, or nearly $78\frac{3}{10}$ wheat bolls.

At Paris, a muid = 12 setiers = 144 bushels; and 19 setiers = about an Amsterdam corn last.

Roman Monies.

The most ancient coins among the Romans were of brass. The *as* originally consisted of a pound of brass, but was afterwards reduced

to half an ounce, but the division of it into 12 parts was still continued.

The chief silver coin was the *denarius*, or Roman penny, 96 of which made a *libra* or pondo. If the value of the libra be stated at £3, as it usually is, the denarius is equal to $7\frac{1}{2}$d. and the *as* to $\frac{1}{2}$d. The proportion of the value of silver to brass was 40 to 1.

The proportion of the value of gold to silver was originally 10 to 1, but in the time of the latter empire, became 14 to 1.

BRASS COIN.

1 As	=	$\frac{1}{2}$d.	
1 Semis	=	$\frac{1}{2}$	As.
1 Uncia	=	$\frac{1}{12}$	As.
1 Semiuncia	=	$\frac{1}{24}$	As.
1 Sextula	=	$\frac{1}{72}$	As.
Decusis	=	10	Asses.
Vicessis	=	20	Asses.
Centussis	=	100	Asses.
1 Sestertium (neut. gen.)	=	1000	Sestertii = £7, 16s. 3d.
1 Talent	=	24	Sestertii.

One pound of gold was originally divided into 48 Aurei, and afterwards into 72. Large sums are usually expressed in pounds of gold, the value of which is £40, nearly.

The Roman foot = 11.632 inches. The *actus* quadratus was a square of 120 Roman feet. The jugerum was equal to two of these, and is to the English acre as 10 to 16.097.

The amphora was a cubical Roman foot, and contained 80 Roman pounds of water. It was nearly equal to $\frac{11}{12}$ of an English bushel. The Sextarius contained about $32\frac{1}{4}$ cubical English inches, and was subdivided in the same manner as the *as*.

The modius, used for dry measure, contained 1 peck and about 8 cubical inches.

Measures of Length in different Countries.

The old French League	=	$2\frac{3}{4}$	English miles.
German Mile	=	4	ditto.
Dutch Mile	=	$3\frac{1}{4}$	ditto.
Italian Mile	=	$1\frac{1}{12}$	ditto.
Spanish League	=	$3\frac{1}{2}$	ditto.
Russian Verst	=	$\frac{3}{4}$	ditto.

Proportion of Foreign Weights to Avoirdupois Weight.

	Avoirdupois lb.		Avoirdupois lb.
lb. of Alicant	=1.0092	lb. of Rochelle	=1.1062
Amsterdam	1.09	Rome	.7875
Ancona	.78	Rouen	1.1387
Antwerp	1.0375	Russia	.8825
Avignon	.9083	St. Sebastian	1.08
Basil	1.1122	Saragossa	.6992
Bayonne	1.08	Seville	1.0343
Bergen	1.1443	Bremen	1.0522
Bern	.9819	Breslaw	.872
Besançon	1.08	Cadiz	1.0343
Bilboa	1.08	Calabria	.73
Bologna	.7218	Cologne	1.0375
Bourdeaux	1.08	Constantinople	1.2529
Flanders	1.0375	Dantzic	.9603
Frankfort on Maine	1.1168	Denmark	1.1232
Geneva	1.23	Lucca	.7703
Genoa	.73	Lyons	.9418
Hamburgh	1.0666	Madrid	.9561
Konigsberg	1.018	Malines	1.0375
Leipsic	1.043	Marseilles	.8868
Leyden	1.0283	Messina	.7078
Leghorn	.755	Milan	.6518
Liege	1.04	Nantes	1.08
Lisbon	.9525	Smyrna	.9561
Lisle	.9561	Stettin	.9909
Lubec	1.04	Sweden	.9345
Naples	.6462	Tholouse	.9237
Nuremberg	1.1122	Turin	.7218
Paris	1.08	Valencia	.6877
Prague	1.2048	Venice	.6568
Revel	.9682	Vienna	1.23
Riga	.895		

Proportion of Foreign Measures of Length to the English Foot.

		Eng. Foot.	Eng. Inches.
Amsterdam	foot	= .931	11.172
Antwerp	foot	.946	11.352
Bononian	foot	1.244 5/6	14.938
Bremen	foot	.964	11.568
Brussels	foot	.902 1/3	10.828
Castilian	foot	.920 1/12	11.041
Dantzic	foot	.941 5/12	11.297
Danish	foot	1.038 3/4	12.465
Frankfort on Maine	foot	.948	11.376
Lyons	foot	1.121 1/2	13.458
Mantua	foot	1.569	18.828
Milan	foot	1.302 7/12	15.631
Old Roman	foot	.969 1/3	11.632

			Eng. Foot.	Eng. Inches.
Grecian	foot	=	1.007	12.084
Paris	foot		1.0657$\frac{1}{3}$	12.789$\frac{1}{16}$
Prague	foot		1.026	12.312
Rhinland or Leyden	foot		1.030$\frac{1}{2}$	12.362
Riga	foot		1.831	21.972
Spanish	foot		1.001	12.012
Swedish	foot		.974$\frac{1}{3}$	11.692
Toledo	foot		.899	10.788
Turin	foot		1.062	12.744
Amsterdam	ell		2.233$\frac{1}{3}$	26.8
Antwerp or Brabant	ell		2.264	27.17
Bavarian	ell		.954	11.448
Basil and Bern	ell		1.875	22.5
Bologna	ell		2.076	24.912
Bononian	ell		2.147	25.764
Breslaw	ell		1.8	21.6
Dantzic	ell		2.002	24.03
Danish	ell		2.077	24.93
Frankfort	ell		1.826	21.912
Geneva	ell		3.73	44.76
Hamburgh	ell		.905	22.86
Leipsic	ell		2.26	27.12
Lubec	ell		1.908	22.896
Lyons	ell		3.880	46.57
Norway	ell		2.042	24.51
Nuremberg	ell		2.227	26.724
Paris, used by mercers	ell		3.898	46.786
Do., used by drapers	ell		3.89	46.68
Rhinland	ell		2.26	27.12
Swedish	ell		1.948	23.38
Brussels	ell		2.271	27.26
Bruges	ell		2.295	27.55
Vienna	ell		1.053	12.636
French	toise		6.3945$\frac{1}{2}$	76.7344
Russian	arsheen		2.333$\frac{1}{3}$	28.
Persian	arash		3.197	38.364
Chinese	cubit		1.016	12.192
Cairo	cubit		1.824	21.888
Parma	cubit		1.866	22.392
Florence	brace		1.909	22.91
Florence geographical	brace		1.7975	21.57
Venice	brace		2.205	26.46
Bononian	brace		2.1	25.2
Milan	brace		1.755	21.06
	calamus		6.544	78.528
Naples	palm		.859	10.314
	brace		2.1	25.2
	cane		6.872	82.512
Rome... merchant's	palm		.815$\frac{11}{12}$	9.791
architect's	palm		.731$\frac{7}{12}$	8.779
merchant's	brace		2.855$\frac{1}{2}$	34.27
architect's	brace		2.560$\frac{1}{6}$	30.73
......	cane		6.817	81.81
Lyons and Marseilles	cane		6.435	77.22
Thoulouse	cane		6.	72.
Castilian	palm		.69	8.281
Castilian and Seville	vare		2.76	33.127
Madrid	vare		3.263	39.166

			Eng. Feet.	Eng. Inches.
Gibraltar		vare	= 2.76	33.12
Portuguese		vare	3.669	44.031
		cavedo	2.279	27.354
Turkish	shorter	pike	2.131	25.576
	longer	pike	2.326	27.92
Genoa		palm	.817	9.81
		cane	7.357	88.29

New French Weights and Measures.

During the time of the French republic, a new system of weights and measures was introduced into France, the foundation of which is the measure of a degree of the meridian.

This was determined to be 57027 toises, from the mean of the measurements of 12 degrees, about latitude 45 deg.; hence a quadrant of the meridian is 5132430 toises, and from this number, the weights and measures, now used in France, are deduced.

The metre, which is the base of the new system, is the 10,000,000 part of 5132430 toises, or quadrant of the meridian, and is equal to 39.371 English inches. The other measures of length both ascend and decend, decimally, as follows:

10 Metres	=	1	Decametre.
10 Decametres	=	1	Hectometre.
10 Hectometres	=	1	Kilometre or mile.
10 Kilometres	=	1	Myriametre or league.
1 Metre	=	10	Decimetres.
1 Decimetre	=	10	Centimetres.
1 Centimetre	=	10	Millimetres.

The square of the Decametre constitutes the Are, and that of the Hectometre, the Hectare, or Acre.

The cube of a Metre forms the unit of solid measure, or the *Stere*; and that of a Decimetre, the Litre or *pint*; and the weight of this bulk of water, at its greatest contraction, makes the Kilogramme, or pound; and the weight of the cube of a Centimetre of distilled water is called the Gramme.

The value of these in English measures, are as follows:

The Toise	=	76.7344 English inches.
or 1 Toise	=	1.0657 Fathoms.
1 Metre	=	39.371 Inches.
1 Myriametre or league	=	6.213856 English miles.
1 Are	=	1077.119234 Square feet.
1 Hectare or Acre	=	2.47117 English Acres.
1 Stere	=	35.3171 Solid feet.
1 Litre	=	61.0280 Cubic inches.
1 Kilogramme or pound	=	2.1133 Troy ounces.
1 Gramme	=	15.444 Troy grains.

COMPOUND ADDITION.

1. Place numbers of the same kind under each other, as pence under pence, shillings under shillings, and pounds under pounds.

2. Find the sum of the figures in the right hand column, which divide by as many of that name as make one of the next higher denomination.

3. Set down the remainder under the column added, and carry the quotient to the next column.

4. Proceed in the same manner through all the denominations.

EXAMPLES.

	£.	s.	d.	£.	s.	d.	£.	s.	d.
	76	18	10¾	74	17	7¼	447	19	6¼
	89	19	7½	68	19	6½	869	16	11½
	68	15	8¾	99	18	11¾	353	17	9¾
	47	12	11¼	74	16	8½	529	10	10½
	36	19	9½	48	13	7½	746	12	8½
	99	8	1¾	95	15	9¼	975	16	7½
	68	17	6½	64	14	8¾	463	19	6¼
	27	16	7¼	17	17	11½	859	17	4¾
Sum	£516	9	3¼	£545	14	11	£5247	11	4½

1. In adding the shillings, first find the sum of the column of units, then put down the right hand figure of the amount, and carry the rest to the tens, add the tens, and for every 2 in the sum, carry 1 to the pounds: if 1 remain, put it down on the left

of the figure already put down. This method will be found much easier than adding all the shillings into one sum, and then dividing by 20 to find the pounds.

2. This manner of proceeding may be followed when 1 is to be carried for every 40, 60, &c. for it is only carrying 1 for every 4, 6, &c. in the amount of the column of tens.

3. In adding grains in Troy weight, pounds in Avoirdupois weight, and the like, the learner at first should add the columns of units and tens separately, till he can do that readily, but after some practice, he should accustom himself to add both together.

4. Pounds, shillings, pence, and farthings, may be added at once, without summing them in separate columns, but this method cannot be much recommended, either for accuracy or expedition.

5. As the adding of columns of pence, shillings, &c. occurs so frequently in business, the student ought to practice this operation till he can perform it accurately and readily*.

EXERCISES.

1.			2.			3.		
£.	s.	d.	£.	s.	d.	£.	s.	d.
9	10	6	8	10	9¼	16	8	5¾
5	7	8	2	17	5½	95	17	3¾
7	12	5	4	12	8¾	47	19	9¼
3	16	4	9	16	6½	24	6	4½
9	11	8	8	19	10¼	89	16	8¼
4	19	10	7	13	3¾	78	15	10¼
5	16	8	2	17	8	65	14	9½
9	12	7	3	14	7½	87	17	7¾
8	15	6	9	8	9¼	95	19	8¼
4	9	11	8	9	8½	88	16	5¾

* The observations on Addition, at pages 9 and 10, are equally applicable here.

4. £.	s.	d.	5. £.	s.	d.	6. £.	s.	d.
45	9	10½	91	14	10¾	437	16	8
96	18	5¾	85	17	8¼	285	19	10
87	16	4¼	29	18	7½	394	18	11¾
79	15	8¼	36	16	4¾	687	14	9¼
95	19	7½	98	17	11½	976	15	4¼
38	17	5¾	47	13	3¾	259	19	7½
47	18	6½	59	14	4¾	927	18	4¾
99	12	3¾	93	15	5½	895	17	3½
58	15	11¼	86	19	9½	467	19	11½
36	16	6½	25	16	6	996	18	10¼

TROY WEIGHT.

7. lb.	oz.	dwt.	gr.	8. lb.	oz.	dwt.	gr.
18	9	13	12	94	10	18	23
65	10	15	17	87	9	17	19
87	8	16	21	35	8	16	18
96	7	18	19	49	7	19	19
43	6	14	18	58	9	17	14
96	3	19	19	89	8	14	17
28	5	13	14	35	9	18	23
39	7	11	18	48	7	19	11
56	8	15	19	69	5	17	19
97	9	14	16	27	6	14	22
28	5	13	14	75	8	15	18

AVOIRDUPOIS WEIGHT.

9. tons.	cwt.	qr.	lb.	oz.	dr.	10. tons.	cwt.	qr.	lb.	oz.	dr.
47	18	1	25	11	12	99	19	3	27	14	9
89	17	3	19	14	13	67	13	1	18	9	2
58	19	2	24	9	8	85	17	2	19	12	0
97	16	1	18	8	15	48	18	3	25	8	8
58	17	2	17	13	9	59	14	1	21	9	7
97	15	3	26	10	8	98	15	2	19	5	6
35	18	1	19	8	7	45	18	2	23	8	9
28	16	2	17	9	9	63	17	1	19	7	11
87	15	3	23	13	15	57	16	3	17	11	15
49	19	1	18	8	8	49	18	2	18	9	8
58	16	2	19	12	13	56	19	3	24	11	12

CLOTH MEASURE.

11.

yds.	*qr.*	*nails.*
817	2	1
598	1	2
385	2	3
597	3	1
386	2	2
975	3	3
499	1	2
684	3	3
947	2	1
286	3	2

12.

Eng. ells.	*qr.*	*nails.*
618	2	2
765	3	1
389	2	3
978	3	3
857	2	1
939	1	2
386	3	1
976	4	3
468	3	2
987	2	3

WINE MEASURE.

13.

tuns.	*hhd.*	*gal.*	*pt.*
86	3	45	7
39	2	57	6
47	1	39	5
96	3	55	7
49	2	54	6
87	3	48	5
65	2	59	6
96	3	61	7
99	3	58	7
58	3	49	6

14.

tuns.	*hhd.*	*gal.*	*pt.*
43	3	33	4
16	2	29	5
86	2	17	6
97	3	28	7
89	2	35	5
46	3	19	6
55	3	24	6
94	3	35	5
79	2	54	7
56	3	48	5

DRY MEASURE.

15.

qr.	*bus.*	*peck.*	*pt.*
17	7	3	14
89	6	2	15
46	5	3	12
96	4	1	8
87	6	3	9
45	7	2	8
96	5	3	15
87	3	1	14
49	5	1	11
68	7	3	10
86	7	3	9

16.

qr.	*bus.*	*peck.*	*pt.*
75	2	1	15
47	1	2	14
98	3	3	9
46	7	2	8
57	6	3	12
95	5	2	15
87	7	3	14
79	6	3	12
86	5	1	9
97	7	2	8
86	5	3	14

COMPOUND SUBTRACTION.

RULE.

1. Place the denominations of the same kind under each other.

2. Proceed, as in simple subtraction, to subtract the lower denominations from those above them; only, when a number in any of the denominations exceeds that above it, borrow as many of that name as make *one* of the next higher, from which deduct the under number, and add the upper figure to the remainder; set down this sum, and carry 1 to the under number of the next higher denomination, before subtracting it from the one above it.

EXAMPLES.

	1.			2.			3.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
From	65	18	9	160	0	9	368	12	3
Sub.	41	14	6	83	10	11	109	18	$10\frac{1}{4}$
Rem.	£24	4	3	£76	9	10	£258	13	$4\frac{3}{4}$

1. As custom has established the method of placing the *subtrahend* under the *minuend*, it should be followed, when there is no reason for doing otherwise: but the *minuend* may be placed under the subtrahend with equal propriety, and the student ought to be capable of working either way with equal readiness, as the last method is sometimes more convenient: of which examples will frequently occur in the course of this work.

2. The student should also acquire the habit, when two numbers are marked down, of placing such a number *under* the *les-*

ser; that, when they are added together, the sum may be equal to the greater. The operation is the same as subtraction, though conceived in a different manner, and is useful in balancing accounts, and on some other occasions.

EXERCISES.

	1.			2.			3.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
From	416	9	$10\frac{1}{2}$	20	8	$7\frac{1}{4}$	847	19	$6\frac{1}{4}$
Sub.	387	5	$9\frac{3}{4}$	6	18	$9\frac{1}{4}$	358	13	$10\frac{1}{2}$
Rem.									

	4.			5.			6.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
From	953	12	$9\frac{3}{4}$	99	10	$3\frac{1}{2}$	423	16	2
Sub.	476	14	$10\frac{1}{4}$	25	7	$1\frac{1}{4}$	196	19	$6\frac{1}{2}$
Rem.									

	7.			8.			9.		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
From	1000	0	0	04	9	$11\frac{1}{4}$	78	3	$5\frac{1}{4}$
Sub.	356	15	$10\frac{3}{4}$	28	13	$8\frac{1}{2}$	29	14	$8\frac{1}{2}$
Rem.									

TROY WEIGHT.

	10.	11.	12.
	lb. oz. dwt. gr.	*lb. oz. dwt. gr.*	*lb. oz. dwt. gr.*
From	75 . 10 . 16 . 20	100 . 0 . 10 . 12	21 . 8 . 10 . 12
Sub.	40 . 11 . 18 . 6	75 . 6 . 12 . 21	12 . 10 . 15 . 23
Rem.			

AVOIRDUPOIS WEIGHT.

	13.	14.	15.
	lb. oz. dr.	*cwt. qr. lb.*	*ton cwt. lb.*
From	120 . 10 . 12	370 . 2 . 10	144 . 4 . 21
Sub.	17 . 10 . 15	195 . 3 . 21	97 . 14 . 24
Rem.			

CLOTH MEASURE.

	16.	17.	18.
	yds. qr. na.	*yds. qr. na. in.*	*yds. qr. na.*
From	94 . 3 . 2	32 . 2 . 3 . 1	9 . 1 . 1
Sub.	78 . 2 . 3	16 . 3 . 0 . 2	4 . 3 . 3
Rem.			

WINE MEASURE.

	19.	20.	21.
	hhds. gal. p.	*tuns hhds. gal.*	*tuns hhds. gal.*
From	67 . 59 . 4	110 . 2 . 40	70 . 1 . 17
Sub.	54 . 62 . 7	78 . 2 . 52	19 . 2 . 29
Rem.			

DRY MEASURE.

	22.	23.	24.
	qrs. bus. gal.	*qrs. bus. gal.*	*qrs. bus. pks. pint*
From	15 . 3 . 5	294 . 6 . 2	877 . 3 . 3 . 10
Sub.	6 . 7 . 6	793 . 4 . 3	345 . 5 . 2 . 12
Rem.			

Before the student proceeds to arrange the following exercises, it will often be necessary to place the sums in different

columns, in order to exhibit a clear view of what is required. For example, if the values of several parcels of goods are to be added, and each parcel consists of several articles, the particular article should be placed in an inner column, and the sum of of each parcel extended to the outer column, which, added, gives th total.

If any person be owing an account and has made partial payments, the payments must be placed in an inner column, and their sum extended under that of the account, in the outer column, and subtracted there.

The two following examples will make this quite plain :

EXAMPLE I.

40 yards of cloth, at 1s. 6d.	£3	0	0				
50 ditto, at 2s. 4d.	5	16	8				
				£8	16	8	
100 lb. of thread, at 8s.	£40	0	0				
40 lb. ditto, at 6s. 3d.	12	10	0				
30 lb. ditto, at 5s. 1d.	7	12	6				
				60	2	6	
				£68	19	2	

EXAMPLE II.

May 4, Lent Chas. Carey	£50	0	0			
June 7, Ditto	70	0	0			
				£120	0	0
Aug. 9, Received in part	£62	0	0			
Sept. 4, Do. 4 cwt. of flax, value	10	10	0			
Oct. 3, Do. 6 cwt. ditto, value	13	0	0			
				85	10	0
Balance due to me				£34	10	0

The convenience of such an arrangement is obvious. Sometimes three or four more columns are necessary. The *right* hand column is called the *outer* column; the next the *inner*, or *first* inner column; the next, the *second inner*, and so on.

MISCELLANEOUS EXERCISES.

1. Borrowed £15 15s., whereof I paid at one time £4 10s., at another, £2 5s. 8d.; at another 5 guineas; and, at another, 1 guinea and a half How much is still due?

2. Lent, at one time, £12 17s. 6d.; and, at another, £31 8s. 6d.; whereof I have received, at different times, the following sums: viz. £7 10s.; £5 9s. 6d.; £8 12s. 8d.; and 4 guineas and a half. How much is still due to me?

3. Sold a grocer 2 pieces of linen, amounting to £7 13s. 7½d., and have got from him tea amounting to £1 9s.; sugar to £1 2s. 7½d.; soap to 5s. 4d; porter to 9s. 6d.; and a five-pound note to pay the balance. How much ought he to get back?

4. Required the net weight of 4 hogsheads of sugar, weighing as follows: viz. - No. 1. Gross, 9 cwt. 1 qr. 25 lb.; tare, 1 cwt. 1 qr. 11 lb.—No. 2 9 cwt. 0 qr. 27 lb.; tare, 1 cwt. 0 qr. 27 lb.—No. 3. 9 cwt. 2 qr. 19lb.; tare, 1 cwt. 1 qr.—No. 4. 9 cwt. 1 qr. 26 lb; tare, 1 cwt. 1 qr. 7 lb.

5. Required the net weight of 6 hhds of tobacco, weighing as follows:—No. 1. 18 cwt. 1 qr. 10 lb.; tare, 1 cwt. 2 qr. 19lb. —No. 2. 19 cwt. 2 qr. 12 lb; tare, 1 cwt. 3 qr. 5 lb.—No. 3. 18 cwt. 1 qr. 10 lb.; tare, 1 cwt. 2 qr. 6 lb.—No. 4. 18 cwt. 1 qr. 14 lb.; tare, 1 cwt. 2 qr. 21 lb.—No. 5. 12 cwt. 3 qr. 26 lb.; tare, 1 cwt. 1 qr. 6 lb.—No. 6. 12 cwt. 3 qr. 5lb.; tare, 1 cwt. 1 qr. 15 lb.

6. From a piece of broad cloth, qt. 33 yards, 2 qr., there was sold to A, 3 yards, 3 qr. 2 n.; to B, 7 yards, 2 qr.; to C, 2 yds. 1 qr. 3 n.; to D, 3 yards, 3 qr. 1 n.; and to E, 6 yards, 1 qr. 2 n. How much should remain?

7. A merchant has in cash £37 13s. 5½d., in goods £1538 19s. 10¼d., a house worth £500, and debts due him £683 13s. 5¾d. He owes A £210 10s. 11d.; B £173 16s. 7d.; C £88 17s. 3d.; and D £48 12s. 11¾d. Required his net stock?

8. A bankrupt had in cash, £6 1s. 3d.; in recoverable debts, £147 19s. 8½d.; in goods, value £48 13s. 9d.; a house valued at 100 guineas, and furniture which sold for £79 16s. 8d. He owed to John Greig, £210 10s.; to James Watt, £45 6s. 8d.; to George Dun, £150 17s. 4½d.; to William Gray, £8 5s. 10¾d.;

to David Swan, £105. 4s. 3¼d.; to Thomas Reid, £33. 11s. 5¾d.; to Walter Lyon, £71. 19s. 7d.; and to Charles Walker, £28. 17s. 2¼d. How much did his creditors lose by him?

9. A merchant, on balancing his books, finds he has cash, £8. 17s. 9½d.; bills amounting to £283. 15s. 7¾d.; wines valued at £859. 13s. 4d.; brandy, £795. 16s. 3¼d.; indigo, £113. 14s. 5¼d.; rum, £146. 9s. 8¼d.; sugar, £377. 10s.; and outstanding debts, £728. 1s. 10¾d. He owes James King, £105. 7s. 6d.; Samuel Bell, £582. 10s. 9¼d.; John Brown, £55. 11s. 3¼d.; and Walter White, £317. 19s. 7¾d.; required his net stock?

10. Required the prime cost of 6 chests of congou tea, weighing as follows: No. 1. 1 cwt. 5 lb.—No. 2. 3 qr. 16 lb.—No. 3. 3 qr. 25 lb.—No. 4. 1 cwt. 6 lb.—No 5. 1 cwt. 8 lb.—and No. 6. 1 cwt. 4 lb. (25 lb. of tare, and 1 lb. of draft being allowed on each chest,) at 6s. per lb. net.

11. A merchant became bankrupt who owed the following sums: To George Palmer, per account, £427. 13s. 9d.; Chas. Ranken, per ditto, £716. 12s. 3d.; John Forbes, per ditto, £814. 17s. 9d.; William Wright, per ditto, £219. 18s. 4d.; and the following bills: To Richard Massy, £300. 17s. 2d.; to John Black, £512.; to David Stephenson, £400.; to Charles Cramer, £1260.; also a bond to Thomas Higgen, for £2000. His effects were the following: accounts due to him by John Vanderpot, £42. 19s. 4d.; by Mandalino Costantini, £15. 12s. 6d.; by Thomas Daintry, £207. 3s. 3d.; by John Airth, £416. 6s. 6d.; by David Young, £213. 6s. 2d. Bills due by Charles M'Indoe, £107.; by David Carey, £403. 2s. 6d.; by William Shand, £714. 3s. 2d.; by Robert Gordon, £100., whereof £30. was paid; by Richard Evans, £48. A house, value £520.; another, value £416.; household furniture, £1191. 15s. 5d.; books, £25. 10s. 3d.; and the following goods: cloth, value £176. 9s.; linens, which were sold in 3 lots for £92. 14s. 6d., £72., and £58. 3s. 9d. Six hogsheads of Port wine, at £15. 15s. each; 456 lb. of flax at 1s. per lb.; and sundry other articles, to the value of £98. 14s. Required a state of his affairs, distinguishing each kind of debt and property; and the deficiency, allowing £40. for the expenses of settling them?

COMPOUND MULTIPLICATION.

CASE I.

WHEN THE MULTIPLIER DOES NOT EXCEED 12.

RULE.

PLACE the multiplier under the lowest denomination of the multiplicand, multiply each denomination by the multiplier, and carry as in addition of the same name.

EXAMPLE I.

	£.	s.	d.	
Multiply	43	14	9¼	by 7.
			7	
	£306	3	4¾	

EXAMPLE II.

	cwt.	qrs.	lb.	
Multiply	7	2	13½	by 5.
			5	
	cwt. 38	0	11½	

EXERCISES.

1. What cost 3 boxes of indigo at £25. 18s. 10½d.?
2. What cost 4 cwt. of wax at £11. 15s. 8¾d.?
3. What cost 9 yards of cloth at £2. 3s. 4¼d.?
4. What cost 7 cwt. of soda at £9. 7s. 7½d.?
5. What cost 11 cwt. of nitre at £3. 14s. 8½d.?
6. What cost 5 barrels of tar at £1. 4s. 9½d.?
7. What cost 10 gallons of rum at £3. 9s. 9½d.?
8. Multiply 25 lb., 5 oz., 16 dwt., 11 gr. by 2.

9. Multiply 5 tons, 12 cwt., 1 qr., 13 lb., 9 oz. by 3.

10. Multiply 17 yards, 2 qrs., 1 nail, by 5.

11. Multiply 45 qrs., 3 bus., 2 pecks, 11 pints, by 10.

CASE II.

WHEN THE MULTIPLIER IS A COMPOSITE NUMBER, WHOSE COMPONENT PARTS ARE UNDER 12.

RULE.

Multiply by one of the component parts, and that product by the other*.

EXAMPLE I.

	£.	s.	d.	
Multiply	11	16	8	by 40.
			10	
	118	6	8	
			4	
	£473	6	8	

EXAMPLE II.

		cwt.	qr.	lb.	
Multiply		12	1	27	by 21.
				7	
	4	7	1	21	
				3	
	T. 13	2	1	7	

The factors may be taken in any order; but, on some occasions, one or more of the lower denominations may be exterminated at the first multiplication, by using one of the factors, which would not be the case by beginning the multiplication with another.

The multiplier may sometimes be divided into *different factors*, one of which will effect the extermination of several of the lower denominations at the first multiplication, when another will not.

Thus, 36 may be divided into the factors 12 and 3, or 9 and 4; but it would be more proper to employ 12 in the first multiplication, in the following examples, than 9 or 4.

* When the number is composed of the product of three others, the *factors* are not so easily discovered; therefore this rule is chiefly employed to obtain the product arising from a number composed of two factors.

EXAMPLE I.

	£	s.	d.	
Multiply	7	5	5	by 36.
			12	
	87	5	0	
			3	
	£261	15	0	

EXAMPLE II.

	lb.	oz.	dwt.	gr.
Multiply	4	7	10	20
				12
	55	6	10	0
				3
	lb. 165	19	10	0

The operation may be proved by dividing the multiplier into other factors, when this is possible, or by multiplying by the factors in a different order.

EXERCISES.

1. What cost 15 gallons of rum at 11s. 7¾d.?
2. What cost 18 lb. of indigo at 9s. 8¼d.?
3. What cost 28 yards of cloth at £2. 7s. 6½d.?
4. What cost 36 cwt. of flax at £3. 18s. 8¼d.?
5. What cost 60 tons of hemp at £65. 15s. 6d.?
6. What cost 88 cwt. of saltpetre at £3. 14s. 9d.?
7. What cost 120 cwt. of madder at £5. 4s. 9d.?
8. What cost 132 gals. of brandy at 15s. 7d.?
9. Multiply 13 cwt., 3 qr., 13 lb. by 48.
10. Multiply 4 lb., 10 oz., 12 dwt., 15 gr. by 18.
11. Multiply 417 acres, 2 r., 13 p., 15 yards by 54.
12. Multiply 9 tuns, 3 hhds., 18 g., 7 p., by 64.
13. Multiply 7 qrs., 3 bus., 2 pecks, 6 pints, by 132.
14. Multiply 1 year, 57 days, 15 hours, 9 min., by 81.

CASE III.

WHEN THE MULTIPLIER IS A PRIME NUMBER, AND LESS THAN 156.

RULE.

Multiply by the factors of the composite number nearest to the given number, whether above or below it (as directed in last case), and then the *first line* by the *difference*, this last product, *added* to the product of the composite number, when it is *less* than the given number, or *subtracted* from it when it is *greater* than the given number, gives the answer.

EXAMPLE I.

	£.	s.	d.	
Multiply	35	17	9	by 67.
			8	
	287	2	0	
			8	
	2296	16	0	=64
	107	13	3	= 3
	£2404	9	3	—67

EXAMPLE II.

	lb.	oz.	dwt.	
Multiply	2	11	15	by 19.
			10	
	29	9	10	
			2	
	59	7	0	=20
	2	11	15	= 1
	lb. 56	7	5	=19

EXERCISES.

1. What cost 38 yards of cloth at £2. 3s. 4d.?
2. What cost 34 acres at £732. 9s. 4d.?
3. What cost 29 yards of cloth at £2. 2s. 2½d.?
4. What cost 43 yards ditto at 3s. 10d.?
5. What cost 75 bales of linen at £59. 7s. 11¼d.?
6. What cost 117 cwt. of sugar at £3. 7s. 10¾d.?
7. Multiply 15 cwt. 2 qr. 11 lb. by 112.
8. Multiply 47 tons, 9 cwt., 1 qr. 25 lb. by 136.
9. Multiply 7 lb., 5 oz., 4 dwt., 7 gr. by 142.
10. Multiply 9 acres, 2 roods, 5¼ p. by 149.

CASE IV.

WHEN THE MULTIPLIER EXCEEDS 156.

RULE.

Multiply by 10 as often as there are figures in the multiplier to the left of units; then multiply the last product by the figure of the highest value in the multiplier, the preceding product by the next inferior figure of the multiplier, and so on with the remaining figures of the multiplier. The sum of all these products, obtained by the respective figures of the multiplier, is the answer.

EXAMPLE.

What cost 4567 yards of cloth at 3s. $5\frac{1}{2}$d per yard?

£.	s.	d.		£.	s.	d.		
0	3	$5\frac{1}{2}$	× 7 =	1	4	$2\frac{1}{2}$	price of	7 yards.
		10						
1	14	7	× 6 =	10	7	6	——	60 yards.
		10						
17	5	10	× 5 =	86	9	2	——	500 yards.
		10						
172	18	4	× 4 =	691	13	4	——	4000 yards.
				£789	14	$2\frac{1}{2}$	price of	4567 yards.

EXERCISES.

1. What cost 675 lb. of tea, at 14s. $7\frac{1}{4}$d.?
2. What cost 957 lb. of pimento, at 1s. $1\frac{1}{4}$d.?
3. What cost 3456 lb. of indigo, at 11s. $6\frac{1}{2}$d.?
4. What cost 2758 yards of linen, at 4s. $6\frac{1}{2}$d.?
5. Multiply 85 tons, 15 cwt. 1 quarter, 21 lb. by 478?
6. Multiply 3 dwt. 5 grains troy, by 5555?

7. Multiply 314 qrs. 6 bushels, 3 pecks, by 4083?

8. Multiply 5 dwt. 6 grains troy, by 1250.

MISCELLANEOUS EXERCISES.

1. What is the value of 8 pieces of cloth, at £5. 17s. 8d. per piece?

2. What is the value of 48 bolls of wheat, at 16s. 8d. per boll?

3. What is the value of 72 bags of cotton, at £17. 14s. 8d. per bag?

4. What is the value of 34 pieces of silk, at £9. 18s. 8d. per piece?

5. What is the value of 23 hhds. of wine, at £15. 4s. 9d. per hogshead?

6. What is the value of 758 qrs. of barley, at £1. 3s. 5d. per qr.?

7. What is the value of 36 bolls of wheat, at 17s. 6d. per boll?

8. What is the value of 72 yards of cloth, at 19s. 4d. per yard?

9. What is the value of 17 bales of wool, at £14. 8s. 9d. per bale?

10. What is the value of 625 cattle, at £3. 12s. 4d?

11. What is the value of 2758 yards of linen, at 4s. 6½d. per yard?

12. What is the weight of 7 hhds. at 3 cwt. 2 qrs. 12 lb. each?

13. What is the weight of 24 ditto, at 7 cwt. 2 qr. 2 lb?

14. What is the weight of 37 ditto, at 9 cwt. 3 qr. 11 lb?

15. What is the weight of 119 ditto, at 3 cwt. 2 qr. 15 lb?

16. Gold, at the mint price, is worth £46. 14s. 6d. per lb.; what is that per grain?

17. How much will the wages of 13 men amount to, in seven weeks, at 1s. 10d. per day, each?

18. A person spends 18s. 4½d. per day, and saves 50 guineas a year; what is his income?

19. A person's yearly income is £150, and he spends 3 guineas per week; whether does he save money or run into debt, and how much in the year?

20. By retailing a puncheon of rum, containing 105 gallons, at 14s. 6d., a vintner cleared 20 guineas; what did it cost him?

21. How much does a bankrupt pay a creditor, to whom he owes £500, at 13s. 4d. a pound?

22. How much cloth in 37 pieces, each 25 yds. 2 qrs. 3 nails?

23. What is the weight of 100 guineas, at 5 dwt. 9½ grains?

24. What is the weight of 29 casks, at 8 cwt. 1 qr. 19 lb. each?

25. What is the capital of a trading company, consisting of 180 shares of £1388. 12s. 2¾d.?

26. If the interest of the national debt be 16s. 1½d. each second of time, how much does it amount to in a year?

27. A mill throws off 73726 yards of silk every time the water-wheel turns round, which is 3 times in a minute, for 10 hours every lawful day; required the produce, at this rate, in a day?

Compound Multiplication is one of the most useful rules in Commercial Arithmetic, for by it may be calculated the price, the weight, the measure, &c. of any number of articles in the simplest, *and often* in the easiest, manner possible.

The price of one article is made the multiplicand and the number of articles the multiplier. As the multiplier expresses the number of articles, (or the number of times the multiplicand is to be added to itself,) it is always an abstract number, and has no reference to any value or measure whatever.

The following are the most common cases in which this rule is employed:

1. To find the value of any number of articles, as tons, yards, &c. multiply the price of one of the articles by the number of articles.

2. To find the weight or measure of any number of articles, multiply the weight or measure of a single article by the number of them.

3. To find the amount of work performed, interest incurred, provisions consumed, or any other thing which increases by length of time, multiply the given rate, per day, month, or year, by the number of years, months, or days.

COMPOUND DIVISION.

CASE I.

WHEN THE DIVIDEND ONLY CONSISTS OF SEVERAL DENOMINATIONS.

RULE.

PLACE the divisor on the left hand of the dividend, as in Simple Division: then divide the highest denomination of the dividend by the divisor, and reduce the remainder, if any, to the next inferior denomination, adding to it the parts of the same name in the dividend: divide this number as before, and proceed in the same manner through all the denominations, to the lowest, and the different denominations in the quotient will be the answer required.

2. When the divisor does not exceed 12, the division may be performed mentally, and the different denominations of the quotient placed under the same denominations in the dividend.

3. When the divisor is a composite number, the factors of which do not exceed *twelve,* divide by the factors successively, and the last quotient will be the answer. (See page 20.)

EXAMPLE I.

Divide £1346. 14s. 10½d. by 38.

```
    £    s.  d.
38)1346  14  10½( £35. 8s. 9¼d.
   114
   ———
    206
    190
    ———
     16
      20
     ———
     334
     304
     ———
      30
       12
      ———
       370
       342
       ———
        28
         4
       ———
       114
       114
       ———
```

EXAMPLE II.

Divide 138 cwt. 2 qr. 17 lb. by 41.

```
                  cwt. qr. lb.
41)138  2  17(3    1   14
   123
   ———
    15
     4
   ———
    62
    41
   ———
    21
    28
   ———
   175
   43
   ———
   605
   41
   ———
   195
   164
   ———
    31 Remainder.
   ———
```

EXAMPLE III.

Divide £429, 7s. 5½ by 9.

```
   £     s.   d.
9)429    7    5½
  ———————————————
£ 47    14    1¾—⅞
  ———————————————
```

EXAMPLE IV.

Divide 17 tons, 8 cwt. 3 qr. 14 lb. by 42.

```
        tons cwt. qr. lb.
     ( 7 | 17   8   3   14
  42 {  —|———————————————
     ( 6 |  2   9   3   10
           ———————————————
            0   8   1   6⅓
           ———————————————
```

EXERCISES.

		£.	s.	d.		
1.	Divide	169	15	8¼	by	3.
2.	Divide	659	8	1½	by	5.
3.	Divide	823	12	0	by	7.
4.	Divide	100	10	0	by	9.
5.	Divide	8	16	6	by	12.
6.	Divide	178	16	10¾	by	12.
7.	Divide	2626	18	2	by	19.
8.	Divide	321	17	0½	by	47.
9.	Divide	418	13	2¾	by	65.
10.	Divide	103	14	2¼	by	231.
11.	Divide	1010	14	1¾	by	32.
12.	Divide	576	18	7¼	by	48.
13.	Divide	629	6	3	by	144.

14. Divide 8 tons, 11 cwt. 2 qr. 14 lb. by 9.

15. Divide 14 lb. 11 oz. 9 dwt. 10 gr. by 8.

16. Divide 5781 qr. 3 bush. 3 pecks, 10 pl. by 84.

17. Divide 127 acres, 2 roods, 4 poles, by 72.

18. Divide 1109 cwt. 3 qr. 20 lb. by 81.

19. Divide 294 years, 146 days, by 73.

20. Divide 3168 miles, 30 poles, by 98.

CASE II.

WHEN THE DIVISOR CONSISTS OF SEVERAL DENOMINATIONS.

RULE.

Reduce both the divisor and dividend to the lowest denomination given in either; then proceed, as in Simple Division, and

the quotient will show how many times the divisor is contained in the dividend*.

EXAMPLE I.

Divide £229. 16s. 9d. by £8. 10s. 3d.

```
                 £   s. d.
£8 10   3)229  16   9
  20        20
 ----      ----
 170       4596
  12         12
 ----     -----
 2043     )55161 (27 Answer.
           4086
          -----
           14301
           14301
          -----
```

EXAMPLE II.

Divide 96 cwt. 1 qr. 20 lb. by 2 cwt. 2 qr. 20 lb.

```
cwt. qr.  lb.          cwt. qr. lb.
 2    2   20            96   1   20
 4                       4
 --                     ----
 10                     385
  28                     28
 ---                    ----
 100                    3100
 20                     770
 ----                  ------
  3,00                 )108,00(36 cwt. Answer.
                         9
                        ---
                         18
                         18
                        ---
```

* It is best not to reduce the terms lower than is necessary to make the divisor and dividend of the same denomination. In Example I. it was unnecessary to reduce the divisor and dividend lower than threepences, as each of them consists of an exact number of threepences.

EXERCISES.

1. Divide £13403. 19s. 4d. by £35 7s. 4d.
2. Divide £1055. 15s. 6d. by £25 2s. 9d.
3. Divide £413. 12s. 4d. by $6\frac{7}{8}$.
4. Divide £1589. 12s. $7\frac{1}{4}$d. by $29\frac{4}{5}$.
5. Divide 115 cwt. 2 qr. 13 lb. by 9 cwt. 1 qr. 19 lb.
6. Divide 537 lb. 6 oz. 3 dwt. by 7 lb. 5 oz. 11 dwt. 11 gr.

The use of division is to find either of the factors, by the multiplication of which a given number is produced, when the other factor is given; and is, therefore, the opposite of multiplication. In arithmetic it is of very general application; and, therefore, it is necessary, for those who study arithmetic, to perform it readily in the various examples to which it may be applied. To assist the student in solving questions by this rule, the following directions have been subjoined.

1. When the value, weight, or measure, of any number of articles are given, to find the value, weight, or measure of a *single* article; divide the *whole* value, weight, &c. by the *whole* number of articles: or, to find the *number* of articles, divide the value, weight, or measure of the *whole* by that of a *single* article.

2. To find how much work is performed, provisions consumed, interest incurred, &c., in a single day, (or any given portion of time,) divide the *whole* work, consumption, interest, &c., by the *number* of days: or, to find the *time*, divide the whole amount by the daily allowance.

3. To find the *length* of a rectangular surface, divide the *area*, or superficial measure, by the *breadth*; or, to find the *breadth*, divide the *area* by the *length*.

4. To find the area of the *base* of a rectangular solid, divide the *solid* measure by the *height*; or, to find the *height*, divide the solid measure by the *area* of the *base*.

The operations performed by Compound Division may be proved in the same manner as those in Simple Division. See page 28.

MISCELLANEOUS EXERCISES.

1. Divide £452 16s. $4\frac{3}{4}$d. among 17 men.

2. What is the value of 1 lb. of sugar, at 86s. 6d. per cwt?

3. What is the price of 1 lb. of white lead, at £40 per ton?

4. What is the price of 1 lb. of rice, at £1 12s. 6d. per cwt.?

5. What will the wages of 13 men, for 7 weeks, amount to, at 1s. 10d. per day, each?

6. A person's income is £96 per annum, and he spends 22s. $3\frac{1}{2}$d. per week; how much does he save yearly?

7. Bought 3 cwt. 3 qr. 12 lb. of cotton wool, for £62 10s., what did it cost per lb.?

8. How many yards of cloth, at 11s. 3d. per yard, may be bought for £11 10s. $7\frac{1}{2}$d.?

9. The earth revolves round its axis, from west to east, in 23 h. 56 min. 4 sec., how many miles an hour are the inhabitants of Edinburgh carried by this motion: the circumference of the earth, on the parallel of Edinburgh, being 13911 English miles?

10. By retailing a puncheon of rum, qt. 105 gallons, at 14s. 6d., a merchant cleared 20 guineas; what did it cost him a gallon?

11. A grocer bought 8 cwt. 1 qr. 7 lb. of sugar for £40 14s. $7\frac{1}{2}$d.; of which he sold 2 cwt. 17 lb. at 1s.; but the markets falling, he got only $9\frac{3}{4}$d. a lb. for the rest. Required his gain or loss?

12. Divide 300 guineas among 5 men, 3 women, and 7 boys, so that each man may have half as much again as each woman, and each woman twice as much as each boy.

13. Divide 1000 guineas among A, B, and C; so that A may have 156 more than B, and B 62 less than C.

14. By selling barley at 30s. a boll, a merchant cleared $\frac{1}{4}$ of what it cost him; what did he pay for it a boll?

15. By selling flax at £72 12s. a ton, a merchant cleared $\frac{1}{7}$ of what it cost him; what did he pay for it a ton?

16. By selling a certain commodity at £6, a merchant cleared $\frac{7}{9}$ of what it cost him; what did he pay for it?

17. A privateer makes a prize, which, after charges are deducted, amounts to £5876 13s. 4d. How must this sum be divided among the crew, which consisted of a captain, 3 officers, and 36 privates; allowing the captain $\frac{1}{8}$, each officer $\frac{1}{24}$ of the remainder, and the rest to be equally divided among the private men?

18. A hogshead of tobacco, placed at the distance of 2 inches from the fulcrum of a steelyard, is equipoised by a weight of 32 lb., hung at 96 inches distance, on the other side. Required the weight of the hogshead?

19. A grocer bought 2 hogsheads of sugar, weighing net, 24 cwt. 2 qr. 16 lb., for £103 10s.; their difference in weight was 1 cwt. 3 qr. 6 lb., and in value £7 11s. 6d. Required the weight and value of each?

20. A vintner buys 20 gallons of spirits, at 9s. per gallon, which he mixed with a certain quantity of water, and then sold the mixture for 9s. 4d. per gallon, by which he gained £2 4s. on the whole. How much water did he put in?

REDUCTION.

Reduction is the changing of numbers from one denomination to another, without altering their value.

The rule for performing operations of this kind, admits of three *varieties*; viz. to bring numbers from a *higher* to a *lower*; from a *lower* to a *higher* denomination; and when the *higher* denomination does not contain an *exact* number of the *lower*.

CASE I.

TO BRING NUMBERS FROM A HIGHER TO A LOWER DENOMINATION.

RULE.

Multiply the highest denomination by the number which will reduce it to the next lower denomination, adding to the product the number of the same name to which the highest is reduced, if any: continue this mode of proceeding till the given denominations be reduced to the denomination required.

EXAMPLE I.

Reduce £154 18s. 6½d. to farthings.

	£154 18s. 6½d.	
	20	
(add 18s.)	3098	shillings
	12	
(add 6d.)	37182	pence
	4	
(add ½d.)	148730	farthings

EXAMPLE II.

Reduce 1789 guineas to pence.

1789	
21	
1789	
3578	
37569	shillings
12	
450828	pence.

EXAMPLE III.

Reduce 3 tons, 8 cwt. 1 qr. 16 lb. to pounds.

	tons cwt. qr. lb.			*cwt. qr. lb.*
Method 1st.	3 . 8 . 1 . 16		Method 2d.	68 . 1 . 16
	20			68
				68
(add 8 cwt.)	68	cwts.	(add 1 qr. 16 lb.)	6844
	4			
				7660 lb*.
(add 1 qr.)	273	qrs.		
	28			
	2200			
	546			
(add 16 lb.)	7660 lb.			

EXERCISES.

1. Reduce £412 19s. 7¼d. to farthings.
2. Reduce £69 12s. 4½d. to halfpence.
3. Reduce 57 guineas to pence.
4. Reduce £9 13s. 2d. to twopences.
5. Reduce £31 16s. 6d. to threepences.
6. Reduce £76 19s. 8d. to fourpences.
7. Reduce £913. 12s. 6d. to sixpences.
8. Reduce 1737 crowns to pence.
9. In 9 lb. 8 oz. 9 dwt. 11 gr. how many grains?
10. In 8 lb. 11 dwt. how many pennyweights?

---

* This method of reducing cwts. to lbs. is easier, and the arrangement neater, than by Method 1st. The reason of this arrangement producing the same result as Method 1st is, that 1 cwt. being equal to 112 lb. any number of cwts., placed under *itself* and added, produces its double; if wrote one place to the *left*, it is *ten* times greater; if *two places*, it is *one hundred* times greater; therefore, if any number be wrote under itself *three times*, the *first* time directly under itself, the second time removed *one place* to the *left*, and the third *two places* to the *left*, the sum of the whole will be 112 times greater than that number.

11. Reduce 41 lb. 1 oz. 2 dr. to grains.

12. Reduce 8 tons, 11 cwt. to pounds.

13. Reduce 15 cwt. 2 qr. 15 lb. to pounds.

14. Reduce 47 yards, 1 qr. 2 nails, to nails.

15. Reduce 7 miles, 2 furlongs, 17 yards, to inches.

16. Reduce 5 tuns of wine to pints.

17. Reduce 5 bushels, 3 pecks, to pints.

18. Reduce 15 reams of paper to sheets.

19. Reduce 10 solar years to seconds.

20. Reduce 365 days, 5 hours, 48 minutes, to seconds.

CASE II.

TO BRING LOWER DENOMINATIONS TO HIGHER.

RULE.

Divide the given denomination by as many of that name as make *one* of the next higher; continue to divide the quotient in this manner till the given denominations be brought to the one required.

EXAMPLE I.

In 31251 farthings, how many pounds?

```
   4)31251          or,     96,0)3125,1(£32 11s. 0¼d.
    -------                      288
  12) 7812—3                     ----
    -------                       245
 2,0)  65,1—0¼                    192
    ----------                    ----
      £32 11 0¼             48)  531(11s.
    ----------                    48
                                  ---
                                   51
                                   48
                                  ---
                                   3 farthings.
```

EXAMPLE II.

In 753815 ounces, how many cwts.

```
   ⎧4)753815
16 ⎨ ———————
   ⎩4)188453—3
     ————————
   ⎧7) 47113—7
28 ⎨ ————————
   ⎩4)  6730—3
     ————————
  2,0)  168,2 17 7
       ——————————
  cwt.   84 2 17 7
       ——————————
```

EXERCISES.

1. In 51273 halfpence, how many pounds ?
2. In 98760 threepences, how many pounds ?
3. In 126090 fourpences, how many pounds ?
4. In 54325 sixpences, how many pounds ?
5. In 75000 Scots merks, how many pounds sterling ?
6. In 56050 grains of gold, how many oz. and lbs ?
7. In 45000 dwts. of silver, how many lbs.?
8. In 24960 drops, avoirdupois, how many lbs.?
9. In 132160 ounces, avoirdupois, how many lbs.?
10. 11025 lbs. how many cwts.?
11. In 10596 half-nails of cloth, how many yards ?
12. In 50500 inches, how many Flemish ells ?
13. In 180360 inches, how many feet and yards ?
14. In 36590 yards, how many English acres?
15. In 740355 cubic inches, how many wine gallons ?
16. In 89616 pints of wine, how many quarts and gallons ?
17. In 8409600 minutes, how many hours and days ?

CASE III.

WHEN THE HIGHER DENOMINATION DOES NOT CONTAIN AN EXACT NUMBER OF THE LOWER.

RULE.

Reduce both to the same name, and then divide the one by the other.

EXAMPLE I.

In 8643 guineas, how many pounds?

Method 1st.

```
      8643
        21
     -----
      8643
     17286
    ------
 2,0)18150,3
    --------
    £9075 3s.
```

Method 2d.

```
        864,3
 1/20 =  432 3
       -------
     £9075 3s.
```

EXAMPLE II.

In 1649 dollars, at 4s. 6d. each, how many guineas?

Method 1st.

```
         1649
            9
        -----
   ( 7)14841
42 {    -----
   ( 6) 2120—1
        --------
guineas  353—15/42|5/14
```

Method 2d. 4s. 6d. = 9 sixpences

21s. 0d. = 42 ditto

and $\frac{9}{42} = \frac{3}{14} = \frac{1\frac{1}{2}}{7}$

therefore, 1649

$\frac{1}{2} =$ $824\frac{1}{2}$

7)$2473\frac{1}{2}$

guineas $353\frac{5}{14}$

As short calculations are less liable to error than long ones, the denominations concerned should be reduced no *lower* than is absolutely necessary, as in Example II., Method 1st: the

divisor and dividend ought also to be *abridged*, when it is practicable, as in Example II., Method 2d. Operations may also be shortened by adding or subtracting some *aliquot part* of the given number, as in Example I., Method 2d; and in the following instances:

To bring any number of
Guineas to pounds add $\frac{1}{20}$ of the number to itself.
Guineas to moidores, deduct $\frac{1}{9}$.
Crowns to dollars, add $\frac{1}{9}$.
Dollars to crowns, deduct $\frac{1}{10}$.
Yards to English ells, deduct $\frac{1}{5}$.
Flemish ells to yards, deduct $\frac{1}{4}$.
English ells to yards, add $\frac{1}{4}$.
Troy lbs. to Avoirdupois lbs., deduct $\frac{1}{6}$.
Avoirdupois lbs. to Troy lbs., add $\frac{1}{5}$.

EXERCISES.

1. In 37254 pounds, how many guineas?
2. In 756 guineas, how many pounds?
3. In 7668 guineas, how many moidores?
4. In 7880 moidores, how many pounds?
5. In 5095 guineas how many crowns?
6. In 4320 lbs. avoirdupois, how many lbs. troy?
7. In 4320 yards, how many ells English?
8. In 9546 moidores, how many guineas?
9. In 2667000 dollars, at 4s. 6d., how many pounds?

MISCELLANEOUS EXERCISES.

1. If 1 oz. of silver cost 5s. 4d. what will 3 lb. 5 oz. cost?

2. If 1 oz. of gold cost £3 17s. 10½d. what will 3 lb. 7 oz. cost?

3. Required the price of 1 lb. of flax, when 1 cwt. costs 45 guilders, each 2s. 1½d.?

4. What cost 71 lb. of cloves, at 6s. $9\frac{1}{2}$d. per lb.?

5. A person observed the flash of a cannon, and 32 seconds after he heard the report. Required the distance of the cannon, sound moving 1142 feet in a second?

6. A person engages to pay a debt of £9 16s. by weekly payments of 3s. 6d., in what time will the debt be discharged?

7. If there are 703,647.000 souls in the world, and if a generation last 35 years, how many die, at an average, in an hour?

8. A degree of the meredian, in latitude 45 deg., measures 57027 French toises. Required its length in English feet, 4000 French feet being equal to $4263\frac{1}{11}$ English feet?

9. From the aberration of the fixed stars, it appears that light in about 8 minutes, 7 seconds, passes over the semidiameter of the earth's orbit, which is computed to be 95,000,000 miles. How many miles do the particles of light move in a second?

10. Suppose there are $14\frac{3}{4}$ lb. of pressure by the atmosphere on every square inch of the human body; and suppose the surface of the body to contain 15 square feet, how many tons of air press on the human body?

11. If a ship sail 7 miles an hour, how many days will it take to run from Gibraltar to Malta, their distance being 1107 miles?

12. In the mint of England, 1 lb. troy of gold is coined into $44\frac{1}{2}$ guineas. Required the weight of a guinea?

13. How much pure gold does a guinea contain, 1 lb. of standard gold containing 1 oz. of alloy?

14. Required the weight of a shilling, 1 lb. troy of silver being coined into 62 shillings?

15. How much pure silver does a shilling contain, 1 lb. of standard silver containing 18 dwt. of alloy?

16. A dollar, coined in the mint of the United States of America, weighs 416 grains, whereof $44\frac{1}{4}$ are alloy. Required its value, agreeably to the British standard?

17. How would the number 100 be expressed, on the supposition of 5 digits being only in use?

18. On the supposition of 5 digits only, what is the import of the digit 5 thrice expressed in the series?

19. How many half-guineas, quarter-guineas, and of each an equal number, are there in £37. 10s. 6d.?

20. How much will £392. 18s. be worth in Ireland; one shilling being worth 13d. in Ireland?

21. How many English miles are equivalent to 375 geographical miles, $69\frac{1}{2}$ of the former being equal to 60 of the latter?

22. How long would 100 men take to count a billion of guineas, supposing each man to count 100 guineas in a minute, and to work 12 hours per day?

23. How many crowns, half-crowns, and groats, and of each an equal number, are there in 46 guineas?

24. A spring of water, which furnishes 5 gallons, $5\frac{1}{2}$ pints each minute, supplies a town of 1500 families. How much water has each family daily?

25. How many turns do the wheels of a carriage, 18 feet in circumference, make in passing over 5 miles?

SIMPLE PROPORTION.

The subject of Proportion having been already explained, at page 32, and illustrated by a number of examples, it is, therefore, unnecessary to repeat the rule for stating and solving questions in this place, as the rule delivered at page 34 is general, and applies to all questions that can be solved by Proportion; or, as it is often termed, the *Rule of Three**.

When the terms are in more denominations than one, if they can be multiplied and divided, as in Compound Multiplication and Division, this method should be adopted. If not, the terms

* As the questions at pages 36 and 40 are not immediately connected with mercantile affairs, those that follow will be confined entirely to such transactions as occur in business.

must be reduced to the same denomination, and the first and second to the same name.

If all the terms be of the same kind, it is sufficient if either the *second* or *third* be reduced to the same name with the *first;* the fourth term, or *answer,* is of the same name as the *other.*

Operations may often be shortened, and rendered more easy, by reducing the lower denominations to a vulgar or decimal fraction of the highest denomination.

EXAMPLE I.

If 5 cwt. 2 qr. 14 lb. of soap be purchased for £28. 14s. 4d., how much may be purchased for £7 6s. 8d.?

£	*s.*	*d.*		£	*s.*	*d.*		*cwt.*	*qr.*	*lb.*
28	14	4	:	7	6	8	: :	5	2	14
20				20						10
574				146				56	1	0×4
3				3						10
1723				440				562	2	0
										4
								2250	0	0

2d line × by 4 = 225

cwt. qr. lb.

1723)2475(1 1 $20\frac{1520}{1723}$

1723

752

4

3008

1723

1285

28

10280

2570

35980

3446

Rem. 1520

EXAMPLE II.

If 3 cwt. 2 qr. 7 lb. of sugar cost £14. 9s. 9d., what will 7 cwt. 3 qr. 21 lb. cost?

Decimally.

28 | 7 lb.   28 | 21 lb.   12 | 9d.

4 | 2.25   4 | 3.75   2,0 | 9.75

3.5625 : 7.9375 : : 14.4875 = £14. 9s. 9d.
7.9375

724375
1014125
434625
1303875
1014125

3.5625)114.99453125(£32.28
106875

81195
71250

99453
71250

282031
285000 nearly.

20

5,60
12

7.20

£32. 5s. 7d. Ans.

By Vulgar Fractions.

	cwt.		cwt.		£
	$3\frac{9}{16}$	:	$7\frac{15}{16}$	:	$14\frac{39}{80}$
or,	57	:	127	:	$14\frac{39}{80}$
	80				80
	4560				1159

1159
127

8113
2318
1159

456,0)14719,3(

£32. 5s. 7d. Answer.

MISCELLANEOUS EXERCISES.

1. If 11 lb. of sugar cost 9s. 7½d. what will 28 lb of the same cost?

2. Required the value of 87 lb. iron, at £25. 15s. a ton?

3. Required the value of 2 cwt. 1 qr. 14 lb. tea, at 7s. 9d. a lb.?

4. Bought 4 cwt. 3 qr. 14 lb cheese, at £3. 5s. 4d. a cwt.; required the amount of my bill?

5. Required the value of 15 casks, qt. 53 cwt. 1 qr. 21 lb. Montreal potashes, at £2. 5s. a cwt.?

6. Required the value of 5 bundles of Riga Rhine hemp, weighing 3 tons, 6 cwt. 2 qr. 7 lb. at £63. 10s. a ton?

7. A piece of linen is worth £5. 10s. 6d. at 4s. 3d. a yard; how many yards does it contain?

8. I have received £7. 16s. for part of a piece of broad cloth, containing at first 37 yards, value £29. 12s. how much remains unsold?

9. A bankrupt's debts amount to £1565. 10s. and his effects to £853. 15s. 6d.; how much will his creditors receive per pound?

10. A bankrupt, whose debts amount to £830. 10s. 6d. compounds with his creditors for 14s. 3d. per £, how much do they receive in all?

11. A merchant bought a quantity of Holland, at 4s. 6d. per ell Flemish, what is that per ell English?

12. A linendraper has on hand a quantity of linen, for which he paid at the rate of 2s. 9¼d. a yard, and means to retail it so as to gain 1½d. on each shilling it cost him; how much must he charge for a yard of it?

13. If a merchant, who charges 1½d. profit on every shilling prime cost, draw £900 in the course of a year; what is his yearly income?

14. If 2d. be charged on a shilling prime cost, to what extent must a merchant deal, to clear £100.?

15. Required the value of $13\frac{5}{8}$ yards of sarsenet, at 6s. 9d. a yard?

16. If the interest of £100 for a year be £5, what is the interest of £316. 10s. 6d. for a year?

17. A gentleman buys an estate for £7500, by which he has $4\frac{1}{4}$ per cent for his money. Required the yearly rent of the estate?

18. If 3 cwt. 13 lb. of curd soap cost £14. 2s. 9d. what will 7 cwt. 2 qr. 25 lb. cost?

19. If 5 cwt 2 qr. 15 lb. of ashes cost £7. 3s. $5\frac{1}{4}$d. how much may be bought for £9. 10s. $5\frac{1}{4}$d.?

20. Bought 2 cwt. 3 qr. 14 lb. of sugar for £13. 2s. 7d.; at what must I retail it a lb. to clear £2. 6s. on the whole?

21. Bought 52 ells English of broad cloth at 20s. 6d. per ell, how may I retail it per yard to clear 7 guineas on the whole?

22. Bought a quantity of linen at the rate of 9s. 6d. for 3 yards, by selling which at the rate of 24s. 6d. for 7 yards, I gained as much as 24 yards cost me. Required the quantity?

23. If 18 cwt. of sugar cost £76, what will 32 cwt. cost?

24. If 7 ounces of silver be worth 38s. 6d. what is the value of 3 lb. 9 oz. 23 dwt.?

25. If 50 yards cloth be purchased for £36. 17s. 6d.; how much may be purchased for £76?

26. If one gain £1. 15s. on a hogshead of wine worth £15, how many must he sell, and to what extent must he deal, in order to gain £100.?

27. Bought 7 yards of cloth for 17s. 8d.; what must be given for 5 pieces, each containing $27\frac{1}{2}$ yards?

COMPOUND PROPORTION.

The rule for stating and resolving questions belonging to Compound Proportion, has been stated at page 38*.

EXAMPLE.

If the freight of a ship, of 170 tons, for 3 months, be £90, how much should the freight of a ship, of 118 tons, be for 5 months?

```
 tons     tons        £
 17,0  :  118   : :  9,0
  3         5
 ----     ----
 51        590
             9
          ----
     51)5310(£104. 2s. 4 4/17d.
         51
        ----
          210
          204
         ----
            6
           20
          ----
          120
          102
          ----
           18
           12
          ----
          216
          204
          ----
        12/17 | 4/17
```

* As this rule is of the greatest use in comparing the monies, weights, and measures of different countries with each other, the student should make himself perfectly well acquainted with it.

MISCELLANEOUS EXERCISES.

1. If 12s. be given for the carriage of 2 cwt. 3 qr. 192 miles, how much should be given, at that rate, for the carriage of 8 cwt. 1 qr. 128 miles?

2. If the interest of £100, for 12 months, be £3. 10s., how much is the interest of £415, for 5 months?

3. A person put out a sum of money to interest, at 4 per cent, by which he cleared £35, in a year and 9 months; required the sum?

4. If cloth, 5 quarters wide, cost 14s. 6d. a yard, what ought cloth of the same quality, 3 quarters wide, to cost per Scots ell?

5. Bought a cask of butter, containing 50 lb. of 24 oz. each, at 1s. 3d. a lb., how must I retail it, by the Dutch lb. of $17\frac{1}{2}$ oz. to clear 3d. on each shilling it cost me?

6. Bought a quantity of Scots cheese, at $5\frac{1}{2}$d. a lb. of 22 ounces; how may I retail it, by the avoirdupois lb., to clear $12\frac{1}{2}$ per cent.?

7. If 1 lb. of gold be 15 times more valuable than 1 lb. of silver, and the proportion of its weight, to that of silver, be as 196 to 110, what is the value of an ingot of gold, equal in bulk to one of silver, worth 50 guineas?

8. If the interest of £100 for a year be £5, what will the interest of £236, for 78 days, be?

9. If the freight of a ship of 120 tons for 7 months, be £350, what ought to be paid for a ship of 150 tons, for 4 months?

10. If a ship's company of 120 men consume £50 value of wine in three weeks, at 1 pint per day each, when the price is £45 per pipe; how long will £1000 value serve 250 men, when each man is allowed $1\frac{1}{2}$ pint per day, and wine at £48 per pipe?

RULES FOR PRACTICE.

THE rules and observations, contained in the foregoing part of this work, comprehend the whole system of arithmetic, and are sufficient to enable any one, who understands them perfectly, to perform every species of computation that can occur. In many cases, however, the work of calculation may be abridged, by attending to the particular circumstances of the calculations to be performed; such as the kind of numbers that are given, the relation they bear to each, and to certain integers, &c.

As this manner of performing computations is of the utmost consequence to those who are engaged in commercial affairs, the most useful methods which practice has suggested, for rendering mercantile computations easy, will be noticed in the remaining part of this work.

TABLE OF ALIQUOT PARTS.

Of a Pound.

s.	d.		
10	0	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
3	4	=	$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
2	0	=	$\frac{1}{10}$
1	8	=	$\frac{1}{12}$
1	4	=	$\frac{1}{15}$
1	3	=	$\frac{1}{16}$
1	0	=	$\frac{1}{20}$
0	8	=	$\frac{1}{30}$
0	6	=	$\frac{1}{40}$
0	4	=	$\frac{1}{60}$
0	3	=	$\frac{1}{80}$
0	2	=	$\frac{1}{120}$
0	1	=	$\frac{1}{240}$
0	$0\frac{1}{2}$	=	$\frac{1}{480}$
0	$0\frac{1}{3}$	=	$\frac{1}{720}$
0	$0\frac{1}{4}$	=	$\frac{1}{960}$

Of a Shilling.

d.		
6	=	$\frac{1}{2}$
4	=	$\frac{1}{3}$
3	=	$\frac{1}{4}$
2	=	$\frac{1}{6}$
$1\frac{1}{2}$	=	$\frac{1}{8}$
1	=	$\frac{1}{12}$
$\frac{3}{4}$	=	$\frac{1}{16}$
$\frac{1}{2}$	=	$\frac{1}{24}$
$\frac{1}{4}$	=	$\frac{1}{48}$

Of a Cwt.

qrs.		lbs.		
2	or	56	=	$\frac{1}{2}$
1	—	28	=	$\frac{1}{4}$
—		16	=	$\frac{1}{7}$
$\frac{1}{2}$	—	14	=	$\frac{1}{8}$
—		8	=	$\frac{1}{14}$
$\frac{1}{4}$	—	7	=	$\frac{1}{16}$
$\frac{1}{7}$	—	4	=	$\frac{1}{28}$
$\frac{1}{14}$	—	2	=	$\frac{1}{56}$

CASE I.

WHEN THE GIVEN PRICE IS AN ALIQUOT PART OF A POUND.

RULE.

Divide the given quantity or number of articles by that aliquot part, and the quotient is the answer in pounds.

If the quantity contain fractional parts, reduce them to a decimal; or take parts of the price for them; or substitute such a part of a *pound* for them as the fraction is of a unit, considering the integral part of the quantity as pounds.

EXAMPLE I.

What cost 6841 yards at 6s. 8d. per yard?

6s. 8d.$=\frac{1}{3}$)6814

£2271 6 8

EXAMPLE II.

What cost 3564 yards at 8d. per yard?

8d.$=\frac{1}{30}$)3564

£118 16 0

EXERCISES.

1. What cost 4673 yards at 10s.?
2. What cost 7884 yards at 6s. 8d.?
3. What cost 4565 yards at 5s.?
4. What cost 5169 yards at 4s.?
5. What cost 3585 yards at 3s. 4d.?
6. What cost 7648 yards at 2s. 6d.?
7. What cost 7658 yards at 2s.?
8. What cost 1035 yards at 1s. 8d.?
9. What cost 3657 yards at 1s.?
10. What cost 2394 yards at 8d.?

11. What cost 4873 yards at 6d.?

12. What cost 6315 yards at 4d.?

13. What cost 2967 yards at 3d.?

14. What cost 7639 yards at 2d.?

15. What cost $9835\frac{1}{3}$ yards at 6s. 8d.?

16. What cost $7581\frac{5}{8}$ yards at 3s. 4d.?

17. What cost $3415\frac{5}{6}$ yards at 1s. 8d.?

18. What cost $6916\frac{5}{11}$ yards at 8d.?

19. What cost $2545\frac{7}{16}$ yards at 6d.?

20. What cost $3577\frac{9}{16}$ yards at 4d.?

21. What cost $4753\frac{9}{10}$ yards at 2d.?

CASE II.

WHEN THE PRICE IS ANY NUMBER OF SHILLINGS.

RULE.

1. Multiply the quantity or number of articles by the number of shillings, and divide the product by 20.

2. If the price be an even number of shillings, multiply by half the number and divide by 10; or, which is the same thing, multiply by half the number of shillings, and double the right hand figure of the product for shillings, and the other figures of it are pounds*.

* This method may be taken when the number of shillings is odd; but in some examples, there will be a remainder of 1 in dividing by 2, which is 1s., and must be added to the double of the right hand figure.

EXAMPLE I.

What cost 2794 lbs. at 9s. per lb.?

```
    2794
       9
  ------
2,0)2514,6
  ------
£1257 6 0 Answer.
```

EXAMPLE II.

What cost 4688 lbs. at 14s. per lb.?

```
  4688
     7
  ----
£3281 12 0 Ans.
```

EXAMPLE III.

What cost 8385 yards at 11s. per yard?

```
  8385
    5½
  -----
 41925
  4192+1
  ------
£4611 15 0 Ans.
```

EXAMPLE IV.

What cost 5779 yards at 19s. per yard?

```
  5779
    9½
  -----
 52011
  2889+1
  ------
£5490 1 0 Ans.
```

EXERCISES.

1. Value 8496 yards at 3s.
2. Value 5143 yards at 7s.
3. Value 4164 yards at 11s.
4. Value 3566 yards at 13s.
5. Value 4787 yards at 15s.
6. Value 5898 yards at 17s.
7. Value 5779 yards at 49s.
8. Value 4298 yards at 21s.
9. Value 7650 yards at 23s.
10. Value 8751 yards at 3s.
11. Value 9322 yards at 5s.
12. Value 5353 yards at 7s.
13. Value 2794 yards at 9s.

14. Value 8385 yards at 11s.

15. Value 654 yards at 2s.

16. Value 5764 yards at 4s.

17. Value 9735 yards at 6s.

18. Value 8466 yards at 8s.

19. Value 4688 yards at 12s.

20. Value 4688 yards at 14s.

21. Value 2532 yards at 16s.

22. Value 9854 yards at 22s.

If the price be the sum of two or more aliquot parts of a pound, these may be calculated for, as directed in Case I, and then added.

CASE III.

WHEN THE PRICE IS SO MANY PENCE.

RULE.

Multiply the quantity by the number of pence, which reduce to pounds; or, if the price be an aliquot part of a shilling, or can be divided into two or more aliquot parts of a shilling, work for these, as in Case I., and the result will be shillings, which reduce to pounds*.

EXAMPLE I.

What cost 7423 lbs. at 4d. per lb.?

```
4d. = 1/3)7423
        ------
   2,0)247,4  4
       --------
     £ 123 14  4
       --------
```

EXAMPLE II.

What cost 4856 lbs. at 11d. per lb.?

```
     4856
       11
   ------
 12)53416
   ------
2,0) 445,1 4
    --------
  £ 222 11 4
```

* The quotient, after dividing by any aliquot part, is always of the same name or denomination as that of which the divisor is an aliquot part.

EXERCISES.

1. What cost 4846 lb. at 2d. per lb.?
2. What cost 7484 lb. at 3d.?
3. What cost 5962 lb. at 6d.?
4. What cost 9371 lb. at 5d.?
5. What cost 3952 lb. at 6d.?
6. What cost 7453 lb. at 7d.?
7. What cost 3450 lb. at 8d.?
8. What cost 1791 lb. at 9d.?
9. What cost 7553 lb. at 11d.?
10. What cost 3708 lb. at 10d.?
11. What cost 3781 lb. at 5d.?
12. What cost 7777 lb. at 7d.?

If the *difference* between the given price and a *pound* be any aliquot part of a pound, consider the quantity or number of articles as so many pounds; then subtract the value, at that aliquot part, from the value at a pound, and the remainder will be the value, at the given price.

EXAMPLE I.

Required the value of 8464 yards, at 16s. 8d. per yd.?

	£8464		
3s. 4d. = $\frac{1}{6}$ =	1410	13	4
	£7053	6	8

EXAMPLE II.

Required the value of 4962 yards, at 18s. 4d. per yd.?

	£4962		
1s. 8d. = $\frac{1}{12}$ =	413	10	0
	£ 4548	10	0

For 13s. 4d. subtract $\frac{1}{3}$.
15s. 0d. —— $\frac{1}{4}$.
16s. 0d. —— $\frac{1}{5}$.
17s. 6d. —— $\frac{1}{8}$.
18s. 0d. —— $\frac{1}{10}$.
19s. 0d. —— $\frac{1}{20}$.

For 21s. 0d. add $\frac{1}{20}$.
22s. 0d. — $\frac{1}{10}$.
23s. 4d. — $\frac{1}{6}$.
24s. 0d. — $\frac{1}{5}$.
25s. 0d. — $\frac{1}{4}$.
30s. 0d. — $\frac{1}{2}$.

This mode of proceeding may be extended to the aliquot parts of a pound, as well as a pound itself.

EXERCISES.

1. What cost 7984 cwt. at 15s. each.
2. What cost 4372 cwt. at 16s.?
3. What cost 3942 cwt. at 17s. 6d.?
4. What cost 4921 cwt. at 18s.?
5. What cost 3975 cwt. at 18s. 9d.?
6. What cost 7986 cwt. at 18s. 8d.?
7. What cost 3764 cwt. at 23s. 4d.?
8. What cost 5211 cwt. at 22s. 6d.? ✓
9. What cost 3574¼ cwt. at 19s. 6d.?
10. What cost 3127$\frac{11}{14}$ cwt. at 20s. 6d.?

CASE IV.

WHEN THE PRICE CONSISTS OF PENCE AND FARTHINGS.

RULE.

Find the value at the given number of pence, as directed in Case III., and at the farthings, from the proportion they bear to the pence; add these together, and reduce the sum to pounds.

EXAMPLE I.

What cost 3287 lb. at 5¾d. per lb.?

4d.	$\frac{1}{3}$	3287	
1d.	$\frac{1}{4}$	1095	8
$\frac{3}{4}$	$\frac{3}{4}$	273	11
		68	5¾
	2,0	143,8	0¾
		£ 71 18	0¾

EXAMPLE II.

What cost 2772 lb. at 7½d. per lb.?

6d.	$\frac{1}{2}$	2772	
1½	$\frac{1}{4}$	1386	
		346	6
	2,0	173,2	6
		£ 86 12	6

It will sometimes abridge the labour to join some of the farthings to some of the pence in taking the parts; as in Example II., where the value is first found at 6d. and then at $1\frac{1}{2}$d. which is $\frac{1}{4}$th of 6d.

Example 1. might have been performed by finding the value at 6d. and then subtracting $\frac{1}{8}$ of that (the value at $\frac{3}{4}$d.) from the value at 6d.

EXERCISES.

1. What is the value of 4572 lb. at $3\frac{1}{2}$d.?
2. What is the value of 7692 lb. at $5\frac{1}{4}$d.?
3. What is the value of 459 lb. at $7\frac{1}{2}$d.?
4. What is the value of 479 lb. at $4\frac{3}{4}$d.?
5. What is the value of 8754 lb. at $2\frac{1}{2}$d.?
6. What is the value of 574 lb. at $1\frac{3}{4}$d.?
7. What is the value of 7463 lb. at $3\frac{1}{2}$d.?
8. What is the value of 6472 lb. at $2\frac{1}{4}$d.?
9. What is the value of 7358 lb. at $9\frac{3}{4}$d.?
10. What is the value of 2796 lb. at $11\frac{1}{4}$d.?

CASE V.

WHEN THE PRICE CONSISTS OF SHILLINGS AND LOWER DENOMINATIONS.

RULE.

1. Multiply the quantity by the shillings, and find the value of the pence and farthings, from the proportion they bear to the shillings; add these together and reduce the sum to pounds.

2. Divide the price into aliquot parts of a pound, or of a part already found; calculate the values corresponding to these, as directed, in Case I., then add them, and their sum will be the answer.

EXAMPLE I.

Value 4258 yards, at 17s. 3d. per yard.

```
                 4258
                   17
                 ----
                29806
                4258
                 ----
                72386
at 3d.=¼ of is   1064  6
                 -------
           2,0)7345,0  6
                 -------
           £ 3672  10  6
```

EXAMPLE II.

Value 5482 lb. at 12s. 4½d. per lb.

```
                         5482
                           12
                         ----
                        65784
at 3d.=¼ of 1s. is=1370  6
1½d.=½ of 3d. is=  685  3
                     ---------
                  2,0)6783,9  9
                     ---------
                 £ 3396  19  9
```

EXAMPLE III.

Value 3894 yards, at 17s. 6d. per yard.

```
10s. 0d. | ½ | 3894
             | ----
 5s. 0d. | ½ | 1947
 2s. 6d. | ½ |  973  10
         |   |  486  15
               --------
           £ 3407   5  0
```

EXAMPLE IV.

Value 1765 lb. at 9s. 2d. per lb.

```
6s. 8d | ⅓ | 1765
           | ----
2s. 6d.| ⅛ |  588   6  8
              220  12  6
           -----------
          £ 808  19  2
```

The third example might have been more easily performed by considering the price one pound, and subtracting the value at 2s. 6d. from it, as directed in Case II.

EXERCISES.

1. What is the value of 1864 yards, at 1s. $3\frac{1}{2}$d.?
2. What is the value of 6218 yards, at 1s. $5\frac{1}{2}$d.?
3. What is the value of 7590 yards, at 1s. $10\frac{1}{8}$d.?
4. What is the value of 5670 yards, at 2s. $9\frac{3}{4}$d.?
5. What is the value of 6750 yards, at 3s. $10\frac{1}{2}$d.?
6. What is the value of 6780 yards, at 4s. $6\frac{1}{4}$d.?
7. What is the value of 3657 yards, at 6s. $9\frac{1}{4}$d.?
8. What is the value of 5678 yards, at 8s. $3\frac{1}{4}$d.?
9. What is the value of 2873 yards, at 9s. 4d.?
10. What is the value of 3751 lb. at 9s. $10\frac{1}{2}$d.?
11. What is the value of 8010 lb. at 11s. 3d.?
12. What is the value of 5768 lb. at 12s. 9d.?
13. What is the value of 3736 lb. at 14s. 7d.?
14. What is the value of 2346 lb. at 18s. 9d.?
15. What is the value of 7963 lb. at 17s. $4\frac{1}{2}$d.?
16. What is the value of 5436 yards, at 33s. 4d.?
17. What is the value of 1786 yards, at 56s. 8d.?
18. What is the value of 1805 cwt. at 67s. 6d.?
19. What is the value of $4793\frac{1}{2}$ cwt. at 78s. 4d$\frac{1}{2}$.?

CASE VI.

WHEN THE PRICE CONTAINS A FRACTION.

RULE.

Multiply the price by the denominator, or by any number that will exterminate the fraction; then calculate for this new price and divide the value, thus found, by the same number by which the price was multiplied, and the quotient will be the answer.

EXAMPLE I.

Value 1050 lbs. at $6\frac{2}{3}$d. per lb.

$6\frac{2}{3} \times 3 = 20$d. $= \frac{1}{12}$ of a £.

therefore, 12)1050

3) 87 10

£ 29 3 4 Ans.

EXAMPLE II.

Value 938 lbs. at 4s. $3\frac{3}{7}$d. per lb.

4s. $3\frac{3}{7}$d. $\times 7 =$ £1 10s.

therefore, £938

$\frac{1}{2} = 469$

7)1407

£ 201 Answer.

EXERCISES.

1. Required the value of 525 lb. at $5\frac{3}{8}$d.
2. Required the value of 7895 lb. at $2\frac{1}{3}$d.
3. Required the value of 5742 lb. at $3\frac{1}{5}$d.
4. Required the value of 4321 lb. at $4\frac{4}{9}$d.
5. Required the value of 6581 lb. at $7\frac{5}{8}$d.
6. Required the value of 2745 lb. at $8\frac{7}{9}$d.
7. Required the value of 926 lb. at 1s. $6\frac{2}{7}$d.
8. Required the value of 598 lb. at 1s. $11\frac{2}{5}$d.
9. Required the value of 647 lb. at 2s. $11\frac{3}{7}$d.
10. Required the value of 399 lb. at 3s. $11\frac{4}{7}$d.

Calculations will sometimes be rendered very short, by estimating the *quantity*, the *price*, and the price the quantity, and then finding the value by Compound Multiplication*.

* If the price be pence, the quantity must be considered as pence, and if the price be shillings, the quantity must be considered as so many shillings.

EXAMPLE I.

What cost 107 lb. of rice, at 4d. per lb.?

107 =	£0	8	11
			4
	£1	15	8

EXAMPLE II.

What cost $54\frac{3}{4}$ yards, at 9s. 3d. per yard?

$54\frac{3}{4}$s. =	£2	14	9
			$9\frac{1}{4}$
	24	12	9
		13	$8\frac{1}{4}$
	£25	6	$5\frac{1}{4}$

EXERCISES.

1. What cost 112 lb. at $3\frac{3}{4}$d. per lb.?
2. What cost 127 lb. at $4\frac{1}{4}$d.?
3. What cost $156\frac{1}{4}$ lb. at $4\frac{3}{4}$d.?
4. What cost 58 lb. at $5\frac{3}{4}$d.?
5. What cost $97\frac{1}{3}$ lb. at $8\frac{1}{2}$.?
6. What cost 89 lb. at $11\frac{3}{4}$d.?
7. What cost $215\frac{7}{16}$ lb. at 2s. 11d.?
8. What cost $133\frac{1}{4}$ lb. at 3s. 8d.?
9. What cost $79\frac{1}{2}$ lb. at 11s.?
10. What cost $75\frac{5}{6}$ lb. at 10s. 3d.?
11. What cost $91\frac{1}{8}$ lb. at 4s. 5d.?
12. What cost $147\frac{3}{4}$ lb. at 7s. 3d.?

CASE VII.

WHEN THE PRICE CONSISTS OF POUNDS AND LOWER DENOMINATIONS.

RULE.

Multiply the quantity by the pounds, and take parts, as formerly, for the other denominations.

If the quantity consist of several denominations, reduce the lower denominations to a fractional part of the highest, or take parts of the price for them*.

EXAMPLE I.

What cost 3592 cwt. of soap, at £3 12s 8d. per cwt.?

	3592		
	3		
	10776		
4s. = 1/5 of a £	718	8	0
4s. = 1/5 of ditto	718	8	0
4s. = 1/5 of ditto	718	8	0
8d. = 1/6 of 4s.	119	14	8
	£13050	18	8

EXAMPLE II.

What cost 12 oz. 10 dwt. 12 gr. of gold, at £3 17s. 6d. per oz.

	£3	17	6
			12
	46	10	0
10 dwt. = 1/2 oz.	1	18	9
12 gr. = 1/20 of 10 dwt.	0	1	11¼
	£48	10	8¼

EXAMPLE III.

What cost 13 cwt. 1 qr. 21 lb. of sugar, at £4 7s. 8d. per cwt.?

1 qr. 21 lb.	=	.4375
and 13 cwt. 1 qr. 21 lb.	=	13.4375
		4
		53.7500
6s. 8d. = 1/3 of a £		4.4791666
1s. 0d. = 1/20 of ditto		.671875
		58.9010416 = £58 18 0¼

* On *some* occasions, the value will be most easily obtained by reducing the lower denominations to a decimal; on *others* to a vulgar fraction; and on some by taking proportional parts of the price; but this depends entirely on the numbers by which the lower denominations are expressed. The method by decimals, though not always the shortest, is, nevertheless, the simplest.

In valuing ozs. dwts., and grs. Troy, the ounces may be esteemed pounds, the dwts. shillings, and each grain a halfpenny. Thus, 10 ozs. 15 dwts. 21 gr. = £10 15s. 10½d.

In valuing tons, cwts., qrs., and lbs., the tons may be esteemed pounds, the cwts. shillings, each qr. threepence, and each lb. $\frac{1}{7}$ of a farthing. Thus, 53 tons, 15 cwt. 3 qrs. 21 lb. = £53. 15s. 11¾d.

In valuing cwts qrs. lbs, &c. the cwts. may be esteemed pounds, each qr. 5s., each lb. 2$\frac{1}{7}$d., and so on. Thus, 21 cwt. 2 qrs. 19 lb. = £21. 13s. 4$\frac{1}{7}$d.

Other methods of abridging the calculations will occur to those who consider the subject with attention. To enable the student to accomplish this, and to give him facility, he should perform every calculation more ways than one, which will also serve the purpose of proving the operation.

EXERCISES.

1. What is the value of 12 cwt. 1 qr. 14 lb. at 6s. 6d. per cwt.?

2. What is the value of 14 cwt. 2 qrs. 16 lb. at 17s. 6d. per cwt.?

3. What is the value of 17 cwt. 0 qrs. 21 lb. at 20s. per cwt.?

4. What is the value of 9 oz. 13 dwts. 17 gr. at £3. 17s. 10d. per oz.

5. What is the value of 32 tons, 17 cwt. 2 qrs. at £35. 10s. per ton?

6. What is the value of 9 tons, 18 cwt. 27 lb. at £4. 17s. 6d. per cwt.?

7. What is the value of 112 tons, 13 cwt. 2 qrs. 7 lb. at £84. 10s. per ton?

8. What is the value of 113½ dozen of rum, at £2. 2s. 3d. per dozen?

9. What is the value of 17¾ lb. of silk, at £2. 4s. 9d. per lb.?

10. What is the value of 17 tons, 11 cwt. 3 qrs. of whale oil, at £35. 15s. per ton?

11. What is the value of 31 tons, 8 cwt. 13 lb. of tallow, at £54. 17s. 9d. per ton?

12. What is the value of 5 hhds. 36 gals. of brandy, at £39. 10s. per hhd?

13. What is the value of 38 dozen, 8 pairs of stockings, at £2. 13s. $8\frac{1}{2}$d. per dozen?

14. What is the value of 36 qrs. 3 bushels of wheat, at £2. 9s. 3d. per qr.?

15. What is the value of 127 lb. 3 oz. 13 dwt. 18 gr. of gold, at £3. 14s. 10d. per oz.?

16. What is the value of 325 lb. 5 oz. 15 dwt. 21 gr. of gold, at £46. 10s. 9d. per lb.?

17. What is the value of 857 tons, 12 cwt. 2 qr. 15 lb. 8 oz. $9\frac{3}{4}$ dr. of indigo, at £525. 8s. $10\frac{3}{4}$d. per ton?

18. What is the value of 6485 tons, 12 cwt. 2 qr. 17 lb. 14 oz. $13\frac{4}{7}$ dr. of tea, at £678. 13s. $4\frac{1}{2}-\frac{1}{4}$ of $\frac{1}{3}$ farthing?

DEDUCTIONS FROM WEIGHTS.

There are certain deductions usually made from the weight of goods, on account of the packages. These deductions depend on the nature of the packages, the custom of merchants, and the regulations of public offices. These are distinguished by the names of Draft, Tare, Tret, and Cloff.

The total weight of any quantity of goods, including that of the package which contains them, is called the *gross weight*.

Draft is an allowance on the gross weight, to turn the scale, in favour of the buyer, in order that the weight may not be deficient, when the goods are sold by retail: it is always deducted before the *tare*.

Tare is an allowance for the weight of the bag, box, or cask, or other package, in which goods are contained.

When other allowances are to be made, the remainder, after deducting the tare, is called *suttle*.

Real tare, or *open tare*, is the *real* weight of the package.

Customary tare is an *established* allowance for the weight of the package.

Proportionate tare is an allowance, at a given rate, per cwt., per cent., &c.

Tret is an allowance of 4 lb. on 104, or $\frac{1}{26}$ part of the suttle weight, for *dust, waste*, &c. on goods sold by the avoirdupois pound. It is now nearly discontinued by merchants; at the East India warehouses, in London, it is entirely abolished; and at the *Custom House* neither tret nor draft is allowed.

Average tare is the allowance made, when a few packages only, out of many, are weighed, their mean or average taken, and the same tare allowed on all the others.

Suppertare is an additional allowance, or second tare, when the goods or package exceeds a certain weight.

Cloff is an allowance of 2 lb. for every 3 cwt., for waste, on a few articles, but is now very seldom made; when allowed, the deduction, like draft, is made from the original weight.

When all allowances have been deducted, the remainder is called *net weight*.

As several of these allowances have been abolished, the *net weight* could be very easily ascertained, if the *real tare* were known; but, as there are several kinds of packages which cannot be conveniently separated from the goods they contain, merchants and public offices have established certain customary tares and allowances.

THE FOLLOWING TABLE

exhibits the present Commercial Allowances, on a few of the most common Articles of British Commerce.

GOODS.	Sold by the	ALLOWANCES.
ALMONDS, ...	cwt.	In serons, 2 lb. draft; 12 lb. tare, per seron, under 3 cwt.; 4 lb. draft, and 15 lb. tare, per seron, above 3 cwt.
ALUM, ...	ton	1 lb. draft, per cwt., and real tare.—See page 232.
ASHES, Pot, American	cwt.	2 lb. draft per barrel; 14 lb. tare per cwt.
BARK, Jesuits	lb.	2 lb. draft per chest; 3 lb. per cwt. for dust; and real tare.
—— Oak, ...	cwt.	4 lb. draft per cask, and real tare.
BARILLA, ...	cwt.	4 lb. draft per seron; real tare, or 10 lb. per cwt. when loose; and 12 lb. draft per ton.
BRIMSTONE,.......	ton	12 lb. draft per ton, or 4 lb. per hhd., and real tare.
CINNAMON, ...	lb.	Tare, as per warrants of the East India Company.
CLOVERSEED, ...	cwt.	2 lb. draft per bag and 4 lb. tare per cwt.
CLOVES, ...	lb.	Tare, as per warrants.
COCHINEAL, ...	ditto	Real tare.
COFFEE, West India,	cwt.	1 lb. draft under 1 cwt., otherwise, 5 lb. per cask; 2 lb. per barrel, or 1 lb. per bag; and real tare.
—— East India or Mocha, ...	cwt.	Tare, as per warrants.
COTTON WOOL, W. India & America.	lb.	1 lb. draft, and sometimes 2 lb. cloff per bale; 4 lb. per cwt. tare.
East India.........	ditto	Draft and tare as per warrants.
COTTON YARN.....	ditto	1 lb. draft per bale; 7 lb. tare per cwt.
FIGS..............	cwt.	In chests or casks, 1 lb. draft, real tare, and 1 lb. above it.
GALLS.	cwt.	1 lb. draft per bag; 9 lb. tare, in hair bags; 6 ditto in linen bags; and 2 lb. more, if roped.

GOODS.	Sold by the	ALLOWANCES.
GOATS WOOL.	lb.	In bales, 1 lb. draft per bale, and 4 lb. tare per cwt., for the shirt.
GINGER.	cwt.	In bags, 1 lb. draft, each weigh of 2, 3 or 4 bags, and 4 lb. tare per cwt.
HEMP.	ton	1 lb. draft for 5 cwt.
HIDES.	piece	1 lb. draft for 10 hides.
INDIGO. East India.	lb.	1 lb. draft per chest and real tare.
——— Spanish.	ditto	Draft, 1 lb; tare, per half seron, under $1\frac{1}{2}$ cwt. 17 lb.; per seron, under $1\frac{1}{4}$ cwt. 21 lb., above, 5 lb.
MADDER, Dutch or Mulls.	cwt.	4 lb. draft, per cask; 10 lb. tare, per cwt.
———, Turkey Roots.	cwt.	1 lb. draft and 9 lb. tare, per bag.
MOLASSES........	cwt.	2 lb. draft and 9 lb. cloff, per cask.
NUTMEGS.........	lb.	Tare, as per warrants; when in shells, $\frac{1}{3}$ is allowed for the shells.
PEPPER.	——	1 lb. draft and real tare.
PIMENTO..........	lb.	In bags, 1 lb. draft per bag; 4 lb. tare per cwt.; and tret as usual.
PITCH.	cwt.	2 lb. draft per barrel; tare according to package.
RICE.............	cwt.	Draft, 2 lb. per barrel, and real tare.
SALTPETRE.	cwt.	Tare as per warrants.
SHUMAC.	cwt.	1 lb. draft per bag; 1 lb. tare per cwt.
SOAP.............	cwt.	2 lb. draft for 3 cwt.; above 3 cwt. 4 lb.; real tare.
SUGAR, British West India Isles.	cwt.	2 lb. draft per hhd.; 1 lb. ditto per barrel; real tare for home consumption; for exportation, varies according to the weight of hhd.
TALLOW, Russia.	ditto	2 lb. draft per cask; 12 lb. tare per cwt.
TOBACCO, North America, ...	lb.	8 lb. draft per hhd., from Virginia; 4 lb. per ditto, from Marlyand; shrinkage, 80 lb. per hhd., from Virginia; and 20 lb. ditto from Maryland.
VALONEA, ...	ton	Draft, 12 lb. per ton; in bulk and bags, real tare.
WOODS, in the log,	ton	Draft, 12 lb. per ton.
———, in casks.	ditto	Draft, 4 lb. per cask, and real tare.
WOOL, Spanish,....	lb.	2 lb. draft per bale; 20 lb. tare, per bale of 2 cwt.; 22 lb., per bale, above 2 cwt.

CASE I.

WHEN TARE ONLY IS ALLOWED, AND AT SO MUCH PER CWT, OR 112 POUNDS.

RULE.

1. If the given tare be an aliquot part of 112, divide the gross weight by *this part*, which will give the tare. If the given tare be no aliquot part of 112, divide it into aliquot parts, and proceed as in Practice.

2. Multiply the gross weight by the tare per cwt., and divide the product by 112, for the tare, which, deducted from the gross weight, leaves the net weight*.

In calculating by any of these methods, it may render the calculations shorter, to convert the lower denominations into a vulgar or decimal fraction of the highest.

EXAMPLE I.

What is the net weight of 18 cwt. 2 qrs. 18 lb. of raisins, tare 17 lb. per cwt.?

METHOD I.

lb.			cwt.	qrs.	lb.	
16	$\frac{1}{7}$		18	2	17	Gross.
1	$\frac{1}{16}$		2	2	$18\frac{4}{7}$	
			0	0	$18\frac{1}{2}$	
		cwt.	2	3	9	Tare.
		cwt.	15	3	9	Net weight.

METHOD II.

	cwt.	qrs.	lb.	
	18	2	18 × 1	
			8	
	149	1	4	
			2	
	298	2	8	
	18	2	18	
112)	377	0	26	
cwt.	2	3	9	Tare.
cwt.	18	2	18	Gross wt.
cwt.	15	3	9	Net weight.

* In computing the tare in the following examples, fractions below ½ lb. are neglected.

EXAMPLE II.

What is the net weight of 15 cwt. 3 qrs. 21 lb. of madder; tare, 12 lb. per cwt.?

lb.		cwt.		lb.	
8	$\frac{1}{14}$	15	3	21	
4	$\frac{1}{2}$	1	0	$15\frac{1}{2}$	
		0	2	$7\frac{3}{4}$	
cwt.		1	2	$23\frac{1}{4}$	tare.
cwt.		14	0	$25\frac{3}{4}$	net.

or,

	cwt.	qrs.	lb.					
	15	3	21	= 15.9375				
				12				
				112)191.2500		cwt.	qrs.	lb.
						15	3	21
cwt.				$1.7\frac{1}{14}$	=	1	2	$23\frac{1}{4}$
					net	14	0	$25\frac{3}{4}$

EXERCISES.

1. Find the net weight of 42 cwt. 3 qrs. 25 lb.; tare, 11 lb. per cwt.

2. Find the net weight of 23 cwt. 14 lb. of soap; tare. 15 lb. per cwt.

3. Find the tare on 54 cwt. 3 qrs. 21 lb. at 5 lb. per cent.

4. Find the tare on 280 cwt. 1 qr. 10 lb. at $2\frac{1}{4}$ lb. tare per 100 lb.

5. Find the net weight of 2 casks of madder, weighing 17 cwt. 3 qrs. 24 lb.; tare, 10 lb. per cwt. and the value at 95s. per cwt.

6. Required the value of 11 bags of cotton wool, each weighing 2 cwt. 1 qr. 14 lb.; tare, 9 lb. per bag, at 2s. $3\frac{3}{4}$d. per lb.

7. Required the net weight of 59 cwt. 2 qr. 24 lb. of prunes; tare, 20 lb. per cwt.

8. Required the net weight and cost of 6 chests of tea, weighing 6 cwt. 1 qr. 11 lb. at 6s. 6d. per lb.; tare, 25 lb. per chest.

CASE II.

WHEN OTHER DEDUCTIONS BESIDES TARE ARE TO BE MADE.

RULE.

1. When *tret* is allowed, first calculate and deduct the tare, then divide by 26, and the quotient will be the tret, which, subtracted from the suttle, leaves the net*.

2. When cloff is allowed, deduct the *tare* and *tret* as before, and divide the remainder by 168; the quotient will be the cloff, which, subtracted from the subsuttle, leaves the net weight†.

3. When draft is allowed, it is deducted before the tare.

EXAMPLE I.

What is the net weight of 30 bags of cotton wool, weighing gross 75 cwt. 1 qr. 8 lb.; draft, 1 lb. per bag; and tare, $2\frac{1}{2}$ lb. per 100 lb.?

	cwt.	qr.	lb.	
	75	1	8	
		1	2	Draft.
	75	0	6	
$2\frac{1}{2}$ per 100 lb. $= \frac{1}{40} =$	1	3	14	tare
cwt.	73	0	20	net weight.

EXAMPLE II.

Required the net weight of 213 cwt. 2 qr. of soap; tare, 9 lb. per cwt. and the usual tret and cloff.

	cwt.	qr.	lb.	
cwt.	213	2	0	gross
$\frac{213\frac{1}{2}\times 9}{112} =$	17	0	$17\frac{1}{2}$	tare
26)	196	1	$10\frac{1}{2}$	suttle
	7	2	$5\frac{3}{4}$	tret
168)	188	3	$4\frac{3}{4}$	subsuttle
	1	0	14	cloff*
cwt.	187	2	$18\frac{3}{4}$	net. Answer.

* This is called the *subsuttle*, when cloff is allowed.

† This number is 3 cwt. × 112 = 336 ÷ 2 = 2 lb. on 3 cwt.

EXAMPLE III.

What is the value of 49 bags of pimento, containing 39 cwt. 2 qrs. 25 lb.; draft, 1 lb. per bag; tare, 4 lb. per cwt.; and tret, 4 lb. per 104 lb.; at 1s. $1\frac{3}{4}$d. per lb.?

	cwt.	qr.	lb.	
	39	2	25	gross
	0	1	21	draft
	39	1	4	
4 lb. $= \frac{1}{28}$ of cwt. $=$	1	1	$17\frac{1}{7}$	tare
	37	3	15	suttle
$\frac{1}{26} =$	1	1	23	tret
	36	1	20	net
	36			
	36			
	3648			
	4080 lbs. at 1s. $1\frac{3}{4}$d.			
1d. $= \frac{1}{12}$s.	340			
$\frac{3}{4}$d. $= \frac{1}{16}$s.	255			
	2,0)467,5			
	£ 233 15 Answer.			

When the tare is at so much per cwt., the net may sometimes be found very easily, as follows; subtract the tare per cwt. from 112 lb., then say, as 112 lb. is to the remainder, so is the gross weight to the net.

This method will be found very convenient, when the net is wanted in lbs.

EXAMPLE.

Required the net weight of 410 cwt. 2 qr. 12 lb. of sugar; tare, 20 lb. per cwt.

	lb.		lb. lb.		cwt. qr. lb.
	112	:	112—20	::	410 2 12
or,	112	:	92	::	410.6075 : 337.286

Net 337 cwt. 1 qr. 4 lb.

EXERCISES.

1. What is the net weight of 10 bags of Smyrna cotton wool, weighing gross, 42 cwt. 3 qrs. 17 lb.; draft, 1 lb. per bag; tare, 13 lb. per bag?

2. What is the net weight of 221 cwt. 3 qrs. 14 lb. of barilla; tare, 22 lb. per cwt.; and tret, 4 lb. per 104 lb.

3. What is the value of 12 casks of madder, weighing 105 cwt. 1 qr. 9 lb.; draft, 4 lb. per cask; tare, 10 lb. per cwt.; price, at an average, 75s. 6d. per cwt.?

4. Required the net weight of 36 bales of Senna, each weighing 3 cwt. 1 qr. 14 lb.; draft, 1 lb. per bale; tare, 11 lb. per bale; and tret, 4 lb. per 104 lb.?

5. Required the value of 10 bags of pepper, each weighing 2 cwt. 3 qrs. 12 lb.; tare, 4 lb. per bag; and tret, 4 lb. per 104 lb.; at 1s. 2¼d. per lb.?

6. Required the value of 6 chests of congou, weighing 6 cwt. 1 qr. 3 lb.; draft, 1 lb.; and tare, 25 lb. per chest; at 7s. 3d. per lb.?

7. Required the net weight, in lbs. of the 4 following chests of Jesuits bark; viz.

	cwt.	qr.	lb.		qr.	lb.	
No. 1.	2	1	16	tare,	2	11	draft, 2 lb. per chest.
2.	2	1	10	——	2	8	dust, 3 lb. per 100 lb.
3.	2	0	21	——	2	6	tret, 4 lb. per 104 lb.
4.	2	0	27	——	2	6	

8. Required the value of the 8 following chests of soap, at 67s. 6d. per cwt.

		cwt.	qr.	lb.	
No. 7.	weighing	8	2	4	
8.	———	7	3	14	
9.	———	6	2	24	
10.	———	8	1	21	draft, 2 lb. per chest.
11.	———	9	0	18	tare, 12 lb. per cwt.
12.	———	8	3	22	
13.	———	9	1	2	
14.	———	6	2	23	

9. Find the value of the 4 following cases of gum tragacanth, at £25. 10s. per cwt.; viz.

		cwt.	qr.	lb.	lb.	
No. 15.	weighing	2	1	7	tare, 41	
16.	———	2	1	11	——— 42	draft,
17.	———	2	1	7	——— 40	2 lb. per case.
18.	———	2	0	27	——— 39	

10. Find the value of the 6 following bales of Surat cotton wool, at $17\frac{1}{8}$d. per lb. net.; viz.

		cwt.	qr.	lb.	
No. 741.	weighing	3	1	0	
729.	———	3	1	16	lb.
903.	———	3	1	0	tare 19 } per bale.
537.	———	3	2	8	draft 1 }
626.	———	3	1	19	
812.	———	3	0	26	

11. Find the value of the 8 following casks of soap tallow, at 63s. 6d. per cwt.

		cwt.	qr.	lb.	
No. 4.	weighing	9	3	19	
5.	———	9	3	1	
6.	———	10	0	17	
7.	———	9	2	26	draft 2 lb. per cask.
8.	———	9	3	0	tare 10 lb. per cwt.
9.	———	9	3	1	
10.	———	9	3	6	
11.	———	9	3	3	

COMMISSION AND BROKERAGE.

Commission is an allowance of so much per cent. to a factor or agent, employed in buying or selling goods, collecting or accepting bills, or transacting commercial business in general. The usual rate is 2 or $2\frac{1}{2}$ per cent. for purchasing goods, and the same for selling goods, with an addition of $1\frac{1}{2}$ or 2 per cent. when the debts are guaranteed by the factor, which is sometimes called, standing *del credere.* The commission on bills is generally $\frac{1}{2}$ per cent., but sometimes only $\frac{1}{3}$ or $\frac{1}{4}$, when the agent employed does not come under acceptance.

Brokerage is an allowance made to a broker, or person who is employed, to settle the prices and terms of purchases and sales of goods; the negociating of bills, or of Government and other securities; effecting insurances; and of various other transactions of business, between the respective parties. The rate of the allowance varies, on different kinds of goods and transactions, from $\frac{1}{16}$ to 1 per cent.; but $\frac{1}{2}$ per cent. is a common rate of brokerage, on many articles in London.

The brokerage on insurances, and on buying and selling stock, &c. will be found under their proper heads.

TO FIND THE COMMISSION OR BROKERAGE ON ANY GIVEN SUM.

RULE.

1. Multiply the given sum by the rate per cent. and divide

by 100; the quotient is the commission. If the rate has a fraction, take a proportional part for it*.

2. Compute by Practice, taking parts of 100 for the rate†.

EXAMPLE I.

Required the commission on £958. 16s. 6d. at $3\frac{1}{3}$ per cent.?

$$\begin{array}{rrrr}
 & £958 & 16 & 6 \\
 & & & 3\frac{1}{3} \\
\hline
 & 2876 & 9 & 6 \\
\frac{1}{3}= & 319 & 12 & 2 \\
\hline
 & 1{,}00)31{,}96 & 1 & 8 \\
 & 20 & & \\
\hline
 & 19{,}21 & & \\
 & 12 & & \\
\hline
 & 2{,}60 & &
\end{array}$$

By Practice.

$3\frac{1}{3}=\frac{1}{30}$ of 100.

$$\begin{array}{rrrr}
 & £958 & 16 & 6 \\
\hline
\text{therefore } \frac{1}{30}= & £31 & 19 & 2\frac{3}{5} \\
\hline
\end{array}$$

Commission, £31. 19s. $2\frac{3}{5}$d.

EXAMPLE II.

Required the brokerage on £597. 12s. 6d. at $\frac{5}{8}$ per cent.

$$\begin{array}{r}
£597\ 12\ \ 6 = £597.625 \\
5 \\
\hline
8)2988.125 \\
\hline
£\ 3.73515625
\end{array}
\quad = \quad
\begin{array}{rrrrr}
\text{or,} & £597 & 12 & 6 \\
\hline
\frac{4}{8}=\frac{1}{200}= & 2 & 19 & 9\text{—}\frac{12}{20} \\
\frac{1}{8}=\ \frac{1}{4}\ = & 0 & 14 & 11\text{—}\frac{23}{20} \\
\hline
 & £3 & 14 & 8\frac{1}{4}\text{—}\frac{3}{4}
\end{array}$$

---

* These questions belong to proportion. The first term of the proportion being 100, the second the given sum, and the third the rate per cent.

† This method is often the shortest.

EXERCISES.

1. Required the commission on £1645., at 1 per cent.?

2. Required the commission on £1070. 17s. 6d., at $1\frac{1}{4}$ p. ct.?

3. Required the commission on £1987. 6s. 8d., at $2\frac{1}{2}$ p. ct.?

4. Required the commission on £491. 15s. $5\frac{1}{4}$d., at 2 p. ct.?

5. Required the commission on £628. 13s. $8\frac{1}{2}$d., at $2\frac{1}{4}$ p. ct.?

6. Required the commission on £6156. 10s. 2d., at $4\frac{1}{2}$ p. ct.?

7. Required the commission on £479. 19s. 3d., at 5 p. ct.?

8. Required the commission on £798. 15s. 6d., at $7\frac{1}{8}$ p. ct.?

9. Required the brokerage on £1150., at $\frac{1}{8}$ per cent.?

10. Required the brokerage on £965. 10s. at $\frac{1}{4}$ per cent.?

11. Required the brokerage on £730. 10s., at $\frac{3}{8}$ per cent.?

12. Required the brokerage on £657. 16s., at $\frac{3}{4}$ per cent.?

13. Required the brokerage on £1172. 16s. 6d., at $\frac{1}{8}$ p. ct.?

14. Required the brokerage on £175. 12s. at 3s. per cent.?

15. Required the brokerage on £1821. 12s. at 2s. per cent.?

16. Required the commission on £832., at $2\frac{1}{2}$ per cent.?

17. Required the commision on £7564., at $2\frac{1}{2}$ per cent.?

18. Required the brokerage on £7321. 10s. 6d. at $\frac{1}{8}$ per cent.?

19. Required the brokerage on £1436. at $\frac{1}{4}$ per cent.?

20. Required the brokerage on £730. 5s. at $\frac{3}{4}$ per cent.?

21. Required the brokerage on £1140. 16s. at $\frac{1}{16}$ per cent.?

22. Required the commission on £796. 8s. at 2 per cent.?

23. A broker procures an insurance on £1850; how much must he be allowed for his trouble, at 3s. 9d. per cent.?

24. A broker transacted business for his employer to the extent of £8760. 15s.; what is he entitled to, at 6s. 8d. per cent.?

25. Bills were negociated by a broker, for A. B., to the amount of £18765. 16s. 8d.; what has the broker to receive, at $\frac{1}{8}$ per cent.?

26. Sent my employer, in Jamaica, account sales of 50 hhds. of sugar, sold for £2750. 18s. 9d. My commission $2\frac{1}{2}$ per cent. and brokerage $\frac{1}{2}$ per cent. Duty, freight, and other charges, £935. 7s. 6d. Required the proceeds due to him?

27. Transmitted my correspondent account sales of 30 puncheons of rum, which were sold at £605. 11s. The charges of custom, freight, &c., paid by me, came, to £213. 17s. 6d. Commission and guarantee of debts, 4 per cent. What is the net proceeds?

28. Rendered account sales of a consignment, gross amount, £571. 8s. 4d. Charges of freight, cartage, and porterage, £14. 16s. 8d.; commission and *del credere*, or guarantee of debts, 5 per cent. Required the net proceeds?

29. Shipped to my employer, goods which I purchased by his order, to the amount of £561. 4s. 10d. Charges of packing, cartage, and porterage, £3. 15s. 6d. Wharfage, entry bond, and shipping, £5. 10s. 8d. Commission, $2\frac{1}{2}$ per cent. Required the amount of the invoice?

The following concise methods may be useful in business.

For $1\frac{1}{4}$ take $\frac{1}{80}$ of the sum.		For 10 take $\frac{1}{10}$ of the sum.
2 take $\frac{1}{50}$ of the sum,		$12\frac{1}{2}$ take $\frac{1}{8}$ of the sum.
$2\frac{1}{2}$ take $\frac{1}{40}$ of the sum.		20 take $\frac{1}{5}$ of the sum.
4 take $\frac{1}{25}$ of the sum.		25 take $\frac{1}{4}$ of the sum.
5 take $\frac{1}{20}$ of the sum.		50 take $\frac{1}{2}$ of the sum.

SIMPLE INTEREST*.

INTEREST is an allowance given for the use of money by the borrower to the lender, at a certain rate *per cent., per annum*.

The highest rate of interest in Britain, is limited by law to 5 per cent, which is called the *legal* interest, and is always understood, when no other rate is mentioned.

The rate of interest, however, may be under 5 per cent., if so stipulated between the parties; but more than 5 per cent. is termed *usury*, and prohibited by law.

CASE I.

TO CALCULATE THE INTEREST OF ANY SUM FOR YEARS.

RULE.

1. Find the interest for one year, by multiplying the given sum by the rate, and dividing by 100, as in Commission, which multiply by the number of years.

2. Multiply the rate into the time, and esteem the product the rate, for which calculate as in Commission†.

---

* Interest is of two kinds, simple and compound. When interest is charged only for the sum originally lent, though it remain unpaid for years, it is called *simple* interest; but when interest is charged, not only for the original sum, but also for the interest as it becomes due, it is called *compound* interest; which will be treated of in another part of this work.

† The product of the rate into the number of years, is the same as if the interest were required for one year, at that rate. This is often the shortest and easiest method of calculating interest for years.

EXAMPLE.

What is the interest of £432. 5s. for 4 years, at 3¾ per cent?

By Rule 1.

		£	s.	d.
2½ per cent	$\frac{1}{40}$	432	5	0
1¼	$\frac{1}{2}$	10	16	1½
		5	8	0¾
1 year	£	16	4	2¼
				4
4 years	£	64	16	9 Ans.

or thus,

	£432.25
	3¾
	1296.75
	324.1875
1 year	£ 16.209375
	4
=	£ 64.837500

By Rule 2.

per cent × years

$$3\tfrac{3}{4} \times 4 = \frac{15}{100} = \frac{3}{20}$$

then,

	£432	5	0
			3
2,0)	129,6	15	0
	£ 64	16	9 Ans.

or, 3¾ × 4 = 15 then

		£	s.	d.
10 per cent	$\frac{1}{10}$	432	5	0
5 ——	$\frac{1}{2}$	43	4	6
		21	12	3
		£ 64	16	9

The interest of any sum for a year, at 5 per cent, is as many shillings as there are pounds in the given sum, or $\frac{1}{20}$th part of the sum; therefore, when the rate is 5 per cent, divide the sum by 20, and the quotient is the rate for one year.

If the rate be 4 per cent, the interest for 1 year is $\frac{1}{25}$ of the principal*; at 2½, it is $\frac{1}{40}$; and so on.

* The *principal* is the sum upon which the interest is calculated.

EXAMPLE.

What is the interest of £235. 10s. for 4 years, at 5 per cent?

	£	s.	d.	
2,0)	23,5	10	0	
	£11	15	6	
			4	years
	£47	2	0	Ans.

or, $5 \times 4 = 20 = \frac{1}{5}$)

	£	s.	d.
$\frac{1}{5}$)	235	10	0
£	47	2	0

EXERCISES.

1. What is the interest of £1265. for 2 years, at 5 per cent?

2. What is the interest of £815. 10s. for 3 years, at 5 per cent?

3. What is the interest of £789. 5s. for 5 years, at 5 per cent?

4. What is the interest of £950. 15s. for 7 years, at 5 per cent?

5. What is the interest of £612. 10s. for 6 years, at $4\frac{1}{2}$ per cent?

6. What is the interest of £780. 15s. for 5 years, at 4 per cent?

7. What is the interest of £725. 13s. 3d. for $3\frac{1}{2}$ years, at $1\frac{1}{4}$ per cent?

8. What is the interest of £273. 3s. $6\frac{1}{2}$d. for $13\frac{3}{4}$ years, at 4 per cent?

9. What is the interest of £1475. for $\frac{1}{2}$ a year, at 5 per cent?

10. What is the interest of £385. 17s. 6d. for 8 years, at $4\frac{3}{4}$ per cent?

CASE II.

TO FIND THE INTEREST OF ANY SUM FOR DAYS.

RULE.

1. Multiply the principal by the number of days, and divide the product by 7300, for the interest at 5 per cent; and for any other rate, take a proportional part of the interest at 5 per cent*.

2. Multiply the principal by the number of days, and by *double the rate;* the last product, divided by 73000, quotes the interest.

NOTE. The day computed *from*, is not reckoned, but the day computed *to*, is included, and in leap years, 29 days are reckoned in February, in computing the days.

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* Questions of this kind, strictly belong to Compound Proportion, and this rule is abridged or reduced to a question of Simple Proportion, the first term of which is a constant quantity, viz. 7300, which is obtained as follows: Let it be required to find the interest of £350, for 219 days, at 5 per cent. per annum.

	£		£		In.		
	100	:	350	: :	5	:	to interest for 1 year
then,	365 days		219 days.				
1st abridged	20	:	——	: :	1		
	73,00		)766,50				
			£10 10 Interest for 219 days.				

Here the 1st term 100, and the 3d term 5, are each of them divided by 5, which makes the first term 20, and the third term 1; therefore the principal is only to be multiplied by the days, and divided by 20 times 365, or 7300, which is the method given in the rule. Or the rule may be illustrated thus:

	days		days		£	
	365	:	219	: :	5	and dividing 1st and 3d terms by 5,
it will be	73	:		: :	1	
	100	:	350			
	7300		)76,650			
			£10 10			

From these proportions, it appears, that the interest of any sum for 1 day, is the $\frac{1}{7300}$ part of that sum; therefore, the interest for any number of days must be obtained by multiplying the interest for one day by the number of days, or the principal by the number of days and dividing by 7300.

A constant divisor for any other rate per cent. may be found, in the same manner, by dividing $365 \times 100 = 36500$, by the given rate per cent.

EXAMPLE I.

Required the interest of £435. for 80 days, at 5 per cent. per annum.

```
        £435
          80 days
      --------
73,00)348,00
      --------
      £4 15  4 8/73 Interest.
```

EXAMPLE II.

Required the interest of £619. 10s. 9d., for 97 days, at 4½ per cent.

By Rule 1.

```
  £619 10  9×1
          12
  --------------
  7434  9  0
           8
  --------------
 59475 12  0
   619 10  9
  --------------
73,00)600,95  2  9
  --------------
  £8  4  8 nearly, at 5 p. cent.
   0 16  5¼ deduct 1/10.
  --------------
  £7  8  2¼ int. at 4½ p. cent.
```

or thus,

```
            619.5375
                 100 days
            ------------
          61953.7500
3 days sub. 1858.6125
            ------------
      73,00)60095.1375( £8.23321
            584            .23321
            ---            -------
             169           7.40200
             146                20
             ---           -------
              235          8.18000
              219               12
              ---          -------
               161            2.16
               146
               ---  £7  8  2 Ans.
                153
                146
                ---
                 77
                 73
                 ---
                 45 Rem.
```

By Rule 2.

$$£619.535\times97\times9=\frac{5408562375}{73,000}=£7\ \ 8\ \ 2\tfrac{1}{4}\ \text{Ans.}$$

When the interest is at any other rate than 5 per cent., it is best to use Rule 2.

To find the interest of any sum for months, at 5 per cent., divide the sum by 240 for 1 month; by 120, for 2 months; by 80, for 3 months; by 60, for 4 months; by 48, for 5 months; by 40, for 6 months; by $34\frac{2}{7}$, for 7 months; by 30, for 8 months; by $26\frac{2}{3}$, for 9 months; by 24, for 10 months; by $21\frac{9}{11}$, for 11 months; by 20, for 12 months; by $17\frac{1}{7}$, for 14 months; by 15, for 16 months; and by $13\frac{1}{3}$, for 18 months.

TABLE OF DAYS IN A YEAR.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

USE OF THE PRECEDING TABLE.

EXAMPLE I.

To discover the number of days, from the beginning of the year, to any given day in any month throughout the year.

This is at once apparent by inspection; for instance, required the number of days from 1st of January to 8th July. Under July, and on a parallel with 8, we find 189 days, the number required.

EXAMPLE II.

To discover the number of days betwixt an assigned day, in any month, to the end of the year.

This is found by a single subtraction. For instance, required the number of days from 8th July to the end of the year.

The days in a year	365
Under July, and on a parallel with 8	189
Remainder,	176 days.

EXAMPLE III.

To find the number of days betwixt two given days in different months of the same year.

This is likewise effected by subtraction.—Required the number of days betwixt 11th May and 18th November.

From the number below November, and opposite to 18, viz.	322
Subtract that under May, and opposite to 12, viz.	132
Remainder,	190 days.

EXAMPLE IV.

To find the number of days betwixt any given day, in any month of one year, to any assigned day, of any month, in the following year.

To the days to run in the year, discoverable by Example II., add the days to the next assigned period, by Example I., and the sum is the answer.

NOTE. When leap-year enters into the question, and the month of February betwixt the assigned periods, 1 must be added to the reckoning, as the table goes no farther than 28 for February.

EXERCISES.

1. What is the interest of £1209., for 57 days, at 5 p. ct.?

2. What is the interest of £182. 10s., for 40 days, at 5 per cent.?

3. What is the interest of £292., for 25 days, at 5 per ct.?

4. What is the interest of £348. 16s., for 85 days, at 5 per cent.?

5. What is the interest of £3175. 9s. 3d., for 113 days, at 4 per cent.?

6. What is the interest of £819. 10s. 9d., for 98 days, at $4\frac{1}{2}$ per cent.?

7. What is the interest of £60. 7s. $3\frac{1}{2}$d., from 1st February, 1806, to 21st July 1809, at $4\frac{1}{2}$ per cent.?

8. What is the interest of £212. 13s. $8\frac{1}{4}$d., from 8th December, 1806, to 13th February, 1810, at $4\frac{1}{4}$ per cent.?

9. What is the interest of £854. 13s. 6d., from 15th May, 1805, to 3d April, 1812, at 4 per cent.?

10. What is the interest of £2496. 10s. 6d., from 2d February, 1804, to 1st August 1812, at 5 per cent.?

CASE III.

TO CALCULATE INTEREST ON A DEBT, WHEN PARTIAL PAYMENTS ARE MADE.

RULE.

Subtract the several payments from the original sum, in the order of their dates, then multiply the sum and the several balances by the number of days each sum has been at interest, which is found by ascertainiug the number of days from the time of one payment to that of the next, and divide the sum of the products by 7300, gives the interest at 5 per cent. If the interest be required at any other rate, take a proportional part of the sum of the products before dividing by 7300; or of the quotient, after dividing by that nnmber. Or proceed by Rule 2. Case II.

EXAMPLE.

Lent James Swan (per bill, dated 2d February, 1809, payable one day after date.) £500.; which I received back in the following partial payments: viz. 9th March, £80.; 15th May, £115.; 1st June, £25.; 14th August, £130.; 7th November, £50.; and 28th December, the balance of the principal. What interest is due at 5 per cent?

Dates.		Transactions.			Days.	Products.
1809.						
Feb.	2.	Lent per bill, at 1 ddt.	£500	×	35 =	17500
March	9.	Received in part	80			
		Bal.	420	×	67 =	28140
May	15.	Received in part	115			
		Bal.	305	×	17 =	5185
June	1.	Received in part	25			
		Bal.	280	×	74 =	20720
Aug.	14.	Received in part	130			
		Bal.	150	×	85 =	12750
Nov.	7.	Received in part	50			
		Bal.	100	×	51 =	5100
Dec.	28.	Received in full	100			
					7300)	89395
					£12	4 11 Ans.

Before performing calculations of this kind, the accountant should try if the last balance be the same as the balance of the account; and if the sum of the column of days be equal to the number of days between the date of the account and the day it was settled.

EXERCISES.

1. On a bill of £158. due 19th September, 1773, of which £25. was paid 13th October; £17. 19th November; £45. 18th January, 1774; £27. 12th of March; and the balance, 30th April; required the interest at 5 per cent.

2. On a bill of £640. due 19th July, of which £40. was paid the first day of each month, till the whole was paid up; required the interest at 5 per cent.

3. Required the interest on £378. due 12th May, of which £10. was paid on 12th June; £12 on 17th August; £158. on 19th September; £47. on 18th October; and the balance, 23d November; interest at 5 per cent.

4. Required the interest on £170. due 12th August, of which £54. was paid on 18th September; £56. on 17th October; and the balance on 14th November; interest at 5 per cent.

5. Lent John Adams £1000. per bill, 1st January, 1816, and received the following payments: on 1st March, £250.; 5th May, £100.; 20th July, £315.; 27th September, £155.; and on 17th October, £100. The account is to be settled on 31st December. What sum have I to receive at that time: interest at 5 per cent.

6. On 5th January, 1807, lent John Jay, per bill, at one day's date £496. 15s. 6d. which I received back in the following partial payments; viz. 29th January, £15. 10s.; 5th March, 215 guineas; 5th May, £21. 5s.; 17th July, £120.; 16th August, £30. 5s. 6d.; 30th October, £63.; and the balance, 11th November. How much interest had he to pay me at 4½ per cent.?

CASE IV.

TO COMPUTE THE INTEREST ON CURRENT OR CASH ACCOUNTS.

RULE.

Add and subtract the sums on the Dr. and Cr. sides of the account, in the order of their dates, as they become due. Multiply the several balances by the days, as formerly, and if the balance be sometimes due to one party, and sometimes to the other, extend the products in different columns.

Add the products, and, when the rates of interest are different, multiply each sum by double its rate, and divide the difference of the products by 73000, for the interest.

EXAMPLE I.

Required the interest on the following account, at 5 per cent, up to the 31st December.

Dr. Mr. Geo. Carey his Account Current with J. Airth, Cr.

May 15,	To Bal.	£182	0	July 5,	By Cash	£198	10
June 29,	To Bill.	136	10	Oct. 10,	By Cotton	145	15
Sept. 16,	To Goods	275	16	Nov. 18,	By Rum	126	0

CALCULATION.

			days	Prod.		
May 15th, To	£182	0	×45	=8190	Dr. side,	£594 5
June 29th, To	136	10			Cr. side,	470 5
	318	10	× 6	=1911	Balance,	£124 0
July 5th, By	198	10				
	120	0	×73	=8760	May	16 days to run.
Sep. 16th, To	275	15			June	30
					July	31
					Aug.	31
	395	15	×24	=9498	Sep.	30
Oct. 10th, By	145	15			Oct.	31
					Nov.	30
	250	0	×39	=9750	Dec.	31
Nov. 18th, By	126	0				
						230 days.
Balance,	£124	0	×43	=5332		

days 230

73,00)434,41

Interest, £5 19 0

Here the sums, on either side of the account, are introduced according to the order of the dates. The sums on the Dr. side are added to the former balance, and those on the Cr. side subtracted.

1. As multiplying the odd shillings and pence by the days, gives but a very small increase to the interest, and as they are very troublesome in accounts of this kind, in practice, it is reckoned sufficiently exact, to neglect the shillings, when they are below 10, and if they are 10, or above, to add 1 to the pounds.

2. The two following methods of computing the interest on Accounts Current are, perhaps, more generally followed, particularly in England, than the above, though less expeditious.

METHOD II.

Dr. Side.

From		days		£	s.	£	s.	d.
May 15th,	to Dec. 31, is	230	on	182	0=	5	14	8
June 29th,	to ditto	185		136	10=	3	9	2
Sept. 16th,	to ditto	106		275	15=	4	0	1
						£13	3	11

Cr. Side.

From		days		£	s.	£	s.	d.
July 5th,	to Dec. 31, is	179	on	198	10=	4	17	4
Oct. 10th,	to ditto	82		145	15=	1	12	9
Nov. 18th,	to ditto	43		126	0=	0	14	10
						7	4	11

Interest due by Geo. Carey, £ 5 19 0

In this method the interest is first computed on the sums of the Dr. side, and then on those of the Cr.; reckoning the time from the day which the sums become due, to the time of settlement. The sum of the interest, on the Dr. side, is then subtracted from that of the Cr. side, and the difference, £5. 19s., is the interest due to J. Airth.

METHOD III.

					Dr.	Cr.
May 15th,	To £182	0	for 230	days	41860	
June 29th,	To 136	10	185	do.	25252	
Sep. 16th,	To 275	15	106	do.	29230	
July 5th,	By £198	10	for 179	days	——	35531
Oct. 10th,	By 145	15	82	do.	——	11952
Nov. 13th,	By 126	0	43	do.	——	5418
					96342	52901
					52901	
				Balance	43441=£5 19 0	

Here the sums are multiplied by the respective number of days, reckoning the time, as formerly, and extending the products of the Dr. side, in one column, and those of the Cr. in another. The difference, 43441, when divided by 7300, or by any of the foregoing methods, gives the interest as before.

1. Of the three preceding methods, the first is not only shortest and most expeditious, but it exhibits the balances at the different dates. The second method is more tedious than the third, occasioned by interest being calculated on each product separately, which requires as many divisions as there are sums in the account. In the third method, the labour is greatly shortened, by the interest being calculated only on the difference of the products, which reduces the whole to one division.

2. Men in business, who follow the second method of computing interest, generally use *interest tables*; but neither this method, nor the use of tables, ought to be preferred to the other methods. Those who accustom themselves to calculate, without the use of tables, soon acquire such facility, in calculations of this nature, as would almost appear incredible, to persons who have had little practice in similar calculations.

EXAMPLE II.

Required the interest and balance on the following cash account, allowing 5 per cent., when the balance is due, to J. S., and 4 per cent. when due to A. B.

Dr. Mr. A. B. his Cash Account with J. S. Cr.

Jan. 25,	To balance		£230	Feb. 14,	By cash paid		£185
Mar. 16,	To	cash	175	Apr. 10,	By	ditto	140
June 1,	To	ditto	25	May 20,	By	ditto	145
Aug. 2,	To	ditto	340	July 4,	By	ditto	300
Sep. 12,	To	ditto	176	Oct. 22,	By	ditto	120
Nov. 7.	To	ditto	100				

			£		Due J. S.	Due A. B.
Jan.	25	To bal.	230	20	£4600	
Feb.	14	By	185			
		Dr.	45	30	1350	
Mar.	16	To	175			
		Dr.	220	25	5500	
April	10	By	140			
		Dr.	80	40	3200	
May	20	By	145			
		Cr.	65	12	—	780
June	1	To	25			
		Cr.	40	33	—	1320
July	4	By	300			
		Cr.	340	29	—	9860
Aug.	2	To	340			
			....	41		
Sep.	12	To	176			
		Dr.	176	40	7040	
Oct.	22	By	120			
		Dr.	56	16	896	
Nov.	7	To	100			
		Dr.	156	286	22586	11960=5 p. ct.
					9568	sub. 2392=1 ditto

Bal. due J. S. £156	0	0	73,00)130,18		9568=4 p. ct.
Int. due to do. 1	15	8			
Due do. £157	15	8	£ 1	15	8 interest.

In this account the balance is sometimes due to the one party, and sometimes to the other. At the beginning, there is a balance of £230. due to J. S., and, on the 10th April, there is £80. due to him. On the 20th May, A. B. pays £145, which

discharges what he owed, and leaves a balance of £65. due to him. The balances continue in A. B's. favour till the 2d Aug., when he received £340., the sum for which he stood Cr. In the transactions which follow, the several balances are due to J. S., as A. B's receipts exceed his remittances.

The days are computed, as before, and the products extended in different columns. As A. B. is allowed only 4 per cent. 1-5th is deducted from the sum of the products in his favour, and the remainder subtracted from the sum of those in favour of J. S. The interest is then calculated on the difference, and as the balance of principal, as well as interest, is due to J S., the interest is added to the principal; the sum of which is £157. 15s. 8d., the balance due to J. S., including interest till the 7th November.

EXERCISES.

1. What interest is due on the following account to 31st Dec. allowing 5 per cent., when the balance is due to T. Coutts and Co. and 4 per cent., when due to W. Jay.

Dr. Mr. W. Jay's Account with T. Coutts and Co. Cr.

Jan. 1.	To balance	£120	Mar. 21.	By cash	£250
Feb. 15.	To cash	100	June 13.	By do.	200
May 20.	To do.	350	Sept. 9.	By do.	300
Aug. 9.	To do.	100	Oct. 3.	By do.	50
Nov. 10.	To do.	220	Dec. 20.	By do.	150

2. Required the interest on the following account to the 31st December.

Dr Mr. J. R. his Account Current with D. C. Cr.

Dec. 31.	To balance	£150	Apr. 9.	By cash	£ 70
Mar. 12.	To cash	120	May 12.	By ditto	300
June 17.	To ditto	165	June 3.	By ditto	240
Sept. 24.	To ditto	242	Aug. 2.	By ditto	10
Oct. 9.	To ditto	178			

3. How much interest is due on the following account.

Dr. Mr. E. Grove his Account Current with G. Street and Co. Cr.

1804	Jan. 10.	To Bal.	£160	1804	April 21.	By goods	£344
	May 6.	To Bill	148		July 25.	By ditto	196
	Aug. 31.	To ditto	216		Sept. 17.	By ditto	297
	Oct. 9.	To ditto	435		Nov. 28.	By ditto	310

4. Required the balance, including interest, on the following account to 1st April, 1806.

Dr. Messrs. Laing and Co. their Account Current with Jay and Vanderpot, Cr.

1805.				1805.			
April 8.	To Goods	£256	14	June 12.	By Bill	£325	8
July 19.	To ditto	164	11	Sept 16.	By ditto	149	12
Oct. 2.	To ditto	201	7	Oct. 22.	By ditto	237	10
Nov. 30.	To ditto	272	9	Jan. 18.	By ditto	460	4
Mar. 15.	To ditto	240	15				

5. What balance is due on the following account, 1st January, 1816, with interest?

Dr. Mr. John Jay his Account Current with W. Bowden.

1815.				
June 30,	To balance, as per account rendered	£1204	8	9
Aug. 12,	To your bill, fav. Stewart, pd. this day	190	5	0
Oct. 6,	To ditto, fav. White............	67	4	0
Nov. 4,	To goods per Union, as per invoice ..	150	11	3
28,	To your bill, fav. Holmes, due 31st Jan.	525	0	0
Dec. 7,	To ditto ditto 10th Feb.	473	14	0

1815.	*Contra, Cr.*			
July 4,	By Bill on Fordyce & Co. London, due	£497	18	4
Sept. 17,	By net proceeds of 20 bales cotton per account sales, due this day,	350	10	0
Nov. 14,	By ditto 30 hhds of sugar, per ditto	821	19	0
Dec. 3,	By bill on J. Andrews, due 6th Feb.	459	6	9
——31,	By net proceeds of 25 hhds. sugar, due 7th March,	692	7	8

CASE V.

TO FIND THE INTEREST ON BILLS OR BONDS, WHEN THE INTERVALS BETWEEN THE PAYMENTS EXCEED A YEAR.

RULE.

Add the interest due at the time of the first payment to the principal, and deduct the payment from the amount, the remainder is a new principal, to which add, in like manner, the interest due at the next payment, and deduct the payment, and so on, adding the interest up to the date of each payment to the principal, and then deducting the payment from the amount.

EXAMPLE.

Borrowed on bond, dated 14th March, 1803, £1140. bearing legal interest; of which I paid £465. on the 2d February, 1805; £206. on the 12th of August, 1806; and £455. on the 20th May, 1808. What balance must I pay to retire my bond, 30th December, 1809?

1803.	March 14.	Borrowed on bond...........	£	1140	0	0
		Interest for 1 year, 325 days		107	15	0½
		Amount....		1247	15	0½
1805.	Feb. 2.	Paid in to account.............		465	0	0
		Balance....		782	15	0½
		Interest for 1 year, 191 days		59	12	4½
		Amount....		842	7	5
1806.	Aug. 12.	Paid in to account............		206	0	0
		Balance....		636	7	5
		Interest for 1 year, 282 days		56	8	0¼
		Amount....		692	15	5¼
1808.	May 20.	Paid in to account............		455	0	0
		Balance....		237	15	5¼
		Interest for 1 year, 224 days		19	3	8¼
1809.	Dec. 30.	Amount due..............	£	256	19	1½

1. In computing the interest, the odd shillings and pence should be reduced to the decimal of a pound*.

2. In leap years, when the 29th day of February is among the odd days, it is *reckoned*, otherwise it is not.

3. Although the above method, of accumulating the interest with the principal, be not strictly conformable to the laws of England, yet the practice prevails in Scotland, and has repeatedly met with the sanction of the Supreme Court in that part of the island.

EXERCISES.

1. Borrowed on bond, 21st May, 1809, £1000, at 5 per cent., and made the following partial payments: £350. on 18th July, 1810; £150. on 29th September, 1811; £180. on 19th December, 1812; £120. on 18th July, 1813; £50. on 18th September, 1814; and the balance on 21st May, 1817. Required how much money will be necessary to retire the bond.

2. Received on bond, £650. bearing interest at 5 per cent., since January, 1806, of which there were paid £80. on 25th April, 1807; £115. on 29th July, 1809; and £195. on 25th May, 1810. Required the interest and balance due, the 8th October, 1811.

3. An estate was purchased for £6000. one half of which was payable on 22d November, 1810, and the other half on the 26th May, 1812; but was only paid as follows: £1200. on 1st March, 1811; £1200 on 21st September, 1813; £1200. on 14th October, 1814; £1200. on 25th February, 1815; and £1200. on 18th April, 1816. Required how much interest is due, at 5 per cent.

4. A bond for £360. was due 1st April, 1802, of which there were paid £50. on the 1st of July, 1803; £15. on 4th January, 1804; £25. on 31st October, 1805; £205. on 1st March, 1807. What balance will be due 2d February, 1809, when the account is to be settled; interest at 4½ per cent.?

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* When the shillings exceed 10, they are usually reckoned another pound; and when under 10, they are neglected.

DISCOUNT.

DISCOUNT is the allowance that *ought* to be made for receiving payment of a debt before it becomes due, and is equal to the interest of its present value, from the time when the money is *paid*, to the time when the debt falls *due**.

CASE I.

TO FIND THE TRUE DISCOUNT AND PRESENT VALUE OF A DEBT DUE AT ANY FUTURE PERIOD.

RULE.

Find the amount of £100., for the given rate and time; then say, as the amount is to the interest of £100, so is the debt to the discount;

And, as the amount is to £100, so is the debt to the *present value*.

EXAMPLE I.

What discount ought to be paid, on receiving present payment of a debt of £500, due 4 years hence; interest, at 5 p. ct.?

Year p. ct. £100
4 × 5 = 20

£ £
120 : 20 :: 500
20

12,0)1000,0

£ 83 6 8 discount.

* As interest and discount are often confounded, even in some *modern works*, which profess to be complete systems of mercantile arithmetic, it is proper that the student should, at least, *know* the distinction between them, whatever be the practice of merchants.

EXAMPLE II.

What sum of ready money is equivalent to £463., due 219 days hence; interest at 5 per cent.?

$$100 \div 20 = 5 \div 4 = \pounds 1\tfrac{5}{100} \text{ interest for 3 months.}$$

$$
\begin{array}{rcrcl}
 & & \pounds & & \pounds \\
101\tfrac{5}{} & : & 100 & :: & 463 \\
4 & & 4 & & \\
\hline
405 & : & 400 & :: & 463 \\
 & & & & 400 \\
\hline
 & & & 405) & 185200 \\
\hline
\end{array}
$$

£457 5 $8\frac{12}{81}$ Ansr.

To find the discount of any sum, for months, reckoning interest at 5 per cent.; divide the sum by 241, for 1 month; by 121, for 2 months; by 81, for 3 months; by 61, for 4 months; by 49, for 5 months; by 41, for 6 months; by $35\frac{2}{7}$, for 7 months; by 31, for 8 months; by $27\frac{2}{3}$, for 9 months; by 25, for 10 months; by $22\frac{2}{11}$, for 11 months; by 21, for 12 months; by $18\frac{1}{7}$, for 14 months; by 16, for 16 months; and by $14\frac{1}{3}$, for 18 months.

EXERCISES.

1. What discount ought to be allowed, on receiving present payment of a debt of £100., due 4 years hence; interest, 5 per cent.?

2. How much ready money is equivalent to £100, due at the end of 90 days; interest, 4 per cent.

3. How much ready money is equivalent to £100, due at the end of 10 years; interest, 5 per cent.

4. What discount ought to be allowed, on receiving present payment of £68. 3s. 6d., due 118 days hence; interest, 5 p. ct.?

5. In what time will £70. amount to £100., at $4\frac{1}{2}$ per cent.?

6. At what rate of interest will £100. amount to £101. 12s., in 146 days?

CASE II.

TO FIND THE DISCOUNT OR INTEREST ON BILLS.

RULE.

Find how long the bill has to run, reckoning from the time it is discounted to the day it is payable, including the days of grace; then calculate the interest on the *amount* of the bill, for *that time*, which is what *bankers* call the *discount**.

EXAMPLE.

A bill of £300., dated 30th April, 1817, and payable 3 months after date, was discounted 2d May; commission, $\frac{1}{2}$ per cent; how much did the holder receive?

A bill drawn 30th April, at 3 months date, is due 3d August; and from 3d May to 3d August is 93 days.

Content of the bill 300 0 0

Interest $300 \times 92 = \dfrac{27600}{7300} = £3\ 15\ 7\frac{1}{4}$

Commission $\dfrac{300}{200} = 1\ 10\ 0$

5 5 $7\frac{1}{4}$

£294 14 $4\frac{3}{4}$

* To discount a bill, is to receive *cash* for it before it becomes due; and a bill is said to be discounted, when it is indorsed to any person, or banking company, who pays the value, after deducting *interest*, from the time the money is advanced, to the day that the bill falls due.

The Bank of England never discounts a bill at a longer date than 2 months.

Private bankers, in London, sometimes discount bills at 3 months, or a longer date, according as money is scarce or plenty.

Many brokers, in London, are employed to get large bills discounted, for which they receive from $\frac{1}{12}$ to $\frac{1}{4}$ per cent., according to the length of the time which the bill has to run, and the plenty or scarcity of money at the time.

This method of computing discount is what is employed by bankers and others, who discount bills. They charge interest at 5 per cent., on the sum of the bill, for the time it has to run, by which they have more than 5 per cent. on the *sum they advance;* and, though the highest interest allowed by law, in Britain, is 5 per cent., yet, in this instance of advancing money, the practice is sanctioned by the general practice of bankers and others, who discount bills.

Some bankers also charge $\frac{1}{2}$ per cent. in name of commission, which is added to the discount, and the amount deducted from the sum of the bill.

Though it be usual among men of business to consider *interest* and *discount* as the same, there is a material difference, the *former* being to the latter as 21 to 20, for 1 year, at 5 per cent.; or discount is always *less* than interest, by the interest of the true discount for the proposed time. To illustrate this, suppose a bill of £105., at 12 months date, is offered to a banker to be discounted, he retains £5. 5s. as the *discount*, which is, in reality, the *interest* of the *sum* of the *bill;* whereas, he ought only to charge interest on the sum he advances, (£100,) or, which is the same, the true discount on £105; and the remainder or proceeds should be such a sum as, if put out at interest, would amount to £105, at the end of 12 months, when the bill becomes due. But, by deducting £5. 5s. from £105., the proceeds are only £99. 15s.; and this sum will amount to no more than £104. 14s. 9d., in 12 months, at 5 per cent., which is 5s. 3d. less than £105, the sum of the bill. If he take only £5, which is the *real discount*, the remainder, £100., will amount to £105; and if the £5. of discount be improved at interest, it will, at the end of 12 months, amount to £5. 5s.

When the term of a bill is expressed in months, calendar months are always understood. Thus, if a bill be dated the 1st January, and made payable at *one month* after date, the term, or month, expires on the 1st February. And if a bill be dated 28th, 29th, 30th, or 31st January, and payable one month after date, the term expires on the 28th February, in common years, and in the three latter cases, in leap-year, on the 29th.

If there be more bills than one discounted at a time, add the several products (of the sums by the days) together, and find the discount, or interest on the sum, either dividing by 7300, or by any of the foregoing methods, which will give the whole discount. This method is much shorter than finding the dis-

count separately, on each bill, as it reduces the whole to one division.

EXERCISES.

1. A bill of £250., dated 12th May, 1816, payable 3 months after date, was discounted 26th May. Required the proceeds.

2. A bill of £250. 7s. 6d., which has 95 days to run, was discounted by Messrs. Coutts and Co. London. Required the proceeds.

3. What deduction ought to be allowed, for present payment, on a bill of £392. 15s., payable 91 days hence?

4. Required the proceeds of a bill of £198. 15s., drawn and discounted 7th January, 1817, payable at 3 months date.

5. A bill of £45. 15s. 6d., dated 9th August, 1816, payable 4 months after date, was discounted 20th September; commission, $\frac{1}{2}$ per cent. Required the proceeds.

6. A bill of £153., dated 11th July 1816, payable 3 months after date, was discounted 1st September; commission, $\frac{1}{3}$ per cent. How much money did the holder receive?

7. R. C. banker, discounted the following bills, on the 12th August. John Bowden's, of £154. 16s., dated 4th May, at 6 months; J. Jay's, of £145. 9s., dated 8th July, at 4 months: the commission was $\frac{1}{2}$ per cent. Required the proceeds.

8. What is the amount of the discount, and net proceeds, of the 4 following bills, discounted 21st December?

1. for	£483	1	6,	Rotterdam,	2d Dec. at $2\frac{1}{3}$ usance.
2.	217	8	9	———	acd. 13th Dec. at 30d sight
3.	721	0	10	———	24th Nov. at 3 months date
4.	165	3	4	———	10th Dec. at 2 months date

9. A house in Hamburgh remits to its correspondent, in London, the following bills:

1. for	£364	1	0,	dated 10th. Nov. at 2 usance
2.	721	0	6,	accepted 13th Nov. at 2 mo. sight
3.	511	10	4,	1st. Nov. at 3 months date

and draws on the 10th Nov. for £511 6 0, at 2 usance
and —— on ditto —— for £946 0 4, at $2\frac{1}{2}$ usance

Required on what side, and the amount of the balance, after allowing interest, reciprocally, at 5 per cent., $\frac{1}{2}$ commission, and 14s. 6d. for postages.

EQUATION OF PAYMENTS.

Equation of Payments is the determination of a mean or equated time, at which two or more sums, due at different dates, may be paid at once, without disadvantage, either to debtor or creditor.

RULE.

Multiply each sum by the time it has to run, before it be due, and divide the sum of the products by the sum of the debts. The quotient is the equated time of payment*.

EXAMPLE.

A merchant owes £89., due this day, 1st March; £72. on the 10th May; £45. on the 4th July; and £54. on the 16th August. Required the mean time of paying the whole.

* This rule is founded on the supposition, that the sum of the interests of the debts payable *before* the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable *after* the equated time, from that time to their terms; an unfair supposition, for though the gain, arising from keeping a sum after it is due, is equal to its *interest* for that time, yet the loss sustained by paying a debt before it is due, is only equal to its *discount* for that time, which is always less than the interest. The rule, therefore, is not accurate, but it affords an approximation exact enough for business; and, being easy, is generally used.

Mar. 1st,	£89 due at present	
May 10th,	72 × 70 days	= 5040
July 4th,	45 × 125 days	= 5625
Aug. 16th,	54 × 168 days	= 9072
	260	260)19737
		76 days, after 1st Mar.;

which is the 16th May, the mean time.

METHOD II.

	£		days		
Mar. 1st,	89	×	70	=	6230
May 10th,	72				
	161	×	55	=	8855
July 4th,	45				
	206	×	43	=	8858
Aug. 16th,	54				
	260				)23943

92 days previous to the 16th August; which brings the mean time to the 16th May, as before.

Here the first sum is multiplied by the number of days to the second sum falling due. The amount of the first and second sums by the time between the second and third falling due, and so on with the rest. The sum of the products, divided by the whole debt, gives 92 days, the time to be reckoned prior to the date of the last sum, as the mean time. This method is more generally used in business than the first.

These methods, being simple and easy, are, in most cases which occur, very near the truth, affording a good approximation. Besides, these are strictly agreeable to the custom of men in business, who allow interest on money paid *before* it be due, as well as on money retained *after* it is due, and their practice regulates all commercial computations.

In the same manner may the average or mean price of goods, at different prices, be found; but this more properly belongs to Alligation Medial.

EXERCISES.

1. The sum of £95. is due as follows: £25. in 6 months; £30. in 7 months; and £40. in 10 months; at what time ought the whole to be paid at once?

2. The sum of £120. was to have been paid as follows: £45. at 4 months; £60. at 9 months; and the rest at 12 months; but the debtor agrees to pay the whole at once. Required the average time.

3. A debt is to be paid thus; one half at 8 months; one fourth at 12 months; and the balance at 15 months. Required the time of paying the whole.

4. A owes B £560. of which £60. is to be paid ready money, and the rest in 5 payments of £100. each, every 3 months. Required the equated time for paying the whole.

5. Bought goods payable as follows; £50. the 1st of May; £64. the 4th of June; £86. the 1st August; and £90. the 5th of September. Required the mean time for paying the whole.

6. A sells for B a parcel of goods. The following is a state of the sales: £110. due 1st of March; £108. 15s. due the 25th ditto; £107. 12s. April 10th; £217. 14s. May 2d; £218. 7s. the 29th ditto; £110. 12s. 6d. July 4th; and £110. 11s. 6d. due November 10th. They wish to settle accounts; required the mean time.

VARIETIES IN PROPORTION*.

PROFIT AND LOSS.

MERCHANTS have found it necessary, in estimating their profits and losses, to have some common standard by which the gain

* This rule may be considered merely as a variety in Proportion, most of the questions being solved by that excellent rule.

or loss actually made, or proposed to be made, on any article of trade, may be tried or expressed, and the standard which has been fixed upon, by universal consent, for this purpose, is, the *centum*, or hundred, almost every gain or loss which arises from the sale or purchase of goods being stated at so much per cent. This rule, therefore, explains the methods of calculating the gain or loss, per cent. on the purchase or sale of any article of merchandise.

CASE I.

WHEN THE BUYING AND SELLING PRICES ARE GIVEN, TO FIND THE GAIN OR LOSS PER CENT.

RULE.

As the buying price is to the gain or loss on it; so is 100 to the gain or loss, per cent.

EXAMPLE I.

Bought cloth at 3s. 4d. per yard, and sold it at 4s. per yard. Required the gain per cent.

	d.		d.		£		
	40	:	8	: :	100		
or,	5	:	1	: :	100	:	20 per cent. gain.

EXAMPLE II.

Sold indigo for 9s. 7d. per lb., which cost 10s. 5d. per .; what was the loss per cent.?

s.	d.		d.		£
10	5	:	10	: :	100
12					

125) 10 × 100 = 1000(8 per cent. loss.
1000

EXERCISES.

1. If 2d. be gained on each shilling, prime cost, what is the gain per cent.?

2. Bought barley at 30s. per boll, which, having received damage, was sold again for 27s. per boll; what was the loss per cent.?

3. Bought 3 tons of hemp, for £245. 19s. 6d. and sold the whole, immediately, for £295. 19s. 6d.; what was gained per cent. by the transaction?

4. Bought cotton cloth at 2s. 10d. per yard, ready money, and sold it at 3s. 6d. per yard, at 6 months credit. Required the gain per cent.

CASE II.

TO FIND THE PRICE AT WHICH AN ARTICLE SHOULD BE SOLD, TO GAIN OR LOSE SO MUCH PER CENT.

RULE.

1. As 100 is to 100 *plus* the gain, or *minus* the loss, so is the prime cost to the gain or loss per cent.

2. Take parts of the prime cost for the rate per cent; add the result, in the case of gain, but deduct it, when there is loss.

EXAMPLE.

Bought sugar at 1s. 3d. per lb.; what must it be sold for, per lb. to gain 15 per cent.?

$$\begin{array}{llll} \pounds & \pounds & & \text{s.} \\ 100 & : 115 & :: & 1\tfrac{1}{4} \\ & \tfrac{1}{4} = & 28.75 & \\ \hline & 1{,}00) & 143.75 & \\ \hline & & 1.4375 & = 1\text{s. } 5\tfrac{1}{4}\text{d. per lb.} \end{array}$$

$$\begin{array}{rrr} & \text{s.} & \text{d.} \\ & 1 & 3 \\ 10 \text{ p. ct.} = \tfrac{1}{10} = & 0 & 1\tfrac{1}{2} \\ 5 \text{ p. ct.} = \tfrac{1}{2} = & 0 & 0\tfrac{3}{4} \\ \hline = & 1\text{s.} & 5\tfrac{1}{4}\text{d.} \end{array}$$

EXERCISES.

1. Bought cotton at 3s. 4d. per lb.; at what must it be sold per lb. to gain 20 per cent.?

2. Gained 20 per cent on sugar, which was bought for 75s. per cwt; at what was it sold per cwt.?

3. A quantity of flax was bought for £63. per ton, which was sold at 10 per cent loss; what was the selling price?

4. The prime cost and charges of 640 lb. of Bohea tea amounted to £84. 13s. 4d.; what must it be sold at per lb. in order to gain $17\frac{1}{2}$ per cent.?

5. Bought cloth at 18s. per yard; at what must it be sold to clear 20 per cent and allow 8 months credit?

6. Bought sugar at 65s. per cwt., at 4 months credit; at what must it be sold, per cwt. to gain $16\frac{2}{3}$ per cent., and allow 6 months credit?

CASE III.

TO FIND THE PRIME COST, WHEN THE SELLING PRICE AND GAIN OR LOSS, PER CENT, ARE GIVEN.

RULE.

As 100 plus the gain per cent, or minus the loss per cent, is to 100; so is the selling price to the prime cost.

EXAMPLE I.

Sold 12 cwt. of sugar for £43. 15s. by which I gained 20 per cent.; what was the prime cost?

100								
20								
			£ s.		£	s.	d.	
£120	:	100	:: 43 15	:	36	9	2	prime cost.

EXAMPLE II.

By selling cloth at 9s. 7d. per yard, I lost 8 per cent.; what was the prime cost?

$$\begin{array}{rcccccccl} 100 & & & & & & & & \\ 8 & & & & & & & & \\ \hline & & £ & & s.\ d. & & s.\ d. & & \\ £\ 92 & : & 100 & :: & 9\ \ 7 & : & 10\ \ 5 & & \text{prime cost.} \end{array}$$

EXERCISES.

1. Sold 500 lb. of indigo for £135. by which I cleared 15 per cent.; what did it cost me per lb.?

2. Sold a quantity of cotton wool at £13. 6s. per cwt., by which I lost at the rate of 5 per cent. Required the prime cost, per lb.

3. Received £18. 1s. 1½d. for a piece of cloth, by which I gained 1s. 6d. per yard, being at the rate of 12½ per cent. Required the length of the piece and the prime cost, per yard.

CASE IV.

TO FIND A PROPORTIONAL RATE PER CENT ON AN ADVANCED PRICE.

RULE.

As the price, whose rate per cent is given, is to the other price, so is 100 with the gain added, or loss subtracted, to a fourth number; which, if greater than 100, shows the gain, and, if less, the loss per cent.

EXAMPLE I.

Sold nutmegs at 15s. per lb. by which I gained 5 per cent., the price of the remainder was afterwards raised to 18s. 6d. per lb.; how much was that per cent.?

$$\begin{array}{lccccc} & s. & & s. & & £ \\ & 15 & : & 18\frac{1}{2} & :: & 105 \\ \text{or} & 1 & : & — & :: & 7 \end{array}$$

then $18\frac{1}{2} \times 7 = 129\frac{1}{2} - 100 = 29\frac{1}{2}$ per cent. Ans.

EXAMPLE II.

When tea was sold at 6s. 8d. per lb., there was gained 12½ per cent. Required the gain or loss at 5s. 4d. per lb.

	d.		d.		£
	80	:	64	: :	$112\frac{1}{2}$
or	10	:	—	: :	8

$$\begin{array}{r} 100 \\ \text{then } 112\tfrac{1}{2} \times 8 = 900 \div 10 = \quad 90 \\ \hline 10 \text{ per cent. loss.} \end{array}$$

EXERCISES.

1. A merchant sold part of a cargo of damaged flax at 40s. per cwt. by which he lost $2\frac{1}{2}$ per cent.; he sold the rest at 35s. per cwt.; what did he lose per cent. by this last sale?

2. By selling sugar at 80s. per cwt. I gained $10\frac{3}{4}$ per cent.; how much per cent. would I have gained or lost, had I sold it at 70s. per cwt.?

3. A loss of 5 per cent. was sustained by selling barley at 20s. a boll; what would have been gained or lost per cent. had it been sold at 22s. a boll?

4. A loss of 10 per cent. was sustained by selling flax at £72. per ton; what would have been gained or lost, per cent. had it been sold at £78. per ton?

CASE V.

TO RAISE THE PRICE SO AS TO ADMIT OF ANY PROPOSED DISCOUNT.

RULE.

As 100 minus the proposed rate per cent. is to 100; so is the value of the article to the advanced price.

EXAMPLE.

What must rice be rated at per cwt. to get 32s. 6d. per cwt. after allowing a discount of $2\frac{1}{2}$ per cent.?

$$\begin{array}{lllllll} 100 & & & & & & \\ \underline{2\frac{1}{2}} & & \pounds & & s. & & s.\ \ d. \\ \pounds 97\frac{1}{2} & : & 100 & :: & 32\frac{1}{2} & : & 33\ \ 4 \text{ Answer.} \end{array}$$

This method is general; but, in many instances, the operation is shortened by particular methods; such as taking aliquot parts of the given price for the discount and adding the same to the given price; as, in the above example, $2\frac{1}{2}$ is $\frac{1}{40}$ part of 100, therefore, 32s. 6d. might have been multiplied by 40 and divided by 39, or $\frac{1}{39}$ part of itself added, thus:

$$\begin{array}{rr} & s.\ \ d. \\ & 32\ \ 6 \\ \frac{1}{39} = & \underline{0\ 10} \\ & 33s.\ 4d. \text{ as before.} \end{array}$$

EXERCISES.

1. A merchant has his usual profit by selling 13 cwt. 2 qrs. of sugar at £46. 5s.; how much must he raise the price to allow a discount of $7\frac{1}{2}$ per cent.?

2. What must cloth be sold at, which cost 15s. per yard, that 20 per cent. may be gained upon it?

3. Bought Dutch linen at 2s. 3d. per ell, Flemish; at what must it be sold per yard to gain 25 per cent.?

4. Bought silk at 16s. $7\frac{1}{2}$d. per lb. at what must I rate it per lb. to have $12\frac{1}{2}$ per cent, profit, after allowing a discount of 5 per cent.?

5. Bought wax at 2s. 3d. per lb. ready money; at what price per lb. must I rate it, to have $17\frac{1}{2}$ per cent. profit, after allowing a discount of 5 per cent. and six months credit?

6. At what price per yard must I rate a piece of broad cloth, which cost me 15s. per yard, that I may have 15 per cent. profit, after allowing a discount of 5 per cent. and 9 months credit?

BARTER.

BARTER is the exchanging of one article for another, and teaches how to proportion the quantities and prices of the commodities to be exchanged, (according to the conditions of the barter,) so that neither party may sustain loss.

The solution of questions of this kind depends on the general principle, that the value of the commodity *given*, and that of the commodity *received* shall be equal.

EXAMPLE.

How much sugar, at 67s. 6d. per cwt. should be given in barter for 640 lb. of tea, at 7s. per lb.?

s.		s.		lb.
$67\frac{1}{2}$	:	7	: :	640
				7

$$67.5)\overline{4480}$$

$$66.3704 = 66 \text{ cwt. } 1 \text{ qr. } 13 \text{ lb.}$$

EXERCISES.

1. How much tea, at 6s. per lb. should be given in barter for $25\frac{1}{2}$ yards of linen, at 4s. per yard?

2. How many yards of Irish linen, at 2s. 3d. should be delivered in barter for 80 pieces of Holland, of 20 yards each, at 3s. per yard?

3. A and B barter. A gives B 2 puncheons of rum, the one containing 113 gallons, at 16s. per gallon, and the other 118 gallons, at 18s. for which he received from B 13 cwt. 1 qr. 11 lb. of cotton wool, and £6. 12s. $2\frac{1}{4}$d. in cash. Required the value of the cotton wool per lb.

C and D barter.—C has 56 cwt. of barilla, worth 30s. per cwt., ready money; but, in barter, valued at 31s. 6d. D has flax worth 80s. per cwt., ready money; required the barter value of D's flax, and what quantity he ought to give C, for his 56 cwt. of barilla.

5. A has 200 yards of linen, worth 2s. 6d. a yard, which he barters with D, at 2s. 7½d., taking in return yarn, at 3s. 9d. a spindle, which D commonly sells at 3s. 4d. ready money. Required which of the two has the better bargain, and how much yarn A received for his linen.

6. A has flax, worth 76s. a cwt. which he offers to barter with B for Osnaburghs, worth 7¼ a yard. B agrees to take flax in part for his Osnaburghs, but rates them at 8¼d., and insists on having ⅓ of that value in cash. To this A consents, but rates his flax so as to be on equal terms with B. What does he charge for it per cwt.?

PARTNERSHIP.

When two or more persons unite, to carry on any branch of business, they are said to be in *partnership*, or in *company*; and, if their shares of the capital are all equal, the *gain* or *loss* of each partner is obtained, by dividing the whole gain or loss by the number of partners concerned in the business.

But, as the shares of the capital are seldom equal, the shares of the gain or loss will also be unequal. The most common cases which occur, in this species of business, are included in the three following varieties:

1. When a company's capital is divided into a certain number of equal shares, but some of the partners possess more shares than others. To find the respective share of the gain or loss, divide by the number of shares, and then multiply the quotient by the shares which each partner possesses.

2. When the sums advanced by the partners are different;

say, as the whole capital is to each partner's stock, so is the whole gain or loss to that partner's share of the same.

3. When the partners put in or withdraw their respective stocks at different periods. Multiply each partner's stock by the time it was employed; then say, as the *sum* of the products is to the product of each partner's stock, into the time, so is the whole gain or loss to that partner's *share* of the gain or loss.

EXAMPLES.

VAR. 1.—A, B, and C, were concerned in an adventure to Virginia; whereof A had $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{6}$ share. The gain on the adventure, after paying all expences, was £1000. Required the gain of each partner.

£
6)1000

£	166	13	4	C's share
	333	6	8	B's ditto
	500	0	0	A's ditto
£	1000	0	0	Proof.

Here the whole number of shares is 6, of which A has 3, B 2, and C 1; therefore the whole gain is divided by 6, and the quotient multiplied by these numbers, which gives the share of each partner.

VAR. 2.—A, B, and C, freight a ship to Jamaica; A puts in goods to the value of £475. 10s.; B to the value of £675. 3s. 4d.; and C to the value of £834. 6s. 8d.: they gained £547. 19s. on the voyage. What is the dividend to each, in proportion to his share of the capital?

	£	s.	d.						
	A=475	10	0						
	B=675	3	4						
	C=834	6	8						
Capital	1985	0	0				547.95 gain		
Abridged	397	0	0	:	$475\frac{1}{2}$	::	109.59	:	131.259=A.
	397	0	0	:	$675\frac{1}{6}$	::	109.59	:	186.376=B.
	397	0	0	:	$834\frac{1}{3}$	::	109.59	:	230.315=C.
								£	547.95 Proof.

Sometimes the answer can be found more expeditiously, by finding the share of gain, loss, or proceeds per cent., and performing the rest of the operation by practice, especially where there are few fractions.

A, B, and C, freight a ship to Jamaica; A contributed to the adventure £500, B £1200, and C £1300; they had returns in tobacco, the net proceeds whereof amounted to £3800: what is the dividend to each?

Sum of shares 3000 : 3800 :: 100

Abridged 3 : 38 :: 10
10

3)380

£126 13 4 per cent.
5

500 = 633	6	8	A draws	£633	6	8
500 = 633	6	8				
200 = 253	6	8	B draws	1520	0	0
1200 = 1520	0	0				
100 = 126	13	4	C draws	1646	13	4
			Proof	£3800	0	0

It will have the same effect, and sometimes the process will be still shorter, to find the proportional share of gain, loss, or proceeds, on £1, and do the rest by practice.

VAR. 3.—Four merchants, A, B, C, and D, enter into partnership thus:

A put in £	64	10	0	for $4\frac{1}{2}$ months,
B	78	15	0	for 6 months,
C	112	14	0	for $8\frac{1}{4}$ months,
D	125	5	0	for $5\frac{1}{4}$ months,

They gain £108. 18s. $4\frac{1}{2}$d.; what is due to each, in proportion their stocks, and the time they were respectively employed?

		£	Months	Prod.
First,	A's stock	64.5	×4.5	=290.25
	B's stock	78.75	×6	=472.5
	C's stock	112.7	×8.75	=986.125
	D's stock	125.25	×5.25	=657.5625

The sum of the products 2406.4375

				£	
Then	2406.4375 : 290.25	::	108.91875		
Abridged	19.2515 : 290.25	::	.8719 :	13.137	A's gain.
	19.2515 : 472.5	::	.8719 :	21.3859	B's ditto.
	19.2515 : 986.125	::	.8719 :	44.633	C's ditto.
	19.2515 : 657.5625	::	.8719 :	29.762	D's ditto.
				£108.9179	

METHOD II.

19.2515 : .8719 :: .045261 gain on £1.

Then	290.250	472.500
Multiplier inverted	162540.0	162540.0
	11.6100	18.9
	1.4513	2.3625
	580	945
	174	284
	3	5
	A's gain=£13.1370	B's gain=£21.3859

986.125	657.5625
162.540.0	162.540.0
39.445	26.3025
4.9306	3.2878
.2272	.1315
592	365
10	7
C's gain=£44.633	D's gain=£29.762

The several shares as before.

Questions of this kind seldom or never occur in business; all differences in point of time, are, in company-affairs, settled by an interest account; and, therefore, it is unnecessary to give examples of this kind.

If the partners be allowed only interest for their unequal advances, subtract the amount of the interest of their several stocks from the net profit on the business; then divide the remainder equally between the partners, to which add the interest of their respective stocks.

EXAMPLE.

Three merchants, A, B, and C, enter into partnership for 1 year; A advanced £340, B £290, and C £150. Their net gain is £260. How much will each partner draw, if the gain be divided in proportion to their stocks; or by allowing each only interest for the unequal advances?

A advanced	£340	780 : 340 :: 260 :	£113	6	8	A's share
B ———	290	780 : 290 :: 260 :	96	13	4	B's do.
C ———	150	780 : 150 :: 260 :	50	0	0	C's do.
whole capital,	780		£260	0	0	Proof.

INTEREST METHOD.

The profit on the business, as above					£260	0	0
Interest on A's stock £340 is	£17	0	0				
on B's do. 290 is	14	10	0				
on C's do. 150 is	7	10	0				
					£39	0	0
The remainder is					£221	0	0
One third to each is					£73	13	4

A's $\frac{1}{3}$ is	£73	13	4	B's $\frac{1}{3}$ is	£73	13	4	C's $\frac{1}{3}$ is	£73	13	4
Interest,	17	0	0	Interest,	14	10	0	Interest,	7	10	0
A's,	£90	13	4	B's,	£88	3	4	C's,	£81	3	4

The division of a bankrupt's estate, among his creditors, is performed by Var. 2; or by the directions given at page 280.

EXAMPLE.

Suppose a bankrupt's effects amount to £1739. 13s. 8½d., what dividend would fall to each of the following creditors, in proportion to their respective claims.

To A	he owes £313	7	3		To E	he owes £600	0	0
To B	 290	4	6		To F	 500	0	0
To C	 700	0	0		To G	 381	10	0
To D	 486	13	8		To H	 418	0	0

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	
3689	15	5 :	313	7	3 : :	1739	13	8½ :	147	14	11¼	A's
3689	15	5 :	290	4	6 : :	1739	13	8½ :	136	16	9¼	B's
3689	15	5 :	700	0	0 : :	1739	13	8½ :	330	10	0	C's
3689	15	5 :	486	13	8 : :	1739	13	8½ :	229	9	3¼	D's
3689	15	5 :	600	0	0 : :	1739	13	8½ :	282	17	10¼	E's
3689	15	5 :	500	0	0 : :	1739	13	8½ :	235	14	10½	F's
3689	15	5 :	381	10	0 : :	1739	13	8½ :	179	17	5½	G's
3689	15	5 :	418	0	0 : :	1739	13	8½ :	197	1	8	H's
						Proof,			£1739	13	8½	

When there are many creditors, it is best to find the dividend corresponding to £1, and proceed as in Method II., Var. 3.

In real partnership there are few instances where the shares of the company's capital are undetermined, as in some of the following examples; for, if they are not determined by one common denominator, they are generally even hundreds; or, if one advance more than another, the difference is made up, by an interest account, at balancing the books. But this case is of singular use, in settling compositions in bankruptcy and average losses in *insurance*. See Marine Insurance.

EXERCISES.

1. A, B, and C, buy a ship, for £1200, whereof A had $\frac{1}{2}$, B $\frac{1}{3}$, and C $\frac{1}{4}$; how much money did each pay?

2. The profits of a mercantile house amount to £10561. 17s. 3d.; of which A holds $\frac{8}{21}$, B $\frac{7}{21}$, C $\frac{4}{21}$, and D $\frac{2}{21}$. Required the share of each partner.

3. Three merchants in Co., equally concerned, gained £1567. 12s. 6d.; but A, who had the management of the business, was allowed for his salary $2\frac{1}{2}$ per cent. on the sales, amounting to £12257. 10s. Required the share of each.

4. A, B, C, and D, purchase a ship; A pays for 6 shares, B for 7, C for 3, and D for 2. They receive £315. 14s. 6d. freight, for the first voyage; how much of this sum ought each to receive?

5. Four partners, A, B, C, and D, engaged to trade in company; A's stock was £150, B's £320, C's £350, and D's £500; and, in the space of three years, they gained £730: required how much of the gain each had to receive.

6. A insures on a ship £130, B £75, C £36, D £28, and E £49; the whole value of the ship being £400, and loss be sustained to the amount of £250.; how much must each pay, and how much must the proprietors bear?

7. A, B, and C, are jointly concerned in trade. At commencing, A advanced £500, B £700, and C £1000: on balancing their books, at the end of 12 months, their net stock amounts to £2950. 8s. 4d.; A stands debtor to the company, in his private account, for £100.; B, for £60.; and C is creditor for £90. By their contract, they are to share equally of the gain, after allowing interest for their unequal advances. Required a statement of their accounts.

8. A and B enter into partnership: A advances £1000., to carry on the business; B pays no money, but, having a thorough knowledge of the trade, agrees to be manager. At the end of the year, the stock and effects amount to £1213. 7s. 9d. By agreement, A is allowed $7\frac{1}{2}$ per cent for the interest of his money, and the risk of business succeeding. B's salary, before he engaged in the copartnership, was £50.; and the profits are to be divided in the proportion which the interest of A's money bears to B's salary. Required a division of the profits, according to the contract.

9. Three partners agree to trade in Co., with a joint stock of £7800. A's share is $\frac{6}{13}$, B's $\frac{4}{13}$, and C's $\frac{3}{13}$. At the end of the year, when they balance their books, there appears a loss upon the business, of £1120. 6s. 8d. C, being discouraged at the ill success, desires to withdraw from the concern, and the rest of the partners agree to take the risk of recovering the out-

standing debts, and to advance his stock, on being allowed a discount of 17½ per cent, to which C assents. Required a state of their respective accounts.

10. A father, ignorant of numbers, ordered £311. 15s. to be divided among his five children, thus: give A (says he) $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, D $\frac{1}{6}$, and E $\frac{1}{7}$. It is required to divide the sum among them, according to the father's intention.

11. Divide 739 acres, 1 rood, among A, B, C, and D, according to their rents, which are: A's £200.; B's £133. 6s. 8d.; C's £113. 7s. 9d.; and D's £78. 11s. 2d. per annum.

12. A, B, and C, pay money into one common stock.—A pays £150, which lies 6 months; B, £160, which lies 8 months; and C, £800, which lies 5 months. They gained £510. 15s.; required each man's share of the gain.

13. Three merchants, A, B, and C, pay money into a common stock.—A pays £100, which remains 8 months; B £70, which remains 6 months; and C £50, which remains 10 months. The profits come to £45. Required each partner's share, according to his stock, and the time it was in trade.

14. If a bankrupt's debts amount to £1850. 10s. and his effects to £971. 10s. 3d. Required how much his composition should be per pound.

15. A bankrupt owed £2850, and paid his creditors with £1068. 15s. Required how much he paid per pound.

16. A bankrupt who pays his creditors 10s. 6d., pays in all £1081. 10s. What was the amount of his debts?

17. A merchant, failing in business, offers his creditors 15s. per pound, in 6 months; or equal security, to pay 20s. per pound, at the end of 5 years, without interest. Which of these offers should the creditors accept?

ALLIGATION.

THIS rule is chiefly employed to resolve questions relating to the mixture of several ingredients together*.

CASE I.

WHEN THE QUANTITIES AND PRICES OF SEVERAL INGREDIENTS, MIXED TOGETHER, OR GOODS, AT DIFFERENT PRICES, ARE GIVEN, TO FIND THE MEAN PRICE OF ANY PROPOSED QUANTITY.

RULE.

Divide the whole value by the whole quantity, which will quote the mean price.

Or, as the whole quantity is to the whole value, so is any proposed part to its price.

EXAMPLE I.

If 35 lb. of tea, at 7s. 6d., 28 lb. at 7s., and 17 lb. at 6s. 8d., were mixed together, what would 1 lb. of the composition be worth?

lbs.		s.	d.			s.	d.
35	at	7	6	per lb.	=	262	6
28	at	7	0	———	=	196	0
17	at	6	8	———	=	113	4
80 lb.						8,0)57,1	10
						7s. $1\frac{1}{4}$d.—$\frac{1}{10}$	Answer.

* By this rule the mean or average price of goods sold in different quantities, and at different *prices*, may also be found.

EXAMPLE II.

What is the average price, per cwt. of the following quantities of sugar, sold at the following prices?

		cwt.	qr.	lb.	
30 hhds.	weighing, net	451	2	17	at 63s. 0d. per cwt.
150 ditto	————	2270	3	14	at 67s. 6d. per cwt.
40 ditto	————	623	1	25	at 74s. 4d. per cwt.

cwt.	qr.	lb.		s.	d.			s.	d.
451	2	17	at	63	0	per cwt.	is	28454	0¼
2270	3	14	at	67	6	————	is	153284	1¼
623	1	25	at	74	4	————	is	46344	10
3346 cwt.								3346)228083	0
						mean price is		68s.	2d.

EXERCISES.

1. If 81 gallons of rum at 18s. 6d.; 47 gallons at 17s. 6d.; and 16 gallons at 15s. were mixed together; what would a gallon of the mixture be worth?

2. If 10 oz. of silver at 5s. 6d.; 9 oz. at 5s. 4d.; and 8 oz. at 5s. 3d. were melted down together; what would an ounce of the composition be worth?

3. If 8 oz. of gold, at 76s. 8d.; 9 oz. at 77s.; 10 oz. at 77s. 6d.; 5 oz. at 77s. 7d.; and 8 oz. at 77s. 8d. per oz. were melted down together; what would an ounce of the composition be worth?

4. Required the average price, per quarter, of the following quantities of wheat;

351 quarters at 63s. per quarter;
217 ditto at 65s. per ditto
333 ditto at 67s. per ditto.

CASE II.

WHEN THE PRICES OF THE INGREDIENTS ARE GIVEN, TO FIND WHAT QUANTITY OF EACH MUST BE TAKEN TO COMPOSE A MIXTURE, AT A GIVEN PRICE, NONE OF THE QUANTITIES BEING LIMITED.

RULE.

Place the prices under each other, and the mean price on their left. Link the prices together, so that one greater than the mean price may always be coupled with one less; then place the difference between each price, and the mean on the right of that price with which it is connected. These differences are the respective quantities at the prices on their left.

EXAMPLE.

A person wishes to mix gold of 18 carats fine, with gold of 19, of 21, and of 23 carats fine; what quantity of each must he take, that the composition may be 22 carats fine?

Mean 22,	18	1	at 18,
	19	1	at 19,
	21	1	at 21,
	23	4 + 3 + 1 = 7	at 23.

EXERCISES.

1. One would mix tea at 8s. per lb. with other kinds at 7s. 6d.; 7s.; 6s. 8d.; and 6s. per lb.; what quantity of each kind must he take, that the mixture may be worth 7s. 4d. per lb.?

2. One would mix brandy at 8s. 6d. per gallon, with other kinds at 9s.; 9s. 6d.; and 10s.; so that the composition may be worth 9s. 3d. per gallon. What quantity of each must he take?

3. How much alloy must be mixed with bullion, 11 oz. fine, to render the composition only 9 oz. fine?

4. How much sugar at 7d., at 8d., and at 11d., per lb., must be mixed together, to form a composition worth 9d. per lb.?

CASE III.

WHEN THE MEAN PRICE, THE PARTICULAR PRICE OF EACH INGREDIENT, AND THE QUANTITY OF ONE OF THE INGREDIENTS ARE GIVEN TO FIND THE QUANTITY OF EACH OF THE OTHERS.

RULE.

Find the quantities by last case, as if none of them were given; then say, as the difference on the right of the price, whose quantity is given, is to each of the other differences; so is the given quantity to the several quantities sought.

EXAMPLE.

A goldsmith has 18 oz. of gold, worth 77s. 9d. per oz., and other kinds at 77s. 6d., at 77s. and at 76s. 9d. per oz., of which he wishes to form a mass worth 77s. 4d. per ounce; what quantity of each kind must he take?

Mean 928	933	7	quantities
	930	4	
	924	2	
	921	5	

					s.	d.
then,	7 : 4 :: 18 :	$10\frac{2}{7}$ oz.	at 77s. 6d.	=	797	$1\frac{5}{7}$
	7 : 2 :: 18 :	$5\frac{1}{7}$ oz.	at 77s. 0d.	=	396	0
	7 : 5 :: 18 :	$12\frac{6}{7}$ oz.	at 76s 9d.	=	986	$9\frac{3}{7}$
		18 oz.	at 77s. 9d.	=	1399	6
Proof		$46\frac{2}{7}$ oz.	at 77s. 4d.	=	3579	$5\frac{1}{7}$

EXERCISES.

1. How much sugar, at 8d., at 9d., and at 10d. per lb., must be mixed with 28 lb. at 7d., that the mixture may be worth $9\frac{1}{4}$d. per lb.?

2. A goldsmith mixes 16 oz. of silver, at 5s. 8d. per oz., with other kinds, at 5s. 6d. and 5s. 3d., and as much alloy as reduces the mass to 5s. 2d. per oz. Required the weight of the mass.

3. A vintner mixes 8 gallons rum at 16s., and other kinds at 15s. 5d., at 15s., and at 14s. 6d. a gallon, with as much water as reduces the mixture to 12s. per gallon. Required the quantity of each ingredient in the mixture.

CASE IV.

WHEN THE WHOLE QUANTITY OF THE MIXTURE IS GIVEN, TO FIND THE QUANTITY OF EACH INGREDIENT.

RULE.

Find the quantities, as if the total quantity was not limited; then say, as the sum of the quantities is to the given quantity, so is each quantity found, to its respective quantity sought.

EXAMPLE.

Required to mix brandy at 20s., at 19s., and at 17s. 6d. per gallon, so as to make up 110 gallons, at 18s. 6d. per gallon?

$$\text{mean } 18\tfrac{1}{2}\left\{\begin{array}{l} 20 \\ 19 \\ 17\tfrac{1}{2} \end{array}\right. \qquad \begin{array}{r} 1 \\ 1 \\ 1\tfrac{1}{2}+\tfrac{1}{2}=2 \\ \hline 4 \end{array}$$

		gal.		
4 : 110 :: 1 :		$27\tfrac{1}{2}$	at 20s. 0d.	= 550
4 : 110 :: 1 :		$27\tfrac{1}{2}$	at 19s. 0d.	= $522\tfrac{1}{2}$
4 : 110 :: 2 :		55	at 17s. 6d.	= $962\tfrac{1}{2}$
	Proof	110	at 18s. 6d.	= 2035

EXERCISES.

1. A mass of gold and silver contains 150 inches, and its specific gravity is 16; how many inches of each metal does it contain, the specific gravity of gold being 19 and of silver 11?

2. Suppose gold of 19, of 21, and of 23 carats fine, given in to be coined at the mint; what quantity of each kind will it require to make 178 guineas, when 12 oz. gold, 22 carats fine, are coined into $44\tfrac{1}{2}$ guineas?

3. A vessel, containing 64 cubic inches, is filled with a mixture of brandy and water; what quantity of each does it contain, the specific gravity of water being 1000, of brandy 931, and of the mixture 950?

THE STOCKS, OR PUBLIC FUNDS.

The term Stock was originally used to signify the trading capital of several public companies; as that of the Bank of England, the East India Company, and the South Sea Company; but it is now chiefly employed to denote the Public Funds, or Debts of Government.

The practice of *funding* was introduced by the Venetians and Genoese, in the sixteenth century, and has since been adopted by most of the nations in Europe.

The establishment of the *funds*, in Britain, took place about the year 1690; since that period the funding system has rapidly increased, and has now arrived at an amazing extent.

When the annual expenditure of the State, at any time, exceeded the annual income arising from taxes, the plan which has invariably been adopted by government, for raising the sums deficient, has been to propose terms to the nation, for obtaining the advance of money, by mortgaging the *revenue* of *future years* for their indemnification. These mortgages have been of two kinds, *limited* and *perpetual*; but *perpetual annuities* have been granted to the greatest extent; and, even when the money was originally advanced on other conditions, the lenders have sometimes been induced, by subsequent offers, to accept of perpetual annuities, instead of the original terms.

The debt, for which *perpetual annuities** are granted, is called the *redeemable debt;* and the other is called, the irredeemable debt. The perpetual annuities are redeemable, at the

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* *Annuities* are the dividends, or interest of stock, which are paid twice a year.

option of government, when at *par*; but the irredeemable annuities exist only for a certain number of years, and annually *absorb*, in the interest, a portion of the *capital*.

Although the debts, thus contracted by government, are seldom paid for a long term of years, yet any creditor of the public may obtain money, for what is due to him, by transferring his share of the funds to another; and regular methods are established for transacting these *transfers* in an easy manner.— By this means, the Stocks have become a kind of circulating capital; and, in some respects, answer the same purposes as the circulating money in the nation.

When a stockholder transfers his share, he may sometimes be able to obtain a greater sum for it than the original value, or even what it cost him; and, at other times, he must accept of a less sum.

The value of the funds depends on the proportion between the *interest* they bear, compared with the interest which the money would produce, when employed to other purposes. This fluctuation in the price of Stock depends on many circumstances: particularly the *plenty* or *scarcity* of money; the *quantity* of the *public debt*; and any event in which the government is deeply concerned; such as making *peace* or *war*, &c.

The business of buying and selling stock, or *stock-jobbing*, is founded on the variation of the prices of stock. Persons of real property may buy or sell stock, according as the value is likely to rise or fall, in expectation of making profit by the difference of price. But a practice exists among persons, who often possess *no property* in the funds, to contract for the sale of stock, against a future day, at a price then agreed upon.— For instance, A agrees to sell B £1000. of stock, in the 3 *per cent consols*, to be transferred in 20 days, for £600. A has, in reality, no such stock; but, if the price of the same kind of stock, on the day appointed for the transfer, should only be 58 per cent, A may purchase as much as enable him to fulfil his bargain for £580, and thus gain £20. by the transaction: on the contrary, if the 3 *per cent consols* be 65 per cent, he will lose £50. Transactions of this kind are generally settled without any *actual* purchase or transfer of stock, by A paying to B, or receiving from him, the difference between the current price of the stock, on the day appointed, and the price agreed upon.

Although this practice is contrary to law, yet it is carried on

to a great extent. In the language of Exchange-alley, where matters of this kind are transacted, the buyer is called a *bull*, and the seller a *bear*. As neither party can be compelled by law to implement these bargains, their sense of honour and the disgrace and loss of future credit, which attends a breach of contract, are the principles by which this business is supported.

The interest or dividend on stock is paid half-yearly; and the purchaser has the benefit of the interest due on the stock he buys from the last term to the time of purchase. Therefore the prices of the stocks rise gradually, *cæteris paribus*, from term to term, and fall at the term when the interest is paid.

It is necessary, however, to observe, that the interest in the different funds is not always equal; the time of paying the dividends makes a difference, and a preference is also given to that stock which is most marketable, and least liable to be redeemed by government.

When a loan is proposed, such terms must be offered to the lenders, as may render the transaction beneficial; and this is now regulated by the prices of the old stocks. If the stocks, which bear interest at 4 per cent, sell at par, or rather above, the government may expect to borrow money at that rate; but, if these stocks are under par, the government must either grant a higher interest, or some other advantage to the lenders, in compensation for the difference. For this purpose, besides the perpetual annuity, another annuity has sometimes been granted for life, or for a term of years. Lotteries have sometimes been employed to facilitate the loan, by entitling the subscribers to a certain number of tickets, for which no higher price is charged than the exact value distributed in prizes, though their market price is generally £6. or £8. higher.

Without offering some further inducement than merely paying a moderate interest, it would be found almost impossible to raise the *immense* sums that are frequently wanted by government to defray the public expences; it has, therefore, been necessary to create a debt for a much larger sum than that which has been borrowed, and this debt is called Stock. Thus, suppose any individual advances £8000. in money, for which the nation is to become indebted to him, £10,000. and is to pay him interest upon that sum, at the rate of 4 per cent, per annum; then, from this transaction, he is said to possess £10,000. stock, in the 4 per cent annuities, and he will receive £[illegible]. annually, by half-yearly payments. Or, if the nation

£400

agree to become indebted to him, £13333. 6s. 8d., and to pay him interest upon it, at 3 per cent, per annum, he would then 400 receive the same yearly interest, (~~£200~~) and would be said to possess £13333. 6s. 8d. stock, in the 3 per cent annuities. This is called raising money at 5 per cent, because, for every £100. that is borrowed, such a quantity of stock is given, that the interest upon it amounts to £5. per annum; but, both the rate of interest that is paid, and the amount of the capital to be created, depend upon the bargain that is made between the *minister* and the *contractors*.

It belongs to the Chancellor of the Exchequer to propose the terms of the loan in Parliament; and he generally makes a previous agreement with some wealthy merchants, who are willing to advance the money on the terms proposed. The subscribers to the loan deposit a certain part of the sum subscribed, and are bound to pay the rest by instalments, or stated proportions, on appointed days, under pain of forfeiting what they have deposited. For this they are entitled, perhaps not only to hold their share in the capital, but to an annuity for 10 years, and to the right of receiving a certain number of lottery-tickets on advantageous terms. This *douceur* is called the *bonus* to the subscribers.

The capital advanced to the public, in the form of transferable stocks, and bearing interest, from taxes appropriated for that purpose, is called the *funded debt*. Besides, there is generally a considerable sum due by government, which is not disposed of in that manner, and therefore is distinguished by the appellation of the *unfunded debt*. This may arise from any sort of national expence, for which no provision has been made, or for which the provision has proved insufficient. The chief branches are, Exchequer bills, Navy bills, and Ordnance bills.

THE FOLLOWING ARE THE PRINCIPAL PERPETUAL GOVERNMENT ANNUITIES.

NAVY FIVE PER CENT ANNUITIES, produced from about 50 millions of stock, partly formed out of navy bills, converted, in 1784, into stock bearing interest at 5 per cent, whence the name.

FOUR PER CENT CONSOLIDATED ANNUITIES, produced from about the same quantity of stock, as the last, bearing interest at 4 per cent, as the name indicates; these annuities are called

consols, or consolidated, from the stock having been formed by the consolidation of several debts of government.

THREE PER CENT REDUCED ANNUITIES, produced by about 170 millions of stock, formed from several debts, that originally bore a higher rate of interest, but which, on various conditions, has been reduced to the rate which the name of the stock expresses.

THREE PER CENT CONSOLIDATED ANNUITIES, produced by above 400 millions of stock, in part formed by the consolidation of several stocks, bearing interest at 3 per cent. When the word consols is indefinitely used, it is always understood to mean these annuities.

THREE PER CENT IMPERIAL ANNUITIES, produced by about 8 millions of stock, created by loans to the Emperor of Germany, with the security of the interest being paid by the government of this country, whenever the emperor should fail in his engagement.

FIVE PER CENT IRISH ANNUITIES, produced by about 2 millions of stock, formed by loans for the use of Ireland, before the union.

THE TERMINABLE ANNUITIES ARE,

BANK LONG ANNUITIES, so called from the annual payment being from their origin made payable at the Bank, and from their being granted for a greater length of time than other terminable annuities. These annuities extend to the beginning of the year 1860, and the annual payments are about 11 hundred thousand pounds.

IMPERIAL SHORT ANNUITIES, formed in the same manner, and upon the same conditions, as the imperial 3 per cent annuities; they extend to May 1819, and amount to two hundred and thirty pounds per annum.

OMNIUM is a term denoting the different stocks formed by a loan, while any part of the loan remains unpaid. For example, suppose 50 millions of money are to be raised, and for every £100 in money are to be given £100 stock in three per cents, £50 stock in the 4 per cents, and 6s. 3d. per cent in the long annuities; then, if any person engage to advance £10,000 in

money; upon paying the first instalment, (for the money is usually advanced at the rate of about 10 per cent per month, until the whole is paid,) he will receive three receipts, which separately contain an engagement to transfer to the person possessing them, £10,000 stock in the 3 per cents, £5,000 stock in the 4 per cents, and £31. 10s. stock in the long annuities, upon the whole of the instalments being paid at or before the appointed time. While these three receipts are sold together, and before the whole of the instalments has been paid, they are called OMNIUM, as they are made up of all or of several of the stocks.

SCRIP is a term given to each of the receipts of the omnium, when they are sold separately: thus, in the foregoing supposition, if the receipt containing the engagement to transfer the £10,000 in the 3 per cents, be sold without the other two receipts, this would be called a sale of *scrip*. Immediately after the whole of the instalments upon any scrip receipt is paid, the transfer of the stock is made to the person who holds it, and there is usually a discount allowed for any prompt payment.

N.B. When the stock created by any loan is formed in only one sort of stock, there is properly speaking no *omnium*, though then by a misnomer, the scrip receipt is called by that name.

STOCKS OF THE PRINCIPAL PUBLIC COMPANIES.

BANK STOCK is a capital of nearly 12 millions, with which the company of the Bank has accommodated Government with various loans, and with which they carry on the banking business, purchase bullion, &c. The dividends on Bank Stock are now 10 per cent, so that the profits of the company are nearly 12 hundred thousand pounds per annum.

INDIA STOCK forms the trading capital of the East India company; this stock (consisting of six millions) produces an annual dividend of 10½ per cent.

SOUTH SEA STOCK AND ANNUITIES consist of, or are produced from, a capital of nearly 20 millions: the greatest part of this is lent to Government, for which the South Sea Company receive 3 per cent; but, from the increase of other profits, the dividends to the proprietors are 3½ per cent.

Besides the permanent loans to government, which have cre-

ated the perpetual and terminable annuities, various sums have been raised from time to time, as temporary loans, on what are called Exchequer Bills, from their being made payable at the treasury of the Exchequer.

Exchequer Bills are issued for different hundreds or thousands of pounds, and bear an interest of 3½d. per cent, per diem, from the day of their date to the time when they are advertised to be paid off.

Navy Bills are merely bills of exchange, drawn at 90 days date, and are given by the Commissioners of the Navy for the amount of supplies, for the use of that department, and of the interest upon those amounts, at 3d. per cent, per diem.

India Bonds are issued by the East India Company, and bear interest at 5 per cent, per annum.

SINKING FUND.

A portion of the revenue has been set apart for the redeeming stock, or paying off the debts of government, which operates on the principle of *compound interest*, and has been termed the Sinking Fund. In the year 1786, it was raised to 1 million, annually, and in 1792 it was raised to £1,200,000. A grant was likewise added of 1 per cent, per annum, on every new loan that has been raised since.

The Sinking Fund, or, as it is now generally called, the Consolidated Fund, is under the direction of a certain number of commissioners, who regularly apply it in buying up or redeeming stock, and the interest, arising from the stock thus redeemed, goes immediately to the increase of the fund, and also all the *terminable* annuities, as they become extinct.

By the operation of this excellent plan, which was presented by Dr. Price to the late Mr. Pitt, more than 160 millions of public debt have been paid off by this fund; and, if no new loans were contracted, and this fund not encroached upon, the whole national debt, which now amounts to about 900 millions, would be redeemed in less than 60 years.

The prices of the stocks, &c. are stated in the lists that are published, as follows:

1.	3 per Cent Consols	$63\frac{1}{4}$, $64\frac{3}{4}$, $\frac{1}{2}$
2.	Bank Long Annuities	$16\frac{1}{4}$, 1—16
3.	Exchequer Bills	2 3 premium.
4.	India Bonds	1 pr. 2 dis.
5.	Omnium	$3\frac{3}{4}$ premium.

The *first* of these signifies that the value of £100. of this kind of stock, sold, on the day this price was quoted, for £63. 5s. in money, at the beginning of the market, that this stock rose to £64. 15s. and left off at £64. 10s.

The *second* signifies that any annual payment of these annuities was worth $16\frac{1}{4}$ years purchase, at the beginning, and left off at 16 and $\frac{1}{16}$ years purchase, at the conclusion of the market.

The *third* signifies that every £100. in Exchequer Bills, bore a *premium* of 2s. at the beginning, which advanced to 3s. towards the end of that day.

The *fourth* of these signifies that every £100. in India Bonds, sold at first at 1s. premium, and afterwards sold at 2s. discount.

The *fifth* signifies that Omnium sold for a premium of £3. 15s.

Sometimes the word *shut* is placed after the name of some kinds of stock; when this is the case, it signifies that the transfer books were *closed;* when *blanks* occur after the name of any kind of stock, in any quotation of the state of the stocks, it signifies that *none* of that kind of stock was sold on that day.

The transfer books of any stock are shut about a month before the dividends on that stock become due, and they remain so about six weeks.

The Dividends on the 3 per Cent Consols, 3 per Cent, 1720, South Sea Stock and Annuities, 3 per Cent, 1751, Navy 5 per Cents, 1784, 3 per Cent Imperial Annuities, and on Imperial short Annuities, are due Jan. 5th and on July 5th; on all other stocks they are due April 5th, and on Oct. 10th. These days, before the year 1800, were old Midsummer, Michaelmas, Christmas, and Lady Days.

The Interest on India Bonds is computed from March 31st and from Sept. 30th.

The BROKERAGE upon the Perpetual Annuities is $\frac{1}{8}$ per cent upon the quantity of stock purchased or sold; on Terminable Annuities it is $\frac{1}{4}$ per cent, upon the amount of the purchase money; on Exchequer bills and India bonds, it is 1s. per cent, and on Lottery tickets it is 6d. each before the time of drawing, and 1s. during that time.

In the following calculations, every purchase or sale is considered to be made through the medium of a Broker.

PERPETUAL ANNUITIES.

CASE I.

TO FIND THE VALUE, OR NET PROCEEDS, OF ANY KIND OF STOCK.

RULE.

1. Multiply the rate, or current price, of the stock, by the quantity of stock, and divide by 100.

2. Or, when the stock is not in even hundreds, work as in Practice.

EXAMPLE I.

What is the cost of £500., 3 per cent consolidated annuities, at 63 $\frac{3}{8}$ per cent?

£	s.	d.	
63	7	6	
		5	
316	17	6	value
	12	6	brokerage
£317	10	0	net cost

EXAMPLE II.

Sold £350. 10s. Bank stock, at 154$\frac{4}{7}$ per cent; what is the net proceeds?

£154.8				
350.5	or,	250	$\frac{1}{4}$	154.8
7740				387.
7740		100	$\frac{1}{10}$	1548
4644		10s.	$\frac{1}{100}$	774
£542.5740	=			£542.574 = £542 11 5¼
				Brok. 0 8 9
				£542 2 8¼

EXERCISES.

1. What is the cost of £650. stock, bought at 65¾ per ct.?

2. What are the net proceeds of £627. stock, sold at 62⅞ per cent?

3. What is the cost of £872. 16s. 7d. stock, at 65¾ p. ct.?

4. What are the net proceeds of £575. stock, in the Imperial 3 per cents, sold at 65½ per cent?

5. What is the cost of £37. stock, in the Navy 5 per cents, purchased at 95⅜ per cent?

7. What are the net proceeds of £371. stock, in the Irish 5 per cents, sold at 97¼ per cent?

CASE II.

TO FIND WHAT QUANTITY OF STOCK MAY BE BOUGHT FOR, OR SOLD TO PRODUCE, A GIVEN SUM.

RULE.

Add the brokerage to the current price, when the stock is to be purchased, but subtract it when it is to be sold: then multiply the given sum by 100, and divide the product by the current price of the stock, thus increased or diminished.

EXAMPLE.

What quantity of stock, at $62\frac{1}{4}$ per cent, must be sold to produce £500?

$$\begin{array}{r} £500 \\ 100 \\ \hline 62\frac{1}{4}-\frac{1}{8}=62\frac{1}{8}=62.125)50000 \\ \hline £804\ 16\ \ 7 \text{ stock.} \end{array}$$

This case may be proved by the last case, and *vice versa.*

EXERCISES.

1. What quantity of stock, at $61\frac{3}{4}$ per cent, can be bought with £700?

2. What quantity of India stock, at $195\frac{1}{4}$, can be bought for £1610. 16s. 3d.?

3. What quantity of 4 per cent stock, at $84\frac{7}{8}$ will £6178. 18s. purchase?

4. What quantity of 5 per cent Navy stock, at $100\frac{7}{8}$, will £1606. purchase?

5. If an annuity, of £300, produced by stock in the 3 per cent consols, were sold at $66\frac{5}{8}$ per cent; how much stock, in the Navy 5 per cents, at $97\frac{3}{4}$ per cent, could be purchased with the net proceeds?

CASE III.

TO FIND THE RATE OF INTEREST ARISING FROM MONEY VESTED IN THE STOCKS.

RULE.

As the current price of any kind of stock is to £100, so is the dividend on £100, of that kind of stock, to the rate of interest, arising from money vested in it.

EXAMPLE.

What rate of interest arises from money vested in the 3 per cent consols, when the price is $65\frac{1}{2}$; the dividend being $3\frac{1}{2}$ per cent?

$$\pounds 65\tfrac{1}{2} : 3\tfrac{1}{2} :: 100$$

$$\text{or, } 131 : 7 :: 100$$

$$7$$

$$131)700$$

$$\pounds 5 \quad 6 \quad 10\tfrac{1}{4} \text{ per cent.}$$

When allowance is to be made for the *proportionate interest**, deduct the proportionate interest from the day the last dividend was paid, to the day the stock was purchased; then proceed as directed in the rule.

EXAMPLE.

If the 4 per cent consols sell for $81\frac{1}{4}$ per cent, on the 12th January; what rate of interest, per cent, is obtained from money vested in that kind of stock?

From Oct. 10th to January 12th, is 94 days
From do. — to April 6th, is 178 days

	days		days		£	
then,	178	:	94	::	2	half yearly dividend.
			2			

$$178)188$$

$$\pounds\, 1.06 = \pounds 1.\ 1s.\ 3d. \text{ proportionate interest}$$

$$81.25$$

£		£		£
£80.19	:	100	::	4
		4		

$$80.19)400.00$$

$$\pounds 4 \quad 19 \quad 9 \text{ rate of interest}$$

---

* This is the interest of the stock, at the given price, from the last day the dividends were paid, to the day the stock was bought. See page 298.

In this example the *true price* of the stock is £80. 19s., and the interest arising from money laid out on it, at this price, is £4. 19s. 9d. per cent.

EXERCISES.

1. What rate of interest arises from money vested in the 3 per cent consols, when the price is at $67\frac{5}{8}$ per cent?

2. What rate of interest arises from money in Bank stock, when it sells at 175; the dividend being at 7 per cent?

3. What rate of interest arises from money vested in India stock, when the price is 180; the dividend being $10\frac{1}{2}$ per cent?

4. Bought £3000. stock, in the 3 per cent consols, when at 63, and sold out when at $67\frac{3}{8}$. Required the gain, allowing $\frac{1}{8}$ per cent for brokerage?

5. What is the rate of interest that is made of money, laid out in the 3 per cent consols, selling for $63\frac{1}{2}$ per cent, on the 18th February?

6. If the following stocks sell for the following prices, on the 1st June, which produces the greatest annual interest per cent, of money laid out in them; viz. 5 per cent Navy, $97\frac{3}{8}$; 3 per consols, $62\frac{1}{8}$; and 3 per cent reduced, $60\frac{7}{8}$.

CASE IV.

TO FIND WHAT SUM MUST BE LAID OUT IN THE PURCHASE OF ANY KIND OF STOCK, TO PRODUCE A GIVEN INCOME.

RULE.

As the rate of interest, on the proposed stock, is to the given income, so is £100 to the quantity of stock to be purchased; the cost of which may be found by Case I.

EXAMPLE.

What sum of money must be laid out in the Irish 5 per cents, at $97\frac{1}{2}$ per cent, to produce an income of £150. per annum?

£		£		£		£	
5	:	150	: :	100	:	3000	stock.

then, 97.5
3

£2925 0 value of stock
$\frac{3000}{800}$ = 3 15 brokerage

£2928 15 whole cost

EXERCISES.

1. What sum of money must be laid out, in the 3 per cent consols, at $63\frac{3}{8}$ per cent, to produce an income of £400 per an.?

2. What sum must be laid out in the Reduced, at $61\frac{1}{4}$ per cent, to produce an income of £200. per annum?

3. What sum of money must be laid out in the Imperial 3 per cents, at $63\frac{1}{4}$ per cent, to produce an income of £500?

4. If £4000. in the Reduced be sold out, at $64\frac{1}{4}$ per cent, what sum will it require, in addition to the net proceeds, to procure an income of £300, by purchasing stock, in the Navy 5 per cents, at $97\frac{1}{4}$ per cent?

CASE V.

TO FIND THE AMOUNT OF THE HALF YEARLY DIVIDENDS.

RULE.

Multiply the half yearly dividend on £100, of the given stock, by the quantity of stock; then divide the product by 100.

EXAMPLE.

What is the half yearly dividend on £75. stock, in the 4 per cents?

75 × 2 = 150 = £1 10s. dividend.

EXERCISES.

1. What is the amount of the half yearly dividend on £61. 12s. 4d. stock, in the 5 per cents?

2. What is the half yearly dividend on £573. 13s. 10 stock, in the 4 per cent Consols?

3. If £700. 13s. 4d. stock in the 3 per cents, was sold out, at $64\frac{1}{4}$ per cent, and the money laid out in the 4 per cents, at $81\frac{1}{2}$ per cent; what difference, in the net half yearly dividend, would the exchange produce?

TERMINABLE ANNUITIES.

CASE I.

TO FIND THE COST, OR NET PROCEEDS, OF A GIVEN TERMINABLE ANNUITY.

RULE.

Multiply the given annuity by the number of years purchase, at which the annuity is bought or sold, to which add the brokerage, if the annuity is to be bought, but subtract it if to be sold.

EXAMPLE.

Required the net proceeds of a Bank Long Annuity of £500. 12s. 6d., sold at $17\frac{1}{8}$ years purchase.

			£500	12	6×1	
					4	
	£500.625	or,	2002	10	0	
	$17\frac{1}{8}$				4	
	3504375		8010	0	0	
	500625		500	12	6	
	62578	$\frac{1}{8}$ =	62	11	$6\frac{3}{4}$	
	£8573.203	=	£8573	4	$0\frac{3}{4}$	Val. An.
Brok. $\frac{1}{800}$ =	10.7165	=	10	14	4	
	£8562.4865	=	£8562	9	$8\frac{3}{4}$	Proc.

EXERCISES.

1. What is the cost of an Imperial Annuity of £300, bought at 7¼ years purchase?

2. What are the net proceeds of £202. 10s., Bank Long Annuity, sold at 18¼ years purchase?

3. What is the cost of £150. Long Annuity, bought at 18¼ years purchase?

CASE II.

TO FIND WHAT ANNUITY MAY BE PURCHASED WITH, OR SOLD TO PRODUCE, A GIVEN SUM.

RULE.

As £100. *plus, the brokerage* of ⅛ per cent, when the annuity is to be bought, or *minus* the *brokerage,* when the annuity is to be sold, is to the given sum, so is £100 to the value of the annuity*; which divide by the rate of the annuity, and the quotient is the yearly annuity.

EXAMPLE.

What yearly annuity, at 17¼ years purchase, can be bought for £1600, in the Bank Long Annuities?

$$
\begin{array}{rcrcr}
 & £ & & £ & & £ \\
 & 100\frac{1}{8} & : & 1600 & :: & 100 \\
 & 8 & & & & 8 \\
\hline
\text{or,} & 801 & : & 1600 & :: & 800 \\
 & & & 800 & & \\
\hline
 & & 801)1280000 & & & \\
\hline
 & & 17.25) & 1598 \text{ Value Ann.} & & \\
\hline
 & & & £92\ 12\ \ 8 \text{ yearly annuity} & &
\end{array}
$$

* Instead of stating the above proportion, it will be sufficiently accurate, in practice, to deduct $\frac{1}{800}$ part, from the given sum, when the annuity is to be bought, and to add the same part, when it is to be sold.

EXERCISES.

1. What yearly annuity, at $7\frac{1}{2}$ years purchase, may be bought with £1400?

2. What yearly annuity can be purchased for £200, in the Imperial Short Annuities, at $7\frac{5}{17}$ years purchase?

EXCHEQUER BILLS, &c.

CASE I.

TO FIND THE INTEREST ON A GIVEN AMOUNT OF EXCHEQUER BILLS.

RULE.

Find the number of days, from the date of the bills, to the day they are advertised to be paid off, which, at $3\frac{1}{2}$d*. per day, will be the interest of £100.; and this, multiplied by the number of hundreds, will give the interest of the amount of the bills.

EXAMPLE.

What is the interest of 3 Exchequer Bills, of £500. each, dated the 17th Oct, 1816, and to be paid on the 17th Feb. 1817?

From 17th Oct. to 17th Feb. is 123 days.

$$
\begin{array}{rrrl}
123 \text{ at } 3\frac{1}{2}\text{d.} = £1 & 15 & 10\frac{1}{2} & \\
 & & 3 & \\
\hline
5 & 7 & 7\frac{1}{2} & \\
 & & 5 & \\
\hline
£26 & 18 & 1\frac{1}{2} & \text{interest}
\end{array}
$$

---

* See page 297.

The interest due upon a given amount, in India Bonds, is found in a similar manner; only it is necessary to observe, that the interest is reckoned in months and days.

EXAMPLE.

What interest is due, July 10th, on two India Bonds, of £100. each?

From March 31st to July 10th, is 3 months and 10 days.

		£	s.	d.		
		200				
	$\frac{1}{20}$ =	10	0	0	interest	for 1 year
3 months = $\frac{1}{4}$	=	2	10	0	ditto	for 3 months
10 days = $\frac{1}{9}$	=		5	$6\frac{2}{3}$	ditto	for 10 days
		£2	15	$6\frac{2}{3}$	ditto	for whole time

EXERCISES.

1. What is the interest on 4th May, 1817, of 4 Exchequer Bills, of £100. each, dated 30th July, 1816?

2. What is the interest of 2 Exchequer Bills, of £500 each, and one of £1000., on the 8th March, 1817; the bills being dated 3d October, 1816?

3. What is the interest due, on the 12th November, on 2 India Bonds, of £500. each?

CASE II.

TO FIND THE COST, OR NET PROCEEDS, OF ANY GIVEN AMOUNT, IN EXCHEQUER BILLS.

RULE.

Find the interest on the given sums, and add it to the amount of the bills, to which add or subtract the brokerage, according as the bills are bought or sold; and, should the bills be bought at a premium, it must also be added, but discount deducted.

EXAMPLE.

What is the cost of 2 Exchequer Bills, of £1000. each, bought at a premium of 2s. per cent, on the 2d January, 1817, and dated 27th October, 1816?

From Oct. 27th to Jan. 2d is 67 days

67 days				
20 hundreds =	£2000	0	0	
£1340 at $3\frac{1}{2}$d. =	£19	10	10	interest
	2	0	0	premium
2000 at 1s. per cent	1	0	0	brokerage
	£2022	10	10	whole cost.

The cost or net proceeds of a given amount, in India Bonds, may be found in a similar manner.

The amount of a Navy Bill, drawn for a given sum, and the interest, is also found in the same manner*.

EXERCISES.

1. What is the cost of 4 Exchequer Bills, of £1000. each, dated 8th Nov. 1816, and purchased the 20th February, 1817, at a premium of 4s per cent?

2. What is the cost of 6 Exchequer Bills of £100. each, 2 of £200. each, 2 of £500. each, and 1 of £1000; dated the 17th November, 1816, and purchased the 24th Feb. 1817, at a discount of 10s per cent?

3. For what amount should a Navy Bill be drawn for £677. 15s. 7d. and the interest for 90 days, at 3d. per cent, *per diem?*

4. For what amount should a Navy Bill be drawn to include the sum of £2000. and the interest for 90 days, at 3d. per cent, *per diem?*

5. What are the net proceeds of 2 India Bonds of £200. each, sold 1st March, at 2s. premium?

* Navy Bills are now considered only as bills of exchange; and, of course, are not marketable at the Stock Exchange.

6. What is the cost of 8 India Bonds, of £100. each, purchased the 4th July at 2s. discount?

OMNIUM AND SCRIP.

CASE 1.

TO FIND THE COST OR NET PROCEEDS OF A GIVEN QUANTITY OF OMNIUM.

RULE.

Find the amount of the instalments, to which add the brokerage upon buying, but deduct it upon selling; and if a premium has been given or received, it must also be added, but if a discount, it must be deducted.

EXAMPLE I.

What is the cost of £1400 Omnium, on which 65 per cent has been paid, at a premium of $2\frac{1}{2}$ per cent?

			£	s.	
£ 65	× 14	=	910	0	Amount of Instalments.
$2\frac{1}{2}$	× 14	=	35	0	——— Premium.
$\frac{1}{8}$	× 14	=	1	15	——— Brokerage.
			£946	15	Whole cost.

EXAMPLE II.

What are the net proceeds of £5000 Omnium, on which 3 instalments, of 10 per cent each, have been paid, at a premium of $5\frac{1}{4}$ per cent?

10
3
—

		£	s.	
30×50	=	1500	0	Amount of Instalments.
$5\frac{1}{4} \times 50$	=	262	10	Premium.
		1762	10	
$\frac{1}{8} \times 50$	=	6	5	Brokerage.
		£1756	5	Whole cost.

The value of Scrip is ascertained by finding the amount of the quantity of stock that the Scrip receipt will entitle the possessor to have transferred to him, at the market price, and subtracting from it the amount of the instalments that remain unpaid at the price at which the stock was taken, the remainder is the present value of the Scrip, to which the brokerage is to be added upon buying, and subtracted upon selling.

The price at which any stock is taken, is marked upon the Scrip receipt.

EXAMPLE.

What is the cost of £2000. Scrip in the Reduced, valued at 64 per cent, and sold at $66\frac{1}{8}$ per cent, there being 4 instalments, of 10 per cent each, remaining unpaid?

£2000 stock, at $66\frac{1}{8}$ per cent	=	£1322	10	0
£2000 Scrip, at 40 p. ct. is £800 unpaid, at 64 per cent	=	512	0	0
Present value		£810	10	0
Brokerage		2	10	0
Whole cost		£813	0	0

EXERCISES.

1. What is the price of £1000. Omnium, on which 25 per cent has been paid, at a premium of $1\frac{1}{4}$ per cent?

2. What are the net proceeds of £1000. Omnium, on which 20 per cent has been paid, at a discount of 2 per cent?

3. What is the value of £8000. Scrip, in the 4 per cents, valued at 81 per cent, and sold at $83\frac{1}{8}$ per cent, 6 instalments being unpaid, of 10 per cent each?

4. What is the cost of £2000. Scrip, in the Consols, valued at 62 per cent, and sold at $63\frac{3}{4}$ per cent, there being 7 instalments, of 10 per cent each, unpaid?

CASE II.

TO FIND THE AMOUNT OF THE DISCOUNT UPON THE PROMPT PAYMENT OF THE INSTALMENTS, EITHER UPON OMNIUM OR SCRIP.

RULE.

Find the discount upon the whole amount of the instalments for the time the *most distant* has to run*.

EXAMPLE.

What is the amount of discount at 5 per cent, on paying 6 instalments, at 10 per cent, on 11th July, 1816, upon £2000. Omnium, the last instalment being due, on each, 27th January, 1817.

From 11th July to 27th January is 200 days.

£2000 at 60 per cent =	£1200	0	0	Amount of Instal.
$(1200 \times 200) \div 7300$ =	32	17	6	Disct. for 200 days
	£1167	2	6	Net amount to be paid

When the discount is at any other rate than 5 per cent, find it at the given rate by some of the rules already given for calculating Interest, &c.

* This is the usual mode of calculating the discount, and it exceeds the discount of 5 per cent, per annum, calculated upon *each* instalment separately, the difference being an inducement to early payment.

EXERCISES.

1. What is the amount of the discount at 5 per cent, upon the payment of 4 instalments, on the 18th August, of 10 per cent each, on £4000. Omnium; the last instalment being due on 26th November?

2. What is the amount of the discount at 4 per cent, upon the payment of 6 instalments of 10 per cent each, upon £1000. Scrip in the Reduced, taken at 62 per cent, the payment being made 10th June, 1816, and the last instalment being due 20th January, 1817?

3. What is the amount of the discount, at 5 per cent, upon the payment of 6 instalments of 10 per cent each, on 30th July, on £1000. Omnium; the last instalment being due on the 18th November?

4. What is the amount of the discount, at 4 per cent, upon the payment of 7 instalments, of 10 per cent each, on the 6th September, 1816, on £1200. Scrip in the Consols, taken at 63; the last instalment being due, 3d January, 1817?

For an account of the *French* and *American* funds, see Exchange, under the heads of *Paris* and *America*.

EXPLANATION OF THE FOLLOWING TABLE.

Suppose the 3 per cents are at 58½, that is every £100. of that stock costs £58. 10s.; then by looking opposite to 58½ in the first column, South Sea Stock (which bears 3½ per cent) at £68. 5s.; the 4 per cents, £78.; the 5 per Cent Consols, £97. 10s.; Bank Stock, £175. 10s.; India Stock, £204. 15s.; and a sum invested in any of these securities at these prices is equal to buying land, at 19½ years purchase, or 5⅛ per cent.—Again, if the 3 per cents be 58½, and Bank Stock at 180, which of these is most advantageous to the purchaser? The 3 per cents at 58½, bring 5⅛ per cent, as above, and Bank Stock to yield the same interest, ought to be 175½; but Bank Stock at 180, brings only 5 per cent, as appears from the Table, so that the 3 per Cent Consols is 3½ per cent cheaper than Bank Stock.

A TABLE OF THE PUBLIC FUNDS.

Exhibiting, at ONE VIEW, the intrinsic value, per cent, of those securities, and the proportion they bear to each other, by which the most advantageous purchase may be known; and also what proportion such purchases bear to the value of Landed Estates and Life Annuities.

3 per cent Consols.	S. Sea Stock, 3 1-half per ct.	4 per cent Consols.	Navy 5 per cent.	Bank Stock 9 per cent.	India Stock 10 1-hf per ct.	Years Purchase.	Interest per annum. £	s.	d.
48	56	64	80	144	168	16	6	5	0
49½	57¾	66	82½	148½	173¼	16½	6	1	2
51	59½	68	85	153	178½	17	5	17	7
52½	61¼	70	87½	157½	183¾	17½	5	14	3
54	63	72	90	162	189	18	5	11	1
55½	64¾	74	92½	166½	194¼	18½	5	8	1
57	66½	76	95	171	199½	19	5	5	3
58½	68¼	78	97½	175½	204¾	19½	5	2	6
60	70	80	100	180	210	20	5	0	0
61½	71¾	82	102½	184½	215¼	20½	4	17	6
63	73½	84	105	189	220½	21	4	15	2
64½	75¼	86	107½	193½	225¾	21½	4	13	0
66	77	88	110	198	231	22	4	10	10
67½	78¾	90	112½	202½	236¼	22½	4	8	10
69	80½	92	115	207	241½	23	4	6	11
70½	82¼	94	117½	211½	246¾	23½	4	5	1
72	84	96	120	216	252	24	4	3	4
73½	85¾	98	122½	220½	257¼	24½	4	1	7
75	87½	100	125	225	262½	25	4	0	0
76½	89¼	102	127½	229½	267¾	25½	3	18	5
78	91	104	130	234	273	26	3	16	11
79½	92¾	106	132½	238½	278¼	26½	3	15	5
81	94½	108	135	243	283½	27	3	14	0
82½	96¼	110	137½	247½	288¾	27½	3	12	8
84	98	112	140	252	294	28	3	11	5
85½	99¾	114	142½	256½	299¼	28½	3	10	2
87	101½	116	145	261	304½	29	3	8	11
88½	103¼	118	147½	265½	309¾	29½	3	7	9
90	105	120	150	270	315	30	3	6	8

MARINE INSURANCE*.

MARINE INSURANCE, or **ASSURANCE,** is a contract of indemnity, by which one party engages, for a stipulated sum, to *insure* another party from any *loss* or *damage,* which his property may meet with, at sea.

Those persons, who take upon them the risk, are called *Insurers, Assurers,* or Underwriters†; and those, who are protected by the insurance, are called the Insured or Assured.

The sums paid to the underwriters, in consequence of the responsibility they take upon themselves, are called *Premiums;* and, in time of war, are of two kinds: viz. short premiums and long premiums.

A SHORT PREMIUM is received, when the vessel is warranted to sail with *convoy;* but if not, a *long Premium* is received.—See Return of Premium.

PREMIUMS are usually expressed in *guineas per cent:* the only exception is insurance for *time.* The whole premium, however, is not received by the *underwriters,* part being retained by the broker or person effecting the insurance. When the rate is expressed in *guineas,* the underwriters only receive the *pounds,* and the odd shillings are retained by the broker; and when the rate is expressed in pounds, they receive 5 per cent less. To distinguish this diminished premium from the *whole,* it is called the *Underwriters Premium.*

POLICIES are stamped papers, or parchments, containing

* The amount of premium, for insuring houses, &c. against fire, is calculated as in Commission.

† The term *Underwriters* is the most common, and has arisen from the writing of their names under the amount of their risk.

the particulars, or specific terms of insurance, and they are subject to certain duties; see page 318. These are distinguished into two sorts; *valued* policies and *open* policies.

VALUED POLICIES express the *value* of the property insured; and, in case of loss, no further proof is necessary, unless a fraud be suspected.

OPEN POLICIES do not express the value of the property insured; and, in case of loss, the value must be proved.

SHORT INTEREST is a term used to denote the overplus of property insured, or an insurance effected on more property than has actually been shipped.

RETURN OF PREMIUM.

The Assured is, in various cases, entitled to a return of the *whole*, or part of the premium.

These are; *first*,—when it is stipulated in the *policy*; *secondly*,—when it is implied by the nature of the contract.

1. When a Return of Premium is stipulated for in the Policy.

These Returns are, for departing with convoy; for sailing on or before a certain day; for ending the voyage short of its ultimate destination; and, in *general*, for any thing which lessens the risk of the insurer; who, having received a premium for running the *whole* risk of the voyage, agrees to make a proportional *return*, if any specified occurrence take place to lessen that risk.

2. When a Return of Premium is implied.

A Return of Premium is generally made, when the *interest* intended to be insured has never been brought within the terms of the *policy*, and the insurer has run no risk. But, if the risk has commenced, there is no *return* of premium.

If the policy be a *valued* one, and the insurer could, at any time, and under any circumstances, be called upon to pay the *whole sum insured*, there is no return of premium. But, where the insurer could never have been liable, the *whole* premium must

be returned. This is the case, if the risk is not run, though it may have been occasioned by the neglect or folly of the party insuring.

If the *insurer* knew of the ship's *arrival*, when he underwrote the policy, he must *return* the premium. On the contrary, if the *assured* knew of the *loss*, at the time he effected the insurance, the premium is *retained* by the insurer.

When the risk has *commenced*, the *assured* has no longer any power to retract, and the insurer is entitled to the *whole* of the premium, if there be no stipulation for a *return*.

What has just been stated, respecting an *implied* return of premium, relates to those cases where the *whole* premium is to be retained or returned; and where there is no stipulation to the contrary. But, where the insurer could never have been called upon to make good the *whole* of the *sum*, on which he received a premium, he can only retain the proportional part of the premium, on the *interest* for which he is responsible. For example, if he could only have been called upon to make good a loss of *one-half* of the sum, on which he received a premium, he must not retain the premium on a larger proportion than that *half*.—The return of premium, in this case, is called a return for *short interest*, or for *over insurance*. Both these terms are used indiscriminately; but there is obviously a distinction. Return for *short interest* is demanded on *valued policies*, or where the interest is *declared*; but, for *over insurance*, on *open policies*, or where the declaration is *general*.

A return for *short interest* is made, in cases where the property declared in the policy is not all shipped; for example, if 60 tons of *hemp*, valued at £3000., or at £50. per ton, be insured; or if 60 tons of hemp be declared, without any valuation, and should there be only 30 tons on board, or only half the interest, a return of half the premium must be made for *short interest*.

When a return of premium is claimed, for *over insurance*, it is in the case of an *open policy* on goods or on freight; and should the amount underwritten, in such an insurance, be £2000., and the amount of property at risk only £1000., it is plain, there is an over insurance of £1000.; and, in case of loss, the underwriter could have been called upon for no more than one-half the amount *insured*. A return of half the amount of the premium must, therefore, be made for *over insurance*.

In cases where the premium is returnable, either in *whole* or in *part*, (without a stipulation in the policy to that effect) it is customary to allow the insurer a *half per cent*, or 10s. per £100, for his trouble and disappointment; therefore, whenever it is said that the *whole* or *any part* of the premium should be returned, it is with this *deduction*. This principle is acknowledged at *Lloyds*, and is always acted upon, where no stipulation is made to the contrary.

The BROKERAGE for effecting an insurance is 1s. in the pound, or 5 per cent, upon the *underwriter's premium*, when the whole premium is expressed in *guineas*, as is commonly the case; but, when this is expressed in *pounds*, it is 5 per cent upon the *whole premium*. The brokerage for procuring the settlement of a *loss* is $\frac{1}{2}$ per cent upon the amount recovered.

The COMMISSION for effecting an insurance is $\frac{1}{2}$ per cent upon the amount insured, and for procuring the settlement of a *loss*, it is 2 per cent upon the amount recovered.

The STAMP DUTY upon policies of insurance, from one part of the *United Kingdom* to another, is 2s. 6d., on each £100. insured; and any *fraction* of £100 is considered another hundred. Every other *policy* is charged 5s. on each £100. insured; but brokers often charge something more, for the paper, printing, &c. and for keeping stampt policies on hand.

In all the following *exercises* the insurances are supposed to be effected upon commission, and through the medium of a *broker**, except those given in Case I.

CASE I.

TO FIND THE AMOUNT OF THE UNDERWRITER'S PREMIUM, THE BROKERAGE, AND THE POLICY, ON ANY SUM INSURED.

RULE.

When the premium per cent is expressed in *guineas*, calculate the whole amount of the premium (as in commission) at as

* In *performing* the exercises it would very much facilitate the progress of the accountant to fix on the *memory* the foregoing definitions; or to refer to the particular definitions connected with the exercise, to be performed, previous to attempting the solution of it.

many pounds per cent, which is the amount of the underwriter's premium; $\frac{1}{20}$ of which is the brokerage.

Or calculate at the given number of guineas per cent, from which deduct $\frac{1}{20}$ for the brokerage, and the remainder is the underwriter's premium.

EXAMPLE I.

What is the amount of the underwriter's premium, brokerage, and policy, upon insuring £500. on goods, from London to Hamburgh, at 4 guineas per cent?

	£4	0	underwriter's premium per ct.
		5	number of hundreds
	£20	0	whole underwriter's premium
$\frac{1}{20}$ =	1	0	brokerage, 1s. per pound
5 hun. at 5s. =	1	5	policy
	£22	5	whole amount

EXAMPLE II.

What is the amount of the underwriter's premium, brokerage, and policy, upon insuring £3255. on goods, from Liverpool to London, at 1½ guineas per cent?

METHOD I.

	£3255
	1½
	3255
	16275
	£48.825
brokerage $\frac{1}{20}$	= 2.44125
policy ⅛ of 33 h.	= 4.125
whole amount	£55.39125

METHOD II.

	£32,55			
$\frac{1}{100}$ =	32	11	0	
½ =	16	5	6	
	£48	16	6	und. prem.
=	£48	16	6	
=	2	8	10	
=	4	2	6	
=	£55	7	10	

In Example I., the stamp duty upon the policy is 5s., the vessel being bound to a *foreign* port; but in Example II. it is 2s. 6d., the vessel being only bound from one port in the *kingdom* to another. See page 318, article *stamp duty*, which must be referred to, in performing *each* of the following examples:

EXERCISES.

1. What is the amount of the underwriter's premium, brokerage, and policy, upon insuring goods to the amount of £1000., shipped on board a vessel, from London to Jamaica, at $3\frac{1}{4}$ guineas per cent?

2. What is the amount of the underwriter's premium, brokerage, and policy, on £3000, insured on a vessel bound from Bristol to London, at $1\frac{1}{4}$ guineas per cent?

3. What is the amount of the underwriter's premium, brokerage, and policy, upon insuring £1500. on goods, from Cork to Barbadoes, at $4\frac{1}{4}$ guineas per cent?

4. What is the amount of the underwriter's premium, brokerage, and policy, upon insuring £2425. on goods by the Company's ships to Bombay and home, at 10 guineas p. ct.?

CASE II.

TO FIND THE AMOUNT OF THE WHOLE PREMIUM, THE POLICY, AND COMMISSION.

RULE.

1. Multiply the premium, per cent, by the number of hundreds, the product is the whole premium; to which add the commission and cost of the policy.

2. If the sum to be insured contain a fractional part of £100, or be less than that sum, proceed as in Practice.

EXAMPLE.

Required the amount of the premium, commission, and policy, on insuring £550., upon a vessel from London to Lisbon, at $2\frac{1}{4}$ guineas per cent?

METHOD I.		METHOD II.			
		£	s.	d.	
£5.5		2	7	3	
$2\frac{1}{4}$				$5\frac{1}{2}$	
11.0					
1.375					
guineas 12.375		£11	16	3	
$\frac{1}{20}$ = .61875		1	3	$7\frac{1}{2}$	
£12.99375	=	£12	19	$10\frac{1}{2}$	whole premium
$\frac{1}{2}\times4\frac{1}{2}$ = 2.75	=	2	15	0	commission
$\frac{1}{4}\times5\frac{1}{2}$ = 1.375	=	1	7	6	policy
£17.11875	=	£17	2	$4\frac{1}{2}$	cost of insurance.

EXERCISES.

1. What is the whole cost of an insurance, effected on goods to the amount of £628., shipped in a vessel from Cadiz to London, at $1\frac{1}{4}$ guineas per cent?

2. What is the cost of an insurance effected on £1494. value of goods from Liverpool to Jamaica, at $3\frac{1}{4}$ guineas per cent?

3. What is the cost of an insurance, effected on £1275., at $1\frac{1}{4}$ guineas per cent, on goods from London to Leith?

CASE III.

TO FIND THE NET COST OF AN INSURANCE, WHEN THERE IS A RETURN OF PREMIUM UPON THE WHOLE VALUE INSURED.

RULE.

Find the whole cost of the insurance, as in last Case, and also the amount of the return, which deduct from the whole cost of the insurance, and the remainder is the *net* cost of the insurance.

EXAMPLE.

What is the net cost of an insurance effected on £437. at

9 guineas per cent, on goods from London to Leghorn, with a return of £5. per cent for convoy and arrival; the vessel having sailed with convoy, and the goods having arrived safe?

TO FIND THE RETURN.

£437
9
———
3933
$\frac{1}{20}$ = 196 13
———
£41,29 13
20
———
5,93
12
———
1,16

£437
5
———
£21,85
20
———
17,00

£	s.	d.	
41	5	11	premium
2	3	9	commission
1	5	0	policy
£44	14	8	cost of Insu.
21	17	0	amt. of return
Net cost of Ins. is £22	17	8*	

EXERCISES.

1. What is the net cost of an insurance effected on a quantity of sugar, value £2400. from Barbadoes to Liverpool, at 4½ guineas per cent, to return 2 guineas per cent for convoy and arrival; the vessel having sailed with convoy and the sugar having arrived safe?

2. What is the net cost of an insurance effected on 20 casks of sugar, at £45. per cask, from Jamaica to London, the premium being 12 guineas per cent, to return £7. per cent for safe arrival; the vessel having arrived safe and the sugar free from damage?

CASE IV.

TO FIND THE NET COST OF AN INSURANCE, WHEN THE RISK DOES NOT TAKE PLACE.

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* When the whole cost of the insurance is inserted in Account Sales, the return of premium must always be subtracted, to ascertain the net cost.

RULE.

Find the whole cost of the insurance, as in Case II., from which deduct the amount of the *underwriter's premium* and *commission* for the cancelment of the policy, and the remainder is the net cost of the insurance.

EXAMPLE.

Suppose an insurance to be effected on goods to the value of £775. from Smyrna to London, at 9 guineas per cent, but the risk did not commence; what was the net cost of the insurance?

	£	s.	d.					
	9	9						
		7¾ hun.						
	£66	3	0					
	7	1	9					
Whole premium	£73	4	9					
Commission	3	17	6		£	s.	d.	
Policy	2	0	0	......	79	2	3	cost of Ins.

TO FIND THE RETURN.

	£	s.	d.					
Undws. pre. p. ct.	£9	0	0					
Commission ditto	0	10	0					
	£8	10	0	×7¾=	£65	17	6	Amot. of Ins.
				Net cost of Ins.	£13	4	9	

EXERCISES.

1. Suppose an insurance to be effected on 120 tons of flax, from Riga to Hull, valued at £47. 10s. per ton, at 5 guineas per cent, but the risk not to take place; what would be the net charge of the insurance?

2. An insurance was effected on 75 tierces of coffee, valued at £15. per tierce, at 3 guineas per cent, from Jamaica to Bristol, the vessel was warranted to sail on or before the

2d August, but did not sail till the 28th August; what was the net charge made for the insurance?

CASE V.

TO FIND THE AMOUNT OF RETURN OF PREMIUM FOR SHORT INTEREST, THE RETURN PER CENT, AND THE NET COST OF THE INSURANCE UPON A VALUED POLICY.

RULE.

1. Subtract the value of the property upon which the risk actually takes place, from the whole property insured, the remainder is the *short interest*, or amount over insured. Upon this sum find the amount of the underwriter's premium, which is the whole return to be made for *short interest*. Deduct the return thus found from the whole cost of the insurance, and the remainder is the net cost.

2. And, as the whole amount insured is to £100, so is the whole sum to be returned to the return, *per cent*, upon the amount insured.

The return per cent is only calculated to the nearest *penny*, which will occasion a trifling difference on the whole return, if the *whole* return were to be found from the return *per cent*, which may be done for the sake of *proving* the operation.

EXAMPLE.

Required the amount of the return of premium for short interest; and also the net cost of an insurance effected upon 50 bags of cotton wool, at £9. per bag, from Demarara to London, at 7 guineas per cent, of which only 30 bags were shipped.

			£
Insured	50	bags at £9. per bag,	450
Interest	30	ditto at ditto	270
Short interest	20	....	£180

TO FIND THE NET COST OF THE INSURANCE.

	£	s.	d.	
	7	7	0	
			$4\frac{1}{2}$	
	29	8	0	
	3	13	6	
	£33	1	6	Amount of Premium.
	2	5	0	Commission.
$5s. \times 4\frac{1}{2} =$	1	2	6	Policy.
	£36	8	0	Amount of Insurance.
	11	14	0	return for short interest.
	£24	14	0	net cost of the insurance.

TO FIND THE RETURN FOR SHORT INTEREST.

£	s.	
7	0	Underwriter's premium, per cent.
0	10	Ditto commission ditto.
6	10	Return per cent.
5	4	Ditto on £80.
11	14	Whole return.

TO FIND THE RETURN PER CENT.

	£		£		£ s.		£ s.	
	450	:	100	: :	11 14			
or,	9	:	2	: :	11 14	:	2 12	return per ct.

In cases where a return of premium is to be made, either for *short interest*, or for convoy and arrival, it is not only necessary to know the whole amount of the return, but the return on £100; because the risk is generally taken by the underwriters, in shares of *even hundreds* of pounds; as £100, £200, &c., and when the return *per cent* is known, it becomes an easy matter to settle with each underwriter, for his proportion of the return; and which is done to the nearest per centage, excluding farthings, or any fraction of a penny.

EXERCISES.

1. Required the whole return of premium for short interest, the return per cent, and the net cost of the insurance, effected upon 80 hhds. of sugar, at £28. per hhd., from Jamaica to Liverpool, there being only 56 hhds. shipped, and the premium being 6 guineas per cent?

2. An insurance was effected on 500 qrs. of wheat, at £2. 10s. per qr. from London to Cadiz, at 2½ guineas per cent, of which only 350 qrs. were shipped. Required the amount of return of premium, the return per cent, and the net cost of the insurance.

3. An insurance was effected on 105 pipes of Port wine, at £42. per pipe, from ~~Lisbon~~ to London, at 3 guineas per cent, but the number of pipes shipped was only 85; required the amount of premium to be returned for short interest, the return per cent, and the net cost of the insurance.

CASE VI.

TO FIND THE AMOUNT OF RETURN FOR SHORT INTEREST, THE RETURN FOR CONVOY AND ARRIVAL, THE RETURN PER CENT, AND THE NET COST OF THE INSURANCE, UPON A VALUED POLICY.

RULE.

Find the amount of the return for short interest, as in last case, to which add the amount of return for convoy and arrival, on the amount actually shipped, the sum is the whole return; which, subtracted from the whole cost of the insurance, leaves the net cost. The return per cent is found as in last case.

EXAMPLE.

What is the amount of a return of premium, for short interest, for convoy and arrival, the return per cent, and the net cost, of an insurance effected on 90 puncheons of rum, at £15. per puncheon, from Jamaica to London, at 8 guineas per cent; to return £4. per cent, for convoy and arrival: of the 98 puncheons insured, only 72 were shipped, and the vessel sailed with convoy, and arrived safe?

Insured 90 puncheons,	at £15 per puncheon			£1350
Interest 72	ditto	at ditto per	ditto	1080
			Short interest,	£270

TO FIND THE NET COST OF THE INSURANCE.

£13.50
8

108.00
$\frac{1}{20}$= 5.4

£113.40=£113	8	0	premium
	6	15	0	commission
5s.×14=	3	10	0	policy
	£123	13	0	whole cost of insurance
	63	9	0	amt. of return for short int. &c.
	£60	4	0	net cost of insurance.

The return for short interest, on £270, at £7. 10s. per cent, is	£20	5
The return for convoy, &c. on £1080, at £4. per cent, is	43	4
Whole amount of the return, ...	£63	9

TO FIND THE RETURN PER CENT.

	£		£		£	s.		
	1350	:	100	: :	63	9		
or,	127	:	2	: :	63	9	:	£4 14s. return per cent

EXERCISES.

1. An insurance was effected on 80 tons of hemp, at £50. per ton, from Riga to Hull, at 7 guineas per cent; to return £4. 10s. per cent for convoy and arrival; but 60 tons could only be received on board: the vessel sailed with convoy and the hemp arrived safe. Required the amount for short interest, for con-

voy and arrival, the return per cent, and the net cost of the insurance.

2. An insurance was effected on 1500 qrs. of wheat, at £2. 10s. per quarter, from Dantzic to London, at 6 guineas per cent; to return £3. per cent, if the vessel should sail with convoy and arrive safe, which was the case; but, by the neglect of the captain, 1200 qrs. only were taken on board. Required the amount of return for short interest, for convoy and arrival, and also the net cost of the insurance.

CASE VII.

TO FIND THE AMOUNT TO BE INSURED, TO COVER A GIVEN SUM*.

RULE.

As £100. *minus* the amount of the premium, commission, and policy per cent, is to £100, so is the given sum to the sum that covers it.

For a voyage out and home, first find how much will cover the voyage *out*; and, in the same manner, how much must be insured to cover *that* sum.

As insurance is never effected upon *shillings*, if there should be any remainder, after finding the number of pounds to be insured, it is to be reckoned another *pound*, as in the following example.

EXAMPLE.

What sum must be insured to cover £1200, on a vessel from London to Gibraltar, the premium 3 guineas per cent?

---

* By covering any sum, is meant the insuring as much *more* than the *given sum*, as, in case of loss, will pay the premium, commission, and policy.

£3 3 premium per cent
0 10 commission
0 5 policy

		£ s.		£		£
£100 − 3 18	=	96 2	:	100	::	1200
				1200		

96.1)120000

£1248 14 or £1249.

PROOF.

	£	s.	d.
Sum to be covered	£1200	0	0
Premium on £1249. at 3 guineas per cent......	39	6	10
Commission on ditto	6	4	10
Policy duty on £1300	3	5	0
Covered property*, ...	£1248	16	8

EXERCISES.

1. Required the sum that should be insured, to cover £600. 17s. 8d., value of goods, shipped on board a vessel from Malaga to London, at 5 guineas per cent.

2. Required what sum ought to be insured to cover £970., 15s. 5d., on a vessel out and home, from Liverpool to Barbadoes, at 6 guineas per cent each voyage?

3. Required what sum must be insured, to cover £1150. 4s. 10d., value of goods, shipped on board a vessel from Archangel to Leith, at 7½ guineas per cent?

4. Required what sum must be insured to cover £3000, value of goods, on a voyage out and home, from London to Leghorn, at 4½ guineas per cent each voyage?

5. Required what sum must be insured to cover £2250. 8s. 4d., value of goods, from Lisbon to London, at 2 guineas p. ct.

* The policy duty being reckoned on £1300, the proof does not exactly agree with the sum to be covered.

CASE VIII.

TO FIND THE AMOUNT TO BE INSURED, TO COVER A GIVEN SUM, AND DEFRAY THE CHARGES ATTENDING THE RECOVERY OF A LOSS*.

RULE.

Add the premium, commission, and policy for insuring £100. to which add the commission and brokerage, chargeable for recovering the same amount of loss, or add £3. 5s. to the premium (see page 318) and deduct the sum from £100., then say, as the remainder is to £100., so is the given sum to be covered, to the amount to be insured. Commission is only allowed when an agent employs a broker to effect the insurance, and is sometimes *less* and sometimes more than 2 per cent, for settling a loss.

EXAMPLE.

What sum must be insured to cover £800. and defray the charges attending the recovery of the loss of that sum: the premium being 4 guineas per cent, and the policy 5s.?

Premium		£4 4				
Commission.........	£0 10					
Policy	0 5					
Commis. for recovery	2 0					
Brokerage for ditto	0 10		£ s.			
	——	£3 5	100 0			
		——	7 9			
			——	£		£
			£92 11 :	100	::	800
			20	16000		20
			1851	)1600000		16000

Amount insured, £865+

* This case is nearly the same as Case VII. the only difference is in adding an additional 2½ per cent to the premium, in order to defray the expense of recovering the sum insured, should the interest be lost.

The exact sum to be insured, in this example, is £864. 8s. but it is here called another pound, for the reason stated at page 328, and it is on this account that the work does not seem to be proved.

The work may be proved thus,

		£	s.	d.			
	Amount to be covered..............				£800	0	0
Cost of insur.	Prem. on £865 at 4 gs. p. ct.	36	6	8			
	Commission at $\frac{1}{2}$ p. ct.....	4	6	6			
	Policy, 5s. per cent	2	5	0			
					42	18	2
Charges on recov.	Commission at 2 per cent	£17	6	0			
	Brokerage at $\frac{1}{2}$ per cent..	4	6	6			
					21	12	6
					£864	10	8

EXERCISES.

1. What amount ought to be insured to cover £1655. 10s. 8d. and defray the charges attending the recovery of a loss of the sum insured; the premium being 3 guineas per cent, and the policy 5s.?

2. What is the sum which ought to be insured to cover £1230. 9s. and defray the charges of a recovery of a loss; the premium being 2 guineas per cent. and the policy 2s. 6d.?

CASE IX.

TO FIND THE AMOUNT OF A RETURN FOR SHORT INTEREST UPON AN OPEN POLICY, THE INTEREST BEING COVERED.

RULE.

Find the amount which ought to be insured to cover the real interest, subtract this sum from the whole amount insured, and the remainder is the short interest, upon which the return may be found by Case V. as upon a valued policy.

EXAMPLE.

Required the amount of the return for short interest on an insurance effected upon goods, supposed to be shipped to the

amount of £2200. at 7 guineas per cent, from Archangel to London, but it is found that goods have only been shipped to the value of £1480.

```
Prem.   £7  7
Com. ..  0 10    £   s.
Policy..  0  5  100  0
                  8  2
               ———————      £        £
               £92 18  :  100  : :  1480
                   20                 20
               ———————             ———————
                 1858               29600
                                      100
                                  ————————
                              1858)2960000
                                  ————————
```

The sum that covers the real interest is £1593 near. pound

```
Amount insured............. £2200
Real int. covered at 7 gs. p. ct. 1593
                              ————
               Short interest £607
```

TO FIND THE WHOLE RETURN.

```
£7  0 Underwriters. pre. per cent    £6,07
 0 10 Commission ditto                6¼ hundreds
—————                                ————
£6 10 Return on £100.                3642
—————                                 3035
                                     ————      £   s.
                   Whole return £39,455 =    39   9
                                      20
                                    ————
                                    9,100
                                    ————
```

TO FIND THE RETURN PER CENT.

```
  £        £        £  s.
2200  :  100  : :  39  9
  20                20
—————             ————
44000              789
                    100
                  ————
         44,000)78,900
                  ————
          £1 15 10 rate per cent of return.
```

EXERCISES.

Required the amount of return for short interest on an insurance effected upon the freight of a vessel from Malaga to London, to the amount of £380, at 3 guineas per cent; the amount of the freight being only £200*.

CASE X.

TO FIND THE AMOUNT OF A RETURN OF PREMIUM FOR SHORT INTEREST, FOR CONVOY AND ARRIVAL, ON AN OPEN POLICY, THE INTEREST BEING COVERED, INCLUDING THE CHARGES ATTENDING THE RECOVERY OF A LOSS.

RULE.

Find the whole cost of the insurance by Case II., then find the sum necessary to cover the real interest, which subtract from the whole amount insured, and the remainder is the short interest, upon which find the return; then find the return for convoy and arrival on the sum which covers the real interest, and add it to the return for short interest, the sum is the whole return, which, subtracted from the whole cost of the insurance, leaves the net cost of the insurance. The return per cent is found as in Case VI.

EXAMPLE.

What is the amount of the return of premium for short interest, for convoy and arrival, the return per cent, and the net cost of the insurance, effected on an open policy, to the amount of £1875, at 5 guineas per cent, to return £2 per cent, for convoy and arrival, on goods from Riga to Hull, including the charges for recovering a loss. The interest shipped amounted only to £1250, which arrived safe?

---

* As this case is only a combination of Cases V. and VII., it is unnecessary to give many exercises in this place.

	£18.75
	5 gs.
	93.75
$\frac{1}{20}$ =	4.6875
	£98.4375
Commission	9.375
Policy......	4.75
	£112.5625 = £112. 11s. 3d. whole cost of insurance.

TO FIND THE SHORT INTEREST.

Premium £5 5 £
Charges 3 5 100 0
8 10

£ £
£91 10 : 100 : : 1250
2 2

183 0 2500
100

183)250000

Real interest covered is	£1366
Amount insured.........	1875
Short interest	£509
Undws. prem. per cent	$4\frac{1}{2}$
Return for short interest	£20.36
	2.545

		£	s.	d.
	£22.905 =	22	18	1
Return for convey on £1366, at 2 per cent		27	6	4
Whole return		£50	4	5

TO FIND THE RETURN PER CENT.

£		£		£	s.	d.		£	s.	d.	
1875	:	100	: :	50	4	5					
$18\frac{3}{4}$	:	1	: :	50	4	5	:	2	3	7	rate per cent of return

TO FIND THE NET COST OF THE INSURANCE.

Whole cost of insurance as found above	£78	15	0
Whole return of premium	50	4	5
Net cost of the insurance	£28	10	7

EXERCISES.

1. An insurance was effected upon goods, to the value of £3420, from Kingston to Liverpool, at 4 guineas per cent, to return £2. per cent for convoy and arrival; the goods shipped amounted only to £2543. 8s. 4d., and the vessel sailed with convoy and arrived safe. Required the whole return of premium, and the net cost of the insurance, the charges for recovery of a loss being included in the insurance.

2. An insurance, to the amount of £830. 7s. 6d. was effected on the freight of a vessel, from Leghorn to London, at 6 guineas per cent, to return £3. per cent, for convoy and arrival, which took place; but the amount was only £718. Required the amount of return, for short interest, convoy, &c. and the net cost of the insurance.

CASE XI.

TO FIND THE SUM RECOVERABLE FROM THE UNDERWRITERS FOR A TOTAL LOSS, UPON AN OPEN POLICY, AND THE INTEREST OVER INSURED.

RULE.

Find the sum that should have been insured to cover the real interest, including the charges attending the recovery of a loss, and deduct it from the whole amount insured. The first of these sums the underwriters will have to settle as a loss, and on the last they will have to make a return for short interest.

EXAMPLE.

An insurance was effected upon £885. 5s. 9d. on goods from Barbadoes to Bristol, at 5 guineas per cent; but the value shipped

was only £650, and the vessel was lost. Required the sum to be recovered from the underwriters, on account of the loss, and also for short interest; the insurance, including the charges incident on the recovery of a loss.

	£	s.	£	s.
Premium	5	5		
Charges	3	5	100	0
			8	10

£91 10 ; £100 :: £650

2	2
183	1300
	100

183)130000

Real interest covered is	£711
Amount insured	885
Sum over insured	£174
Return per cent	$4\frac{1}{2}$
	696
	87

	£	s.	d.
Return for short interest £7,83 =	7	16	7
To be recovered on account of loss	711	0	0
Whole claim upon the underws. £	718	16	7

TO FIND THE RETURN PER CENT ON BOTH CLAIMS.

	£		£		£	s.	d.		£	s.	d.
1st. For short int.	885	:	100	::	7	16	7	:	0	17	8
2d. For the loss.	885	:	100	::	711	0	0	:	80	7	0
Whole claim per cent on underwriters									£81	4	8

Proof. £885. at £81. 4s. 8d. per cent is £718 18 0

EXERCISES.

1. An insurance was effected upon £1500. on goods from Petersburgh to London, at 6 guineas per cent; but the quantity

shipped being only to the amount of £1180. 9s., and the vessel being lost, it is required to find the amount to be recovered from the underwriters, on account of the loss, and also for short interest, the insurance, guaranteeing, the charges attending the recovery of a loss?

2. An insurance was effected upon £780, on the freight of a vessel from Archangel to Leith, at 7 guineas per cent; but the actual amount of the freight was only valued at £600.; the vessel being lost, it is required to ascertain the whole sum for which the underwriters are liable, in this transaction.

CASE XII.

TO FIND THE NET PROCEEDS OF A LOSS RECOVERED UPON A VALUED POLICY, AND THE NET AMOUNT TO BE RECEIVED BY THE ASSURED.

RULE.

Find the net proceeds of the loss, from which deduct the cost of the insurance, on the amount recovered, and the remainder is the net amount the assured must receive, in the case of a total loss. But when a loss takes place, upon part of an insurance, the amount to be recovered is the product of the whole sum insured, multiplied by the rate per cent of the loss calculated upon the whole sum.

EXAMPLE.

An insurance was effected on 80 hhds of sugar, valued at £20. per hhd., from Jamaica to London, at 7 guineas per cent, to return £4. per cent for convoy and arrival; of the 80 hhds. insured, 25 hhds. were shipped on board a vessel, which was totally lost, and 50 hhds. arrived by a vessel that sailed with convoy, the rest was not shipped. Required the net proceeds of the loss; the net amount that the assured would receive credit for from his agent; the amount of the return of premium, for short interest; for convoy and arrival; and the net cost of the insurance.

TO FIND THE WHOLE COST OF THE INSURANCE.

80 hhds. at £20. each, is £1600.

	£7	7	premium per cent
		8	
	58	16	
		2	
	£117	12	whole premium
£1600. at $\frac{1}{2}$ p. ct.	8	0	commision
ditto at $\frac{1}{4}$ p. ct.	4	0	policy
	£129	12	whole cost of insurance.

80 hhds. insured, valued at £20. per hhd. is		£1600
25 hhds. ditto lost, is	£500	
40 hhds. ditto arrived safe, is	1000	
		1500
5 hhds. short interest		£100

TO FIND THE RATE OF THE LOSS PER CENT.

$$\begin{array}{ccccc} £ & & £ & & £ \\ 16,00 & : & 5,00 & :: & 100 \\ & & & & 5 \\ \hline & & & & 16)500 \\ \hline & & & & £\ 31\frac{1}{4} \end{array}$$

£ $31\frac{1}{4}$ rate of the loss per ct.

TO FIND THE NET PROCEEDS OF THE LOSS.

Loss at $31\frac{1}{4}$ per cent upon £1600. is		£500	0
Charges { Brokerage on £500. at $\frac{1}{2}$ per ct.	£ 2 10		
Charges { Commission on do. at 2 per ct.	10 0		
		£12	10
Net proceeds of the loss,		£487	10

TO FIND THE SUM RECEIVED CREDIT FOR.

			£500 0
Premium on £500, at 7 guineas per ct. is	£36 15		
Commission on do. at ½ per cent, is	2 10		
Policy on do. at ¼ ditto is	1 5		
			40 10
Net sum the assured receives credit for,			£459 10

TO FIND THE RETURN OF PREMIUM.

Return for short interest, on £100. at 6½ per cent ..	£ 6 10
Return for convoy, &c. on £1000. at 4 per cent	40 0
Amt. of return for short interest, and for convoy, &c.	£46 10

TO FIND THE RETURN PER CENT.

£		£ s.		£
16,00	:	46 10	: :	100

16)46 10

£2 18 1½ return per cent

TO FIND THE NET COST OF THE INSURANCE, ON THE WHOLE VALUE, EXCLUSIVE OF THE LOSS.

Premium on £1100, at 7 guineas per cent			£80 17
Charges { Commission on £1100. at ½ per ct.	5 10		
Charges { Policy, on ditto at 5s. p. ct.	2 15		
			8 5
Net cost on £1100. is			£89 2

EXERCISES.

1. An insurance was effected on 150 tons of hemp, at £52. per ton, from St. Petersburgh to London, at 6 guineas per cent, to return £3 per cent for convoy and arrival; of this quantity insured, 50 tons were shipped on board a vessel, which was com-

pletely lost, and 80 tons arrived safe, by a vessel which sailed with convoy the whole of the voyage; the remaining 20 tons were not shipped. Required the whole cost of the *insurance*; the net proceeds of the *loss;* the net amount the assured would receive credit for, from his agent; the amount of the return of premium, for short interest; for convoy and arrival; and the return per cent.

AVERAGES.

The word Average, when applied to Marine Insurance, originally signified a proportionate division of any damage, among the parties concerned in any risk at sea. It is now used, however, for all *losses*, damages, &c. short of a *total loss*.

Averages are divided into three kinds; viz. General, Particular, and Petty Average.

The *first* of these is the only one properly entitled to the *appellation*.

The *second* is made use of by foreign writers, merely in opposition to the first, or as a means of showing that the damage incurred is a *particular* loss, and, therefore, not a subject for general contribution.

The *third* is composed of some of the petty and ordinary charges of the voyage; and might, therefore, bear any other name, as well as that of average.

GENERAL AVERAGES.

A claim for a *general* contribution may arise from two causes:—*first*, from a sacrifice deliberately made of the property of one of the parties, concerned in the adventure, for the benefit of the others, whereby *his loss* is converted to *their gain;* and, of course he has a right to claim restitution. *Secondly*, a claim may arise from expenses incurred, or services performed, by one party; for example, the shipmaster may incur expenses for the general benefit, and, therefore, he has a right to claim a remuneration.

To render a claim of this nature valid, the ship must be in *actual distress;* the thing intended to be destroyed must be expressly selected for that purpose; the sacrifice must be made, upon mature deliberation, by the master, on consulting with his crew, &c.; and he must have no other end in view, than the preservation of the ship and cargo.

The most ancient and well-founded claim, to a *general* contribution, is *Jettison* of the cargo*.

As the limits of this work will not admit of stating, particularly, the laws and precedents regarding the *contributory interest*, it may be sufficient to state, that the *ship* itself, and the *cargo* on board, contribute according to their respective *values;* but whether the *freight* shall be liable to contribute, in *all* cases, is not so satisfactorily settled. When the average is adjusted, after the ship's arrival, and the freight payable at the port of discharge, there is no doubt but it ought to make part of the contributory *interest;* and, on a ship being charted for a voyage, and the average settled at the port of loading, it is agreed, in Lloyd's, that the freight ought to contribute, to make good the average loss.

On the average being adjusted, after the *arrival* of the ship, the *wages* of the *seamen* must be *deducted* from the amount of

* *Jettison* is the act of throwing goods, &c. overboard, either to lighten the ship when at sea, in a storm, or for the purpose of making her float when she gets aground; but, in both cases, the goods thrown overboard must have been regularly stowed under the *hatches*, for, if they are upon deck, no contribution can be demanded for them, though they must bear their proportion, if saved.

the *freight*, and the *remainder* is the sum liable to contribution*. But it is necessary to remark that, before proportioning the loss, *each* of the *interests*, viz. the *cargo*, the *ship*, and the *freight*, after the value is accurately ascertained, must be severally cleared of every charge attached to it.

There is no general rule for settling the value of the *ship*, which ought to contribute to make good a general average loss. But the mode which is held to be the best, is that which approximates nearest to the value of the *ship* when she sailed, after deducting the provisions and stores expended, the wear and tear of the voyage, and any partial loss, by sea damage, incurred up to the time that the general average loss took place.

The interests then that *contribute*, are the *cargo*; (which is valued either at the cost on board at the port of loading, or the net proceeds at the port of discharge); the value of the *ship*, as nearly as it can be ascertained, when the loss was incurred, and the value of the freight, received by the owners of the ship, after deducting the seamen's wages.

In computing a general average, for masts, rigging, &c. cut away, a deduction is made of ⅓ from the cost of replacing them, as the *new* articles are supposed to be so much better than the *old*; but *goods*, thrown overboard, are valued at the sum they would have brought, had they arrived safe†.

A contribution can only take place, when the ship and the remainder of the cargo are preserved, in consequence of the *Jettison*; for, if the goods be thrown overboard in a storm, and the ship afterwards perish in the same storm, there can be no contribution of the goods saved, if any: because the object, for which the goods were thrown overboard, was not accomplished.—But if the ship be preserved by the jettison, or the goods thrown overboard, and she continue her voyage, and is afterwards lost, the effects saved from the last misfortune, if any, must contribute to the loss sustained by the jettison, because their preservation was owing to that act.

---

* It is customary to deduct the master's wages also.

† When the average claim is settled at the port of *loading*, the goods are generally valued at the *invoice* price.

PARTICULAR AVERAGES.

The term *particular Average*, as understood at Lloyd's, signifies a *partial loss* of the *ship* or *cargo*, arising from the common accidents of the sea; and must be borne by the owners of the property that suffer the damage.

Underwriters are liable for general and particular averages, in proportion to the sums which they have underwritten. Thus, if a man underwrite £100. upon a property valued at £600. and a *total loss* happen, he is answerable only for £100.; if a partial loss happen, amounting to £80. per cent, on the whole value, he must pay £80. being his proportion of the loss. In case of a *total* loss, either of the whole cargo or of part of it, capable of a distinct valuation at the outset, the underwriter, as far as his proportion of the insurance extends, must pay the *prime cost*, or value mentioned in the *policy* of the property insured; he has nothing to do with the prices of the markets.

If goods *arrive*, but *damaged*, the underwriter must pay the owners such proportion of the prime cost, or value mentioned in the policy, as corresponds with the proportion of the diminution in value occasioned by the damage. Thus, if the value in the policy be £50. and if the goods, though damaged, sell for £60. when, if sound, they would have sold for £80. the difference in value between the sound and damaged is $\frac{1}{4}$, consequently the underwriters must pay $\frac{1}{4}$ of £50. or £12. 10s. On the contrary, had they come to a losing market, and sold, when damaged for £35. when, if sound, they would have brought £70. the difference is $\frac{1}{2}$, and the underwriters must pay $\frac{1}{2}$ of £50. or £25. When *no valuation* is stated in the policy, the *invoice* of the cost, with the addition of all charges, and the premium of insurance, is the foundation on which the loss is computed.

General averages are *always* paid, however small; but, to prevent the underwriters from being harassed with trifling demands for *particular averages*, there is a memorandum at the bottom of London policies, declaring, "That in insurances on corn, fish, salt, fruit, flour, and seeds, the underwriters will not pay any particular average, unless the ship be stranded: that in insurances on sugar, tobacco, hemp, flax, hides, and skins, they consider themselves *free* from average under 5 per cent; and that, on all other goods, as well as on the ship and freight, they consider themselves free from particular average under 3 per cent, unless the ship be stranded."

SALVAGE LOSS.

A *Salvage loss* is that kind of loss, which, it is presumed, would have become a *total loss*, had it not been for certain services performed. The property saved is termed the *salvage*, the charges incurred, the *salvage charges*, and the difference between the *original value* and the *salvage* (after deducting the charges) is the *salvage loss*.

The services for which salvage charges are claimed, are, for preventing the *loss* of the *vessel* or *cargo*, by shipwreck, fire, pirates, enemies, &c. The expense for *recapture* of the vessel is $\frac{1}{8}$ of the real value, if the recapture be made by any of His Majesty's ships; but $\frac{1}{6}$ of the value, if made by any other vessel.

TO COMPUTE AVERAGES.

GENERAL RULE.

As the whole value subject to *contribution* is to the whole loss, so is each person's share of that value to his proportional average of the loss; or so is £100. to the average loss *per cent*.

EXAMPLE.

A ship, in her voyage to Lisbon from London, was under the necessity of cutting a cable, and of replacing it by a new one, which cost £120. and other charges amounted to £10. The net amount of the cargo was £2000.; the ship at Lisbon was valued at £1200., and the freight, after deducting seamen's wages, and other charges, amounted to £800. Required what part of the loss must be borne by the *cargo*, the *ship*, and the *freight*.

To a new cable................	£120	
Deduct $\frac{1}{3}$ of newness............	40	
		£80
To charges......................		10
	Total	£90

The cargo is valued at	£2000
The ship	1200
The freight	800
	£4000

TO FIND THE LOSS PER CENT.

	£		£		£		s.	d.	
	4000	:	100	::	90				
or,	40	:	1	::	90	:	s.	d.	
	4	:	1	::	9	:	2	6	loss per cent

£2000 at £2. 5s. per cent	=	£45	0	0	cargo must bear.
1200 at ditto ditto	=	27	0	0	ship ditto.
800 at ditto ditto	=	18	0	0	freight ditto.
Proof		£90	0	0	

EXAMPLE II.

It is required to find the whole amount to be recovered from the underwriters, upon the following transaction: £2400 was insured upon 80 hhds. of sugar, valued at £30. per hhd. from St. Vincent's to Bristol, at 7 guineas per cent, to return £5. per cent for convoy and arrival. Only 60 hhds. were shipped, which arrived safe with convoy; but, during the voyage, one of the masts was cut away and some goods thrown overboard, to lighten the vessel, which got aground. The assured had to contribute towards the general average, at the rate of £2. 5s. per cent, upon the 60 hhds. at the value stated in the policy.

80 hhds. at £30. per hhd........	£2400
60 ditto shipped	1800
Short interest	£600

Interest £1800 at £5 per cent	£90	0	ret. for convoy, &c.
Short do. 600 at 6. 10s. ditto	39	0	do. for short int.
Interest 1800 at 2. 5s. ditto	40	10	contribution to general average.
Amount to be recovered	£169	10	

	£		£	s.		£
then,	24,00	:	169	10	::	100
		24)	169	10		
			£7	1	3	rate per cent.

EXERCISES.

1. Suppose three merchants, A, B, and C, have chartered a ship, to convey goods from London to Hamburgh, and that A put goods on board to the value of £600, B to the value of £800, and C to the value of £160; the ship gets aground, and the captain, in order to lighten her, is under the necessity of throwing 40 casks of copperas overboard, the property of C. Required the proportion of the loss to be borne by the different interests bound to contribute.

2. The ship Endeavour, on a voyage from Kingston to London, suffered the following damages; masts, cables, &c. cut away, the expense of replacing which was £450; an anchor lost, which cost £50; 5 puncheons of rum thrown overboard, value £100; sundry charges of pilotage, &c. £50. The value of the ship was £4000; of the cargo per account sales, £7000; gross freight, £1590; portage bill, £90. Required the general average loss per cent, and how much of it the owners of the ship, the proprietors of the goods, and the underwriters, must pay respectively, only £2000 having been insured on account of the ship, and £1500 on the cargo?

3. Insurance was made on 80 hhds. clayed sugars, on board the Vrow Martha, from the island of St. Thomas to Hamburgh, valued in the policy at £30 per hhd. In the course of the voyage, the sea-water got in, and, when the ship arrived at Hamburgh, it appeared that every hhd. of sugar was damaged. The damage the sugars had sustained made it necessary to sell them immediately, and they were accordingly sold for £20. 0s. 8d. per hhd.; whereas, had they been sound, they would at that time have brought, at Hamburgh, £23. 7s. 7d. per hhd. How much in all, and per cent, did the proprietor of the sugar recover from the underwriters?

PARTICULAR AVERAGE.

It has been remarked, at page 343, that by particular average is understood, the damage which particular goods receive, or the injury which the ship sustains, from any cause that does not relate to the benefit or preservation of the whole. As when a ship loses a mast, or any part of her rigging, by stress of weather, or when goods are damaged by accident.

The *rule* for the adjustment of particular average losses is, that the underwriters pay the difference of *prime value* between the *damaged* goods and the *sound*.

CASE I.

TO FIND THE AMOUNT OF A PARTICULAR AVERAGE UPON GOODS INSURED ON A VALUED POLICY.

RULE.

As the estimated value of the goods, when sound, is to the whole loss, so is the prime cost, or value mentioned in the policy, to the amount of the loss which the underwriters have to make good.

EXAMPLE I.

Suppose 50 hhds. of sugar to be shipped at Jamaica for Liverpool, and that the cost, per invoice, is £1500, or £10 each, and that the whole is insured at that price; but, on arrival at Liverpool, are found damaged by salt water, so that they only produce £400, or £8 each hhd. but would have produced £600, or £12 per hhd. had they arrived undamaged. Required the loss per cent which the underwriters have to sustain*.

	£600†						
	400						
£600 :	200	: : £100		£	s.	d.
3 :	1	: : 100	:	33	6	8

If there is a return for convoy stipulated for, the average loss must first be deducted.

In the following example, the return is required upon the whole property insured; but it may sometimes be required to find the return upon the sound and damaged parts separately.

---

* In this example it is not necessary to state the *premium*.

† The value of the property upon which the return is to be made.

EXAMPLE II.

Suppose 40 hogsheads of sugar, valued at £15 per hhd., were insured, from Kingston to London, at 8 guineas per cent, to return £5. per cent, for convoy and arrival; the whole arrived, but 8 hhds. were so much damaged as to sell for £9. per hhd.; whereas, had they arrived safe, they would have sold for £20. per hhd. Required the amount of the return of premium, for convoy and arrival; and the amount of the particular average.

8 hhds. at £20 is £160 would have sold for, if sound
8 ditto at 9 is 72 price sold for

then, £160 : £88 less :: (8×15)=£120
or, 4 : 88 :: 3 : £66 aver. loss

TO FIND THE RETURN FOR CONVOY, &c.

40 hhds. at £15. per hhd. is.... £600
Average loss 66

Sum on which the return must be made, £534

therefore, £534
5 return per cent

£26,70 = £26 14 amount of return
66 0 average loss

£92 14 amount to be recovered from underwriters.

£		£ s.		£		£ s.	
600	:	92 14	::	100			
6	:	92 14	::	1	:	15 9	rate per cent*.

* Though the mode of adjustment here given (and now in general use) has a reference to the market price, it is perfectly understood that the underwriter has no concern with the fluctuation of the markets; and, therefore, whether they be high or low, it is of no importance to him; for, as Lord Mansfield said, "the market is only used as scales to weigh the extent of the damage." The comparison between the price of the sound and damaged goods is instituted, only to ascertain the *quantum* of the damage which the goods have sustained, or the *relative* depreciation.

If the return of premium, upon the 32 hhds. of sound sugar, had been to be settled for separately, the amount to be recovered from the underwriters would have been the same; but the work would have stood thus:

32 hhds. at £15. per hhd. is £480;
and £480, at 5 per cent, is £24. amount of return on 32 hhds.

8 hhds. at £15. per hhd. is	£120	
Average loss	66	
Sum on which the return on 8 hhds. is to be calculated, ...	£54	

£54. at £5. per cent. is..	£2	14	amount of return
Average loss	66	0	
Return on 8 hhds. and average loss	68	14	
Return on 32 hhds.	24	0	
Amount to be recovered	£92	14	as before.

The rate per cent, of each return, might also have been found, but is purposely omitted, as an exercise for the student.

CASE II.

TO FIND THE AMOUNT OF A PARTICULAR AVERAGE LOSS, UPON PROPERTY INSURED, ON AN OPEN POLICY, WHEN THERE IS A RETURN FOR CONVOY AND ARRIVAL.

RULE.

As the selling price of the property, when sound, is to the whole loss, so is the value insured to the average loss; which deduct from the sum insured, and the remainder is the sum upon which the return is to be made. Compute the return, and add it to the average loss, and the sum is the amount to be recovered from the underwriters.

EXAMPLE.

An insurance was effected upon £3000, the value of a cargo of flax, from Riga to Hull, at 5 guineas per cent, to return £2. 10s. if the vessel sailed with convoy and arrived, which was the case; but the flax being damaged, sold only for £2250; whereas, if it had arrived sound, it would have sold for £3400. Required the amount of the particular average loss, the return of premium, for convoy and arrival; the *whole* amount to be recovered from the underwriters; and the *rate per cent.*

TO FIND THE AVERAGE LOSS.

Value, if sound, £3400
sold for 2250

£3400 : whole loss £1150 : : £3000 amt. insu*.

or, 34 : £1150 : : 30
30

34)34500

£1014 14 3 average loss

TO FIND THE RETURN OF PREMIUM, &c.

£3000 amount insured
1015 average loss, to the nearest pound

£1985 amount upon which the return must be made
$2\frac{1}{2}$ return per cent

3970
9925

	£	s.	d.	
£49.625 =	49	12	6	amount of return
	1014	14	3	average loss
	£1064	6	9	amount to be recovered from underwriters.

* In this example the whole cost of the insurance is not required, and as so many examples of doing *this* have *already* been given, it is omitted.

TO FIND THE RATE PER CENT TO BE RECOVERED.

£		£	s.	d.		£
30,00	:	1064	6	9	: :	1,00

3,0)106,4	6	9	
£ 35	9	6	per cent.

FOREIGN EXCHANGES.

EXCHANGE is the paying or receiving of the money of one country for its equivalent in the money of another, by means of a written instrument, termed a *bill* of exchange.

The monies by which *exchange* is regulated, and in which *accounts* are kept, are *generally* the *same*; but, in many places they are *imaginary**, as the *pound sterling* in England.

In commercial transactions with foreign countries, a knowledge of Exchanges is of the greatest importance; for, without this knowledge, it would be impossible to compare the prices of goods, in foreign markets, with each other; to judge of the most profitable mode of drawing or remitting money, or to export or import goods with the least probability of advantage.

The *Par of Exchange* is the intrinsic value of the money of one country, compared with that of another country, which is estimated by the *weight* and *purity* of the gold or silver, which they respectively contain.

The *Course* of *Exchange* is the current value allowed for the money of one country, when reduced to the money of another country. This rate or price is seldom at par, but fluctuates, according to circumstances, like the prices of every other article of trade.

If the demand for bills of exchange, on any place, exceed the supply, this circumstance generally has the effect of rendering

* That is, having no real coin corresponding to them in value.

the course of exchange more favourable to that place, and *vice versa;* this *demand* aud *supply* must arise from the extent of the money transactions between the respective countries, which again are produced by the exportation or importation of goods, subsidies, the payment of armies, the wants of travellers, interest on foreign money, vested in the public funds, &c.

Although these circumstances must have considerable influence on the course of exchange, between two countries, yet they do not wholly regulate it; for the drafts or remittances, from one place or country to another, are frequently negociated through the medium of other countries; for example, the amount of goods, sent from America to the Continent of Europe, is often paid for by bills on London; and goods imported into England, from the shores of the Baltic, are generally paid through the medium of Hamburgh or Amsterdam; therefore, this circuitous mode of making payments must also have considerable effect on the course of exchange.

Sometimes the *value* of the coins of a country is altered, by the order of its government, while it retains the same nominal value. As this changes the *par*, it must also alter the course of exchange with other countries, in which the coins remain unaltered.

The course of exchange also differs, according to the time at which a bill of exhange is payable: those at a *short date* being more valuable than those at a *distant date;* not only by the *interest,* for the difference of *time,* but also on account of the assurance it affords, against risk, for the time the bill has longer to run.

The course of exchange is, nevertheless, confined, by particular circumstances, within certain limits. When this differs much from the true value, the merchants, to whom the difference is unfavourable, have recourse to the transportation of *bullion,* and even *specie,* instead of *bills,* which the vigilance of government cannot altogether prevent, although laws are often made to prohibit the exportation of specie.

When the rate of exchange is given between two places, one of the places always gives a *certain* fixed price, to receive an *uncertain* one. Thus, England gives a certain fixed sum, (a Pound sterling), for an uncertain variable price of the money of Holland, France, &c.; and, sometimes, England gives an uncertain, or variable price, (so many Pence,) for a certain fixed sum

of the money of other countries; as for the Piastre of Spain, the Milree of Portugal, &c.

The *lower* the course of exchange runs between two countries, the more favourable it is considered to be to the place or people in whose money it is reckoned; and *vice versa*. Thus, if the course of exchange, between London and Amsterdam, be at 9 guilders, per pound sterling, it is evident that it would require *more* sterling money to pay a debt due in Amsterdam, and *fewer* guilders to discharge a debt due in London, than when the exchange was at 11 guilders, per pound sterling. The Amsterdam merchant having to procure pounds sterling, to remit to London, buys them at a less price, when the *exchange* is *low*, than when it is *high*; but the London merchant, having to sell pounds sterling, (or to purchase guilders), will receive fewer of these, than when the course of exchange is higher; in this case the exchange is evidently in favour of Amsterdam; but the contrary of this would have been the case, if the course of exchange had been high.

Usance is the usual time at which bills are drawn, between certain places; such as *one*, *two*, or *three* months after date; and double, or half-usance, means double or half of the usual time. If the usance be one month, 15 days are allowed for half usance.

Days of Grace are a certain number of days granted, after the term, mentioned in the bill, is expired: the number of these days are different, at different places. In England, 3 days of grace are allowed; at Hamburgh, 12; and at Madrid, 14; see page 356. Bills at sight, however, must be paid when presented.

ENGLAND.

In England, accounts are kept in pounds, shillings, pence, and farthings; but the pound is *imaginary*, there being no current coin of the country of the value of one pound*.

The principal coins of this kingdom are the following:

			s.	d.	
Of Gold	The Guinea	=	21	0	value.
	— Half ditto	=	10	6	
	— Third ditto	=	7	0	

* Previous to the coinage of 1817.

			s.	d.
Of Silver	The Crown	=	5	0
	— Half ditto	=	2	6
	— Piece of	=	3	0
	— Piece of	=	1	6
	— Shilling	=	0	12
	— Sixpence	=	0	6

Of Copper	The Penny	=	4 Farthings.
	— Half	=	2 Farthings.
	— Fourth	=	1 Farthing.

Gold and silver are weighed by the pound Troy. (See the Table, page 161.)

In every well regulated country, there is a standard fixed by law for the degree of purity or fineness of its coins; that is, the proportion between the quantity of pure metal and alloy, is precisely fixed.

The fineness of gold is generally expressed in carats. Thus the whole weight of any piece of gold is supposed to be divided into 24 equal parts, called carats, and each carat into 4 grains. As the imaginary unit of reference may be any weight whatever, these denominations have no specific weight.

The standard for gold coin in Britain, is 22 carats of pure gold and 2 of alloy; hence the standard gold of this country is said to be 22 carats fine, one ounce of which is worth 3 pounds, 17 shillings, and $10\frac{1}{2}$ pence sterling.

The imaginary unit, in assaying silver, is divided into 12 ounces, and the ounce into 20 dwt.

Standard silver consists of 11 oz. 2 dwt. of pure silver and 18 dwts. of alloy, and is worth 5 shillings and 2 pence sterling, per ounce.

The coinage of gold and silver is regulated as follows:

One pound of standard gold, troy weight, is coined into $44\frac{1}{2}$ guineas; and a pound of standard silver is coined into $12\frac{2}{5}$ crowns, or 62 shillings.

From these regulations, it appears that the guinea weighs 5 dwts. $9\frac{17}{89}$ grs. and contains $118\frac{58}{89}$ grains of fine gold: that the crown weighs $464\frac{16}{31}$ grs. and contains $429\frac{21}{31}$ grains of fine silver, and the inferior coins in proportion.

The proportion between the value of standard gold and standard silver in England, is as $15\frac{1}{14}$ to 1.

The charges of coinage was defrayed by government: therefore, those who took bullion of the standard required, to the mint, received in exchange for each ounce, £3. 17s. 10½d. in gold coin, as nearly as the sum would admit; and in exchange for an ounce of standard silver, 5 shillings in silver and 2 pence in copper coin: in fact, if the first of those sums could have been paid exactly in *gold* and the last in *silver*, each would have exactly weighed an ounce of standard metal*.

These were the regulations respecting the weight and fineness of the coins of this country, previous to the coinage of 1817; but, as that coinage differs from former coinages, it may be useful to explain the principle by which that coinage was regulated.

COINAGE OF 1817.

Gold Coins	Half Sovereign	=	10 Shillings	
	Sovereign	=	20 shillings	or £1.
	Double Sovereign	=	40 shillings	or £2.
	5 Sovereign piece	=	100 shillings	or £5.

The silver coins have the same names as in former coinages.

Gold is considered the standard metal, and there is no alteration, either in weight or fineness, from former coinages†. The sovereign, or 20s. piece, being $\frac{20}{21}$ parts of the weight and value of a guinea, and the other pieces in the same proportion.

The real weight of the sovereign is 5 dwt. 3.274 grs.; and 934 sovereigns and a *half sovereign* weigh exactly 20lbs. troy.

	oz.	*dwt.*	*grs.*
The weight of the half sovereign is........	0	2	13.637
The weight of the double sovereign is	0	10	6.548
The weight of the 5 sovereign piece is	1	5	16.370

* This is not the case in other countries, for there is an allowance made for the fallibility of workmanship; and in some foreign Mints, this is a source of emolument, for the governments issue coins at a rate above their intrinsic value; and the profit thus made is called Seigniorage.

† See page 354.

The silver coins are also of the *old standard fineness*, of 11 oz. 2 dwt. of pure silver to 18 dwt. of alloy; and one pound troy, of this standard, is now coined into 66 shillings, instead of 62 shillings, as was formerly the case. (See page 354.)

	dwts.	grs.
One shilling of this coinage therefore weighs	3	$15\frac{3}{11}$
The sixpence	1	$19\frac{7}{11}$
The crown	18	$4\frac{4}{11}$

All the other pieces are in the same proportion.

This change, in the weight of our silver currency, has been made with a view to keep it in circulation at *home*, and also to prevent our monies from being melted down, as heretofore, by thus returning to the old principle of Seigniorage.

The COURSE OF EXCHANGE between London and the principal trading towns in Europe, is exhibited in the following table; extracted from Lloyd's List for the 3d of December, 1816; and also the Par, Usance, and Days of Grace.

Course of Exchange between London and		Par.	Usance.	Days of Grace.
Amsterdam ....B.2 U.	40 0			
Ditto at sight	39 6			
Amsterdam ............C.F.	12 5	36 7	1 mdt	6
Ditto at sight	12 2			
Rotterdam ....12 U.	12 6	11 cur.	1 mdt	6
Antwerp	12 5	10 16	1 mdt	6
Hamburgh ....2½ U.	36 10	35 cur*	1 mdt	12
Altona ... .2½ U.	36 11	——	1 mdt	12
Paris, 3 day's sight.........	25 50	24 75	30 dt	-
Ditto....... ....2 U.	25 70	——	30 dt	10
Bourdeaux	25 70	145	14 dt	10
Frankfort on Main Ex. M.	152		2 mdt	
Madrid .... effective†	35	39	——	14
Cadiz ....effective	34½	— Same as Mad.	——	6
Bilboa	35	— Same as Mad.	——	14
Seville	34	— Same as Mad.	——	14
Gibraltar	31			
Leghorn	46½	54	3 mdt	none fixed
Genoa	43¾	48	3 mdt	30
Venice	27		3 mdt	6
Malta	46	about 50	——	——
Naples	37½	42	3 mdt	3
Palermo..............per oz.	114	120	3 mdt	none
Lisbon	55½	67½	30 dt	6
Oporto	55½	Same as Lisb	——	——
Rio Janiero	58½	——	——	3
Dublin	10½	8½		3
Cork	10½	8½	——	

* There can be no fixed par with Banco, because the agio is fluctuating.

† In drawing bills of exchange upon Spain, this word is now used in-

By the preceding table, it appears, that the course of exchange on the above day, (3d Dec.) between London and Amsterdam, was *above par;* for Amsterdam gives London 39s. 6d. Flemish, for £1 sterling, and the par is only 36s. 8 gr. Flemish.

Between London and Madrid, it appears, that the exchange is below par; for London gives only 35 pence for 1 dollar, and the *par* is 39 pence per dollar; therefore, in both cases, the exchange is found to be favourable to London; but in all computations of this kind, *interest* should be allowed for the time which the bills have to run.

HOLLAND.

In Holland, there are two kinds of money; *current money* and *bank money,* commonly called *Banco.* Bank money is generally better than the same nominal sum of current money, and bears a premium of from 2 to 5 per cent. This premium, or difference between currency and banco, is called *Agio,* and is at present about 2 per cent. At Amsterdam, many of the leading transactions in trade, as well as foreign exchanges, are transacted in bank money; but accounts are kept both in current money and bank money.

Current money is the real coin which circulates in the country.

Bank money is *imaginary,* being the property which a person possesses in the bank.

The Bank of Amsterdam was established in 1609, by authority of government, and under the guarantee of the city. This bank received foreign coin, as well as the worn coin of the coun-

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stead of *Vales Reales,* otherwise they may be paid in this paper, which is generally at a discount.

To the greater number of places mentioned in the table, London gives the *certain price* (£1) for an uncertain, and to others, as the towns in Spain and Portugal, and some others, an *uncertain* (so many pence) for a certain sum of the money of these places. London exchanges with several other places, which will be noticed in another part of this work.

try, at its real intrinsic value, in the good standard money of the country; deducting only as much as was necessary to defray the expence of coinage, and other necessary expenses of management. For the value, after this deduction, it gave credit in its books.

This credit was called bank money, and, as it represented money exactly according to the standard of the Mint, was always of the same real value, and intrinsically worth more than current money. In consequence of this, it was, at the same time enacted, that all bills drawn upon, or negociated at, Amsterdam, of the value of 600 guilders, and upwards, should be paid in bank money, which took away all uncertainty in the value of those bills*. By this regulation, every merchant was obliged to keep an account with the bank, in order to pay his foreign bills of exchange, which necessarily produced a certain demand for bank money.

Besides the intrinsic superiority and the additional value which that demand necessarily gave bank money over current money, it had some other advantages.—It was secure from fire, robbery, and other accidents; the city of Amsterdam was responsible for it; the law prohibited all arrests, whether direct or indirect, upon sums lodged in the bank; and the confidence which the public had, that the deposits, represented by the receipts, in the hands of the different proprietors, actually existed, in real specie, in the coffers of the bank, and could be realised, should any extraordinary event occasion the dissolution of the establishment.

In consequence of these advantages, bank money seems to have borne an agio from its commencement; and it was generally believed, that all the money, originally deposited in the bank was allowed to remain there, because no person thought proper to demand payment of a debt, which could be sold, at any time, for a considerable premium. For, in withdrawing it from the bank, it could be of no more value than an equal sum of current money; but, while it remained in the coffers of the bank, its superiority was known and acknowledged, and could, at any time, be paid away by a simple transfer without the trouble of counting, or the risk of transporting it from one place to another.

Those deposits of coin, or those deposits that the bank was bound to restore in coin, constituted the original capital of the bank, or the whole value that was represented, by what is called

* In 1643, this sum was reduced to 300 guilders.

bank money. In order to facilitate the trade in bullion, the bank was, for many years, in the practice of giving credit, in its books, upon deposits of *gold* and *silver* bullion.

This credit was generally about 5 per cent. below the mint price of such bullion. The bank granted, at the same time, what was called *a recipice*, or receipt, for the value, which entitled the bearer, or the person who made the deposit, to take out the bullion again, at any time, within the space of six months after it had been deposited, upon transferring to the bank a quantity of bank money, equal in value to that for which he received credit, in the bank books, at the time the deposit was made; and upon paying *one-fourth* per cent, for the keeping of it, if the deposit was in *silver;* and *one-half* per cent, if it was in *gold.*

The bank of Amsterdam was, for many years, the great warehouse of Europe for bullion. The far greater part of the bank money, or of the credits upon the books of the bank, is believed to have been created, for these many years, by such deposits.

The person who made a deposit of bullion, obtained both a bank credit and a receipt: with the former he paid his bills of exchange, as they became due; and either sold or kept his receipt, according as he expected the price of bullion to rise or fall. The receipt and bank credit seldom kept long together; but this was of very little consequence. The person who had a receipt, and who wanted to take out bullion, always found plenty of bank money to purchase, at the ordinary price; and the person who had bank money, and wanted to take out bullion, always found receipts in equal abundance.

The bank also granted receipts and bank credits, upon deposits of current coin of the country; but these were often of no value, and would bring no price in the market. The reason of this was, the bank only allowed a credit of 5 per cent, below the current value, for the sum deposited; and, before it could be again taken out, one-fourth per cent. was to be paid for keeping it, which was real *loss* to the holder of the receipt. When, however, the agio of the bank fell as low as 3 per cent. such receipts were of some value, and might be sold at a small premium. But the agio of the bank being generally about 5 per cent, such receipts were frequently allowed to expire, or, as they expressed it, to fall into the hands of the bank.

The bank of Amsterdam professed to lend out no part of what

was deposited in it; but, for every guilder, for which it gave credit in its books, to keep in its repositories the value of a guilder, either in coin or in bullion; and, at Amsterdam, no point of faith was better established, than that for every guilder, circulated as bank money, there was a corresponding guilder, in gold or silver, to be found in the treasury of the bank. The coins of the State, which it received into its repositories were only of the larger kind; as ducatoons, 3-guilder pieces, and rixdollars; and each of these was received only at the standard of banco: so that the ducatoon was taken only at 60 stivers; the 3-guilder piece, at 57 stivers; the specie rixdollar, at 50 stivers; and the current rixdollar, at 48 stivers; which was 4 or 5 per cent below the current value.

Besides the above-mentioned coins, the bank received several other foreign coins, at a stated price, in parcels of at least 500 or 1000 pieces each; which, when ascertained to be of the due weight, were put into bags, in presence of the proprietor, who put his seal upon the bags, and received a receipt for them; in consequence of which he could, at any time, take back the same pieces, upon paying the charges of keeping, which, as already stated, were one-half per cent for gold, and one-fourth for silver.

To prevent the tricks of stock-jobbing, in raising or depressing the *agio*, by the holders of receipts and the owners of bank money, a resolution was adopted by the bank of selling bank money, at all times, for currency, at 5 per cent agio, and to buy it again at 4 per cent agio. In consequence of this resolution, the proportion between the market price of bank money and currency kept very near to the proportion between their intrinsic values. Before this resolution was adopted, the market price of bank money used sometimes to rise so high as 9 per cent agio, and sometimes to sink so low as *par*, according as opposite interests happened to influence the market.

In order to support the belief, which so generally prevailed, that all the specie which had ever been deposited in the bank still remained there, it was reported that every successive set of directors, on entering upon their office, examined and verified the existence of the specie, and likewise the bullion, constituting the funds of the bank.

When the domestic political troubles, however, raged in Holland, and financial difficulties began to prevail, both in the government of the city of Amsterdam and in that of the republic

at large, suspicions were very generally entertained, that the deposits of coin had not been preserved, and that they no longer existed to the original extent. The aristocratical nature of the government, and the secrecy in which all the transactions were involved, rendered it impossible to ascertain the truth of these reports until the general *revolution*, occasioned by the entrance of the French armies, when it was discovered that the greatest part, if not all, of the specie and bullion had been lent out to the government of Amsterdam, and to the States of the province of Holland, on bonds, bearing interest. The merchants of Amsterdam, being aware of the discredit which this discovery must bring upon their city, determined to take up these bonds, thereby restoring the original basis on which the bank was founded; while, at the same time, a privilege was obtained of frequently examining the affairs of the bank, and other precautions taken, in order to prevent the possibility of the recurrence of a similar culpable abuse. Since the above period, the bank has regained its credit, and bank money still continues to bear an agio of from 2 to 5 per cent.

EXCHANGES WITH AMSTERDAM.

At Amsterdam, accounts are kept in guilders, stivers, and pennings*, but the course of exchange, with London, is often stated in schillings and grotes Flemish.

16 Pennings	=	1 Stiver	=	2 gr. Flemish
20 Stiuvers	=	1 Guilder or Florin	=	$3\frac{1}{3}$ Schil. Flem.
$2\frac{1}{2}$ Florins	=	1 Rixdollar	=	100 gr. Flemish
6 Florins	=	1 Pound Flemish		

Flemish Money	12 Grotes	=	1 Schilling
	20 Schillings	=	1 Pound

In Holland, the Flemish denominations of money are much employed in stating the prices of many of the leading articles of merchandise; as, wine and spirits, grain, East and West India produce, drugs, and dye stuffs; and also, in computing the course of exchange, with many other commercial towns; and it is highly probable that the use of Flemish money, in computing exchanges, and the prices of some of the leading

* This coin is as seldom used, in accounts in Holland, as farthings in England.

articles of merchandise, was introduced from Flanders, in consequence of these branches of trade having been originally carried on there, and subsequently transferred from thence to Amsterdam, where the accustomed denominations of coins were adhered to, in stating those prices.

THE OTHER CURRENT COINS OF HOLLAND ARE THE FOLLOWING:

Copper	Duyt, 8 of which	=	1	Stuiver, or Stiver
	Stiver	=	8	Duyts
Silver	Dubbeltje	=	2	Stivers
	Seathalf	=	5½	Stivers
	Schilling	=	6	Stivers
	Rixdollar	=	50	Stivers
	Half ditto	=	25	Stivers
	Quarter ditto	=	12½	Stivers
	Zealand Rixdollar	=	52	Stivers
	——— half ditto	=	26	Stivers
	——— quarter do.	=	13	Stivers
	Drie Gulden	=	3	Florins
	Ducatoon	=	63	Stivers
Gold	Ryder	=	14	Florins
	Half ditto	=	7	Florins
	Ducaat, or Ducat	=	5¼	Florins

All foreign bills, of the amount of 300 florins, or above, are paid, at Amsterdam, in banco, which, as already mentioned, is generally from 2 to 5 per cent better than the same nominal sum of currency, and the difference is termed Agio.

USANCE.

The USANCE in Amsterdam is, for London, Paris, Antwerp, and Geneva, one month after date; for Germany and Switzerland, 14 days after sight; for Dantzic, Konigsberg, and Riga, *one* month after sight; for Italy, Spain, and Portugal, *two* months after date.

DAYS OF GRACE.

Six DAYS OF GRACE are allowed for the payment of bills of exchange, Sundays and holidays included, for bills drawn in *current* money; but for those drawn in bank money, there are commonly no days of grace allowed.

COURSE OF EXCHANGE.

		Uncertain Prices.			*Certain Prices.*
London	rec.	37 schil. Flem.	for	£1	Sterling.
Idem	rec.	11 Florins	for		ditto.
Hamburgh	rec.	33 Stiv. Banco	for	1	Rixdollar of 2 mark B.
Dantzic	rec.	1 Pound Flem.	for	372	Groshen current.
Paris, Bourdeaux	rec.	54 Grot. Flem. Banco	for	3	Francs.
Geneva	rec.	90 Grotes Flem.	for	1	Crown current.
Genoa	rec.	84 Grotes Flem.	for	1	Pezza of 5¾ Lire di Banco.
Idem	rec.	106 Florins curt.	for	120	Marks Banco.
Leghorn	rec.	89 Grotes Flem.	for	1	Pezza of 8 reals.
Lisbon, Oporto	rec.	44 Grotes Flem.	for	1	Crusade of 400 rees.
Madrid, Cadiz &c.	rec.	98 Grotes Flem.	for	1	Ducat of exchange.
Petersburgh	rec.	54 Grotes	for	1	Ruble.
Vienna	rec.	18 StiversBanco	for	1	Rixdollar current.
Venice	gives	96 Soldi Piccoli	for	1	Florin Banco.
Frankfort	gives	139 Rixdollars	for	100	Rixdollars current.

Antwerp gives 4½ per cent & receives 104½ Florins for 100 Flor. Banco.
Agio of the Bank, 4 per cent, or 100 Flor. B. = 104 Florins current.

TO REDUCE BANCO TO CURRENCY AND THE CONVERSE.

As 100 : (100+Agio) :: Banco : Currency

And as

(100+Agio : 100 :: (Currency : Banco.

EXAMPLE I.

Reduce 1625 Florins Banco to currency, agio, $3\frac{1}{4}$ per cent.

$$100 : 103\tfrac{1}{4} :: 1625$$
$$103\tfrac{1}{4}$$

flor. stiv. pen.

1677.8125 = 1677 16 4 current.

EXAMPLE II.

Reduce 1677 florins, $16\frac{1}{4}$ stiv., current, to banco.

$$103\tfrac{1}{4} : 100 :: 1677\ 16\tfrac{1}{4}$$
$$\text{or } 413 : 400 :: 1677.8125$$
$$400$$

413)671125.0000

1625 florins banco.

TO REDUCE ENGLISH MONEY INTO DUTCH MONEY.

RULE.

As £1 sterling is to the given sum of sterling money, so is the given course of exchange to the Dutch money required.

Dutch money may be reduced to English by reversing the above analogy.

EXAMPLE I.

How many florins banco in £1000 sterling; exchange 34s. 8½gr. Flemish, per £1 sterling?

METHOD I.

	£1	:	£1000 : : 34s. 8½gr.
	20		34
gr.	20		34000
$4 = \frac{1}{3}$		=	333 4
$4 = \frac{1}{3}$		=	333 4
$\frac{1}{2} = \frac{1}{8}$		=	41 8
			34708 4
			6
			2,0)20825,0
			10412 10 Flor. Banco.

METHOD II.

S. gr.
34 8½ = S. 34.708$\dot{3}$
1000

34708.3333
Mult. $\frac{6}{20} = \frac{3}{10} =$ .3

Florins 10412.50000

METHOD III.

s. gr.
34 8½
12

416½
1000

1 florin = 4,0d.)41650,0

10412 10 florins, as before.

METHOD IV.

34s. $8\frac{1}{2}$gr. = £1. 14s. $8\frac{1}{2}$gr.

		1000 at £1	14	$8\frac{1}{2}$	
10s. = $\frac{1}{2}$ of a £ =		£500			
4s. = $\frac{1}{5}$ of ditto =		200			
8gr. = $\frac{1}{6}$ of 4s. =		33	6	8	
$\frac{1}{2}$gr. = $\frac{1}{16}$ of 8gr. =		2	1	8	
		£1735	8	4	Flemish
				6	
		10412	10		Florins.

METHOD V.

34s. $8\frac{1}{2}$gr. = 34s. $4\frac{1}{4}$st.
6

2,0)20,$8\frac{1}{4}$

Florins 10 $8\frac{1}{4}$ = £1.

Then 1000 at 10 florins $8\frac{1}{4}$s.
10

	10000	
4s. = $\frac{1}{5}$ of a florin =	200	
4s. = $\frac{1}{5}$ of ditto =	200	
$\frac{1}{4}$ = $\frac{1}{16}$ of 4s. =	12	10
Florins	10412	10

EXAMPLE II.

In 7000 florins current, how much sterling money; exchange 35 schillings Flemish, Agio 3 per cent.?

METHOD I.

35s.

$\frac{6}{20} = \frac{3}{10} = .3$

			Flor.	
Florins	10.5	:	7000	:: £1
Agio	103	:	100	::
	10815		)700000	
			£647 4 $11\frac{1}{4}$	

METHOD II.

$$
\begin{array}{rcrcr}
\text{F.} & & \text{F.} & & \text{F.} \\
103 & : & 100 & :: & 7000 \\
 & & & & 100 \\
\hline
\end{array}
$$

$$
\begin{array}{ll}
 & 103)700000 \\
\text{gr.} & \\
35 & \text{F}.6796.1165 \\
12 & \qquad\quad 40 \\
\hline
\multicolumn{2}{l}{42,0)27184,4.6600} \\
\end{array}
$$

$$
\begin{array}{lrrr}
 & \pounds & \text{s.} & \text{d.} \\
\text{Florins } 647.249 = & 647 & 4 & 11\frac{1}{4}
\end{array}
$$

In Example I., £1000 is reduced to schillings Flemish, by multiplying by 34s. and taking parts for $8\frac{1}{4}$ grotes, which are reduced to stivers by multiplying by 6, the number of stivers in a schilling, and then to florins, by dividing by 20, the number of stivers in a florin.

The 2d method, by *decimals*, is both simple and easy.

The 3d method, here employed, is to reduce £1000 to grotes by multiplying by the grotes in 34s. $8\frac{1}{4}$ gr., and then to reduce these to florins, by dividing by 40, the number of grotes in a florin. In this instance the calculation is short, by this method; but this is not always the case.

By method 4th the pounds sterling are first reduced to pounds Flemish, and then to florins, by *multiplying* by 6, which might have been done previous to reducing the pounds sterling to pounds Flemish.

In method 5th, the rate of exchange, 34s. $8\frac{1}{4}$ gr. Flemish, is first converted into florins; the pounds sterling are then multiplied by the florins, and proportional parts taken for the stivers. Each of the above methods has its advantages, in performing particular examples, which the attentive student will discover, by a little practice and attention.

In Example II., method 1st (35s. Flemish) is reduced to florins, by multiplying them by .3, which is the same as mult-plying by 6 and dividing by 20; then the proportion is stated

for bringing the given number of florins to pounds sterling; but there is another stating, in proportion, necessary for bringing current money to bank money; therefore, these two proportions are blended together and performed at once; which should always be done, when it is practicable, as it is much neater and *shorter* than the method by *two* statings, which is adopted in method 2d.

EXERCISES.

1. In 6725 florins banco, how many current guilders; agio $1\frac{1}{4}$ per cent?

2. In 4560 current florins, how many guilders banco; agio $4\frac{4}{7}$ per cent?

3. In £640. 10s. sterling, how many current guilders; ex. 36s. $4\frac{1}{2}$d., agio $3\frac{1}{4}$ per cent?

4. In 11240 current guilders, $18\frac{1}{2}$s., how much sterling; ex. 11 guilders, 8 st., agio $2\frac{1}{2}$ per cent?

5. Reduce £586. 10s. sterling, into Dutch money; ex. at 10 florins, 8 stivers, per pound sterling.

6. How much English money in 4583 guilders, 13 stivers, 8 pennings, currency of Holland; ex. 35s. 10gr. Flemish, per pound sterling; agio $4\frac{1}{8}$ per cent.

7. A merchant in Amsterdam owes £1704. 5s. sterling, in London, how much Dutch money is necessary to pay the debt; ex. 10 florins, $6\frac{1}{4}$ stivers, per pound sterling.

8. When flax sells, at Amsterdam, at 35 stivers per head, reckoning 18 heads, 1 cwt., what is the prime cost, in London, per cwt. allowing 4s. per cwt. for freight and charges; ex. being 37s. 5 gr. per pound sterling.

9. A merchant in London having shipped a parcel of coffee, by order, and on account and risk of a house at Amsterdam, amounting, as per invoice, to £733. 2s. 8d., draws for this sum, at $2\frac{1}{2}$ usance, on his correspondent at Amsterdam; and having negociated his draft, in London, at the ex. of 11 guilders, 8 stivers, per pound sterling, what sum, in guilders, will the merchant in Amsterdam have to pay, the agio being at $3\frac{1}{2}$ per cent.?

10. A house in London having received from Amsterdam a cargo of wheat, to be sold on account of the shipper, the net proceeds in London amounted, as per account sales, to £1322. 7s. 5d.; with which sum a bill is bought on Amsterdam, at the ex. of 11 guilders, 9¼ stivers, per pound sterling, and remitted to the shipper: how many guilders ought the latter to receive for the net proceeds of his wheat, the agio of the bank being 3⅝ per cent.?

NOTE. As it is absolutely necessary to be acquainted with the monies by which exchanges are regulated between different countries, before the money of one country can be reduced to its equivalent, in that of another, examples of *foreign* places with *each other*, will be given, after tables of the monies of each, and examples with London have been exhibited.

RABAT AT AMSTERDAM.

Some sorts of goods are sold at Amsterdam with a *Rabat*, which is a discount, generally, of 8 per cent, per annum, for prompt payment, on goods which were formerly sold at a very long credit.

Thus, 15 months, or 10 per cent, are allowed on German, Prussian, and Polish wool; and 18 months, or 12 per cent, on brown Muscovado sugar, potashes, soda, and Italian silks.

Twenty-one months, or 14 per cent, are allowed on Spanish wool and lamb's wool.

Dutch cloths are sold with 4 per cent Rabat, for ready money; or at 9 months credit, without Rabat.

In all sales of goods, except these, a further abatement is made, of 1 or 2 per cent, for prompt payment.

At Amsterdam there are certain allowances made, in the sales of goods, which the limits of this volume will not admit of mentioning.

ROTTERDAM.

At Rotterdam, accounts are kept in guilders, or florins, stivers, &c.; *Current* and exchanges are also transacted in currency.

The Bank of Rotterdam was erected about 25 years after that of Amsterdam, and was constituted on the same plan, except that it receives all the current coins of the province of Holland at their *current* price; its accounts are, therefore, kept in current money. Merchants are, however, permitted to keep *two* accounts in the bank books, the one in current money and the other in Amsterdam banco; but they must allow the agio which Amsterdam banco bears against currency.

Bills drawn on Rotterdam are, however, sometimes made payable in Amsterdam banco; but, when this is the case, the holder then receives the amount in currency, with an allowance for the agio.

The other *current coins*, the *usances*, and *days of grace*, are the same as at Amsterdam.

EXAMPLE I.

In £800 sterling, how many current guilders; exchange, 11 florins, 12 stivers, per pound sterling?

$$\begin{array}{rr} & £800 \\ & 11 \\ \hline & 8800 \\ 10 \text{ st.} = \frac{1}{2} = & 400 \\ 2 \text{ st.} = \frac{1}{5} = & 80 \\ \hline & 9280 \text{ Florins} \end{array}$$

EXAMPLE II.

In 5380 florins current, how many pounds sterling; exchange 10 florins, 16 stivers per pound sterling?

flor. 10.8)5380.0 florins

£498 3 nearly

EXERCISES.

1. In £500. sterling, how many current guilders; exchange 11 guilders, 10 stivers current, per pound sterling?

2. In £480. 17s. 6d. sterling, how many current guilders; exchange at 12 guilders, 6 stivers current, per pound sterling?

3. In 7800 current guilders, how many pounds sterling; exchange, 36s. 4 gr. Flemish, per pound sterling.

4. In 6448 current guilders, how many pounds sterling; exchange, 35s. 6 gr. Flemish, per pound sterling?

5. A merchant in Rotterdam ships 60 mats of flax, by order and on account of a house in Newcastle, and draws for the amount of the invoice, 8794 guilders, 3 stivers, on the latter's correspondent, in London, the exchange being 11 guilders, 18 stivers, per pound sterling; how much will the flax cost the Newcastle importer, in sterling money, exclusive of frieight, insurance, and charges?

HAMBURGH.

There are two kinds of money at Hamburgh, distinguished into *banco* and *currency*, as at Amsterdam*.

* Besides banco and currency, there are two other kinds of money peculiar to Hamburgh; these are,

Specie, or old full weight constitution rixdollars, reckoned ½ per cent better than banco.

Light coin is composed chiefly of foreign coins, which are esteemed about 36 per cent worse than banco.

Hamburgh money was formerly distinguished by the word *Lube*, from

BANK OF HAMBURGH.

The bank of Hamburgh was erected in 1619. It formerly received nothing but the old specie rixdollars of Germany, which it still continues to receive, according to a certain standard of weight and fineness.

Those who bring to the bank 1000 specie rixdollars, receive credit for 1001.

Since the year 1769, the bank also receives silver in bars, which must be 15 loths, 12 grains, or $\frac{47}{48}$ fine.

Those who deposit such bars or ingots, are credited by the amount in the bank books, at the rate of 27 marcs, 10 schillings, banco, for each marc of the same silver, the whole of which is again delivered on demand, without any reference to the time it remained in the bank, upon paying 27 marcs, 12 schillings, *banco*, for each marc of registered weight, which is 2 schillings per marc more than the sum advanced, the amount of the difference being applied to the purpose of defraying the charges of the bank.

The smallest sum that can be paid in or transferred at the bank, is 100 marcs.

The intrinsic value of bank money is $23\frac{1}{13}$ per cent better than current money, which is composed of the effective coins of Hamburgh; but this difference, or agio, as it is termed, is variable, being sometimes above $23\frac{1}{13}$, and at other times below it.

The par between English money and currency is 43s. Flemish, for £1 sterling; but there can be no permanent par with *banco*, for the reason just stated; namely, the agio being variable. If the agio be 23, £1 sterling is equivalent to 34s. 11¼gr.; but, by the estimate of merchants, the par is from 34s. 7gr. to 34s. 10gr. varying according to the price of gold and silver, and the state of the agio.

---

the town of Lubeck, the place where coined; but merchants now use the word Hambro' instead of it, but it is still retained in books of exchange.

RABAT.

Some kinds of merchandize, when sold here in large quantities, have an allowance made upon them of 7, 13, or 16 months Rabat, reckoned at 8 per cent, per annum, which is deducted from the nominal price, when the buyer pays ready money, or even within 4 weeks of the day of sale.

Refined sugars, English and Dutch cloths, are sold with 7 months rabat, or $4\frac{2}{3}$ per cent, that is, $4\frac{2}{3}$ are deducted from $104\frac{2}{3}$, or 7 from 157.

Cotton, cinnamon, cochineal, indigo, ginger, nutmegs, cloves, mace, rice, raw sugars, gall nuts, Russian leather, linen, Italian silks, Silesia cloth, Marseilles soap, almonds, molasses, shumac, Turkish yarn, &c. are sold with 13 months rabat, or, $8\frac{2}{3}$ per cent, that is, 13 is deducted from 163.

The allowances for Tare on the various articles of merchandize are fixed, as is the case at Amsterdam; but the limits of this work will not admit of exhibiting a table of these*.

MONIES OF EXCHANGE.

Accounts are kept in marcs, schillings, and phennings, both in Banco and Currency, and by some in Pounds, Schillings, and Grotes, Flemish; but Flemish money is imaginary, being only used in Exchanges, and in the purchase and sale of a few articles of merchandize.

Payments are made in Banco, and sometimes in Currency.

HAMBRO' BANCO AND CURRENCY.

12 phennings	=1 schilling	= 2	grotes Flem.		
16 schillings	=1 marc	=32	ditto	= $2\frac{2}{3}$	sh. Flem.
2 marcs	=1 dollar of ex.	=64	ditto	= $5\frac{1}{3}$	ditto
3 marcs	=1 rixdollar	=96	ditto	= 8	ditto

---

* The tare on English raw sugar, in hhds. of 1000 lb. and under, is 20 per cent. Of 1500 lb. and upwards, 18 per cent. English white sugar, in hhds. of 1500 lb., 14 per cent. Coffee, per boll of 300 lb., 14 lb.

FLEMISH.

12 Grotes	=	1 Schilling.
20 Schillings	=	1 Pound.

therefore,

6 Schillings Hambro' banco	=	1 Schilling Flemish
3 Marks banco	=	48 Schillings Hambro' banco.

COURSE OF EXCHANGE.

			Uncertain Prices.		*Certain Prices.*
London	rec.	35	Schillings Flem.	for	1 Pound sterling
Amsterdam	gives	34	Stivers banco	for	2 Marcs banco
Ditto	gives	103¼	Rixdollars current	for	100 Rixdollars banco
Bourdeaux	rec.	25	Schillings banco	for	3 Francs
Copenhagen	gives	149	Rixdollars	for	100 Rixdollars banco
Cadiz	rec.	90	Grotes Flem. banco	for	1 Ducateofexchange
Genoa	rec.	80	Grotes Flem. banco	for	1 Pezza
Leghorn	rec.	86	Grotes Flem. banco	for	1 Pezza of 8 reals
Lisbon	rec.	43½	Grotes Flem. banco	for	1 Old crusade
Madrid	rec.	91	Grotes Flem. banco	for	1 Ducate of ex.
Oporto	rec.	43	Grotes Flem. banco	for	1 Old crusade
Paris	rec.	26	Schil. Flem. banco	for	3 Francs
Venice	gives	2	Livres	for	1 Marc banco
Vienna	gives	310	Florins Austrian cur.	for	100 Rixdollars banco

USANCE.

The Usance for bills drawn from England, France, and Holland, is 1 month after date. From Italy, Spain, and Portugal, is 2 months after date.

DAYS OF GRACE.

There are 12 Days of Grace, Sundays and holidays included; but bills due on Sunday, or any holiday, must be paid, or protested, the day before.

TO REDUCE BANCO INTO CURRENCY AND THE CONVERSE.

RULE.

1. As 100 is to 100 *plus* the Agio, so is the given sum of Banco to its equivalent in Currency.

[illegible]

EXAMPLE I.

[illegible] sterling, [illegible] ; the [illegible]

[illegible]

Answer [illegible]

EXAMPLE II.

[illegible] sterling, [illegible] how much banco; the [illegible]

[illegible]

[illegible] 5000

Answer [illegible] 623

EXAMPLE III.

[illegible] how much sterling money; exchange [illegible] per pound sterling?

METHOD II.

```
 S.  gr.  Marcs.
 36   2   3280
      6     16 sch.
 ------  ----   £   s.  d.
    217)52480(241  16  10¼
        434
        ---
         908
         868
         ---
          400
          217
          ---
          183
           20
          ----
          3660
          217
          ----
          1490
          1302
          ----
           188
            12
          ----
          2256
          2170
          ----
            86
             4
          ----
           344
           217
          ----
           127 Rem.
```

METHOD I.

```
  s.  gr.  marcs.
  36   2   3280
   3          8
 ------   -----
 108.5)   26240
 ------   -----      £   s.  d.
     £241.843=241  16  10¼
          -----
```

EXAMPLE IV.

In £225. 17s. 6d. how many marcs banco, &c. exchange, 36s. 3 gr. Flemish, per pound sterling?

METHOD I.

	S.	gr.
	36	3
		12
	435	0
	£ 225$\frac{7}{8}$	
	2175	
	870	
	870	
	97875	
$\frac{7}{8}$=	380$\frac{5}{8}$	
32 { 8	98255$\frac{5}{8}$	
4	12282 nearly	
Marcs	3070	8

METHOD II.

	£225	17	6
16s.=$\frac{4}{5}$=	180	14	0
3gr.=$\frac{1}{80}$=	2	16	6
	409	8	0
	7$\frac{1}{2}$		
	2863		
	204	8	
	3067	8	
8s. =	3	0	
Marcs	3070	8	Banco

METHOD III.

	S.		marcs		S. gr.
	20	:	7.5	: :	36.25
or,	4	:	1.5		1.5

	18125
	3625
	4)54.375
Marcs in 1 pound =	13.59375
	225
	6796875
	2718750
	2718750
$\frac{7}{8}$=	1189458
	M. s.
Marcs	3070.48833=3070 8

EXERCISES.

1. In 5000 marcs banco, how much sterling money; exchange, 35s. 10gr. Flemish, per pound sterling?

2. In 8732 marcs, 12 schillings current, how many pounds sterling; exchange, 34s. 5gr. Flemish, per pound sterling, agio 20 per cent?

3. How many marcs banco in 1950 guilders current, of Holland, agio 4 per cent; exchange $32\frac{1}{2}$ stivers per dollar of Hamburgh?

4. How many current guilders of Holland in 3640 marcs current; agio, at Hamburgh, 20 per cent, exchange 35s. Flemish; agio, in Holland, $3\frac{1}{2}$ per cent?

5. London remitted to Hamburgh £8600, when the course of exchange was 35s. 5gr. Flemish, per pound sterling; what was gained or lost, by drawing for the same sum, when the exchange was at 33s. 4gr. per pound?

6. London drew on Hamburgh for 2000 marcs, when the course of exchange was 36s. 7gr.; but, when the bill became due, London was obliged to remit at 33s. 4gr. What was the difference, in favour of Hamburgh, and how much per cent?

7. A merchant in London executes an order for a person at Hamburgh, amounting to £373. 4s. 7d.; what will the goods cost in Hamburgh, including 236 marcs, 4s. banco, for freight and charges; exchange at 36s. 9gr. Flemish, per pound sterling?

8. Twenty hhds. of sugar were sold in Hamburgh, the net proceeds of which were 4562 marcs, 6s. banco; what sum ought the proprietor to receive, in England, the course of exchange being 35s. 5gr. Flemish, per pound sterling?

9. A merchant in Hamburgh having shipped a quantity of three-crown yarn, for a house in London, amounting to 9364 marcs, 11s. banco; how much sterling money ought he to draw for, in order to receive payment, exchange being 36s. 1gr. Flemish, per pound sterling?

10. A factor at St. Petersburgh ships 20 tons of hemp, for

a merchant in Dundee, and draws on a house in Hamburgh for 4674 marcs, 9s. banco; what sum must the house in Hamburgh draw for, on London, on account of the importer in Dundee, in order to cover the same, with the addition of $\frac{1}{2}$ per cent commission, and £13. 7s. charges; exchange at 36s. 7 gr. Flemish, per pound sterling?

11. A house in London draws on a merchant in Hamburgh, for £535. 10s., at 2 usance, exchange being 35s. 9gr. Flemish, per pound sterling; and, when the bill falls due, is redrawn upon, by the merchant at Hamburgh, at 2 months date, exchange being 36s. 2gr. Flemish. Required the amount of the *first draft*, in marcs banco, and also of the second, in sterling money, including $\frac{1}{2}$ per cent commission, $\frac{1}{8}$ per cent brokerage of bills, and 11s. 8d. postages; and also what sum, and what rate per cent, per annum, does the London house pay for the the accommodation of 4 months?

ALTONA exchanges in the same manner as Hamburgh; but the course of exchange is generally 1 grote, or penny Flemish, higher than upon Hamburgh.

PARIS AND BOURDEAUX.

In FRANCE there are two different systems of monies, weights, &c.; namely, the Old System and the New System.

The Old System is now nearly out of use; it is, therefore, unnecessary to enter upon a description of it in this place.

The New System was established in 1795. See page 178.

Accounts are now kept in Francs, Decimes, and Centimes. The Franc and Livre were formerly synonimous; but, in the coinage of 1795, the 5-livre pieces were, by some accident, made too heavy, being worth $101\frac{1}{4}$ sous, instead of 100; the *new* Franc has, therefore, been adopted in conformity to this accidental value, which makes 80 Francs=81 Livres.

The GOLD COINS of France are pieces of 40 Francs and 20 Francs: the 40-Franc piece is worth £1. 11s. $8\frac{1}{4}$d. of English gold coin; and the value of the 20-Franc piece is in the same proportion.

The SILVER COINS are of 5, 2, and 1 Francs; and also of $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ Francs, or 75, 50, and 25 Centimes.

The COPPER COINS are pieces of 2 and 1 Decimes; and also of 5, 3, 2, and 1 Centimes.

The *fineness*, both of the *gold* and *silver coins*, is inferior to the English coins, in the proportion of 54 to 55; hence the 5-Franc piece is worth 48½d., and, therefore £1 = 24 Francs, 74 Centimes*.

MONEY OF EXCHANGE.

OLD COINS.

12 Deniers	=	1 Sou or Sol
20 Sous	=	1 Livre Tournois
3 Livres	=	1 Ecu.

NEW COINS.

10 Centimes	=	1 Decime
10 Decimes, or 100 Cents	=	1 Franc
80 Francs	=	81 Livres.

Hence, as 81 : 80 :: Livres : Francs.

At Paris and Bourdeaux, Exchanges are computed in Francs and Centimes.

USANCE.

The USANCE for bills, drawn from Spain and Portugal, is 60 days after date.

From England, and other countries, 30 days ditto.

DAYS OF GRACE.

Ten DAYS of GRACE are allowed, on bills payable at one or more Usances; but bills drawn at sight, or at a fixed term, must

* This is without allowing for remedy; which, in the gold, is $\frac{1}{500}$ part of the weight, and $\frac{3}{1000}$ in the alloy; and, in the silver, $\frac{1}{300}$ of the weight, and $\frac{7}{1000}$ in the alloy.

be paid, or protested, after they are due: at Lyons, no days of grace are allowed.

COURSE OF EXCHANGE.

		Uncertain Prices.			Certain Prices.
London	receives	25 Francs	for	1	Pound sterling
Amsterdam	gives	54 Grotes Flem.	for	3	Francs
Copenhagen	gives	67 Rixdollars	for	300	Francs
Genoa	receives	465 Centimes	for	1	Pezza
Hamburgh	receives	185 Francs	for	100	Marcs banco
Leghorn	receives	503 Centimes	for	1	Pezza of 8 reals
Lisbon	gives	460 Rees	for	3	Francs
Naples	receives	$4\frac{1}{4}$ Francs	for	1	Ducat di regno
Petersburgh	receives	340 Centimes	for	1	Ruble
Spain	receives	$15\frac{3}{4}$ Francs	for	1	Doubloon
Vienna	receives	260 Centimes	for	1	Florin current.

EXAMPLE I.

In 4728 francs, 50 cents, how much sterling money; exchange, at 24 francs, 75 cents, per pound sterling?

24.75)4728.50(£191. 1s. $0\frac{4}{33}$d.
2475

22535
22275

2600
2475

125
20

2500
2475

25
12

$$\frac{300}{2475}=\frac{4}{33}$$

EXAMPLE II.

In £187. 15s. 6d., how many francs, &c.; exchange at 25 francs, 5 cents, per pound sterling?

```
     £187.775                    Francs 25.05
Francs  25.05                        £  187
       ------                       ------
       938875                        17535
      938875                        20040
     375550                         2505
     -----------  franc  ct.        ------
     4703.76375=4703 76½            468435
                        10s. = ½ =12525
                         5s. = ¼ =  62625
                         6d. =1/10=  62625
                             ----------  franc  ct.
                        Francs 4703.76375=4703 76½
```

EXAMPLE III.

In 7478 livres, 15 sous, how many pounds sterling; exchange, 24 francs, 85 cents, per pound?

```
£     F.    £
81 :  80 :  7478.15
               80
           -------
     ( 9)598252.00
  81 { ----------
     ( 9) 66472.44
      ----------
24.85)F. 7385.8271(£297.216
         4970          20
         ----         ----
         24158        4.320
         22365          12
         -----        ----
          17932       3.840
          17395          4
          -----       ----
            5377      3.360
            4970      ----
            ----
             4071   £297. 4s. 3¼d. Ans.
             2485
             ----
             15866
             14910
             -----
               956 Rem.
```

EXERCISES.

1. In £208. 18s. how many francs, &c.; exchange 25 francs, 2 cents per pound sterling?

2. In 1954 francs, 18 cents, how many pounds sterling; exchange, 24 francs, 8 cents per pound sterling?

3. In 7500 livres, 12 sous, 6 deniers, how many pounds sterling; exchange, 24 francs, 25 cents, per pound sterling?

4. In 2430 livres, how many guilders of Holland; exchange, 54 gr. Flemish for 3 francs?

5. In 17800 marcs of Hambro' banco, how many francs, &c.; exchange, 24¼ schillings banco, for 3 francs?

6. In 16725 francs, 50 cents, how many marcs of Hamburgh; exchange, 190 francs, for 100 marcs banco?

7. A merchant, in London, remitted to Bourdeaux £4500, when the course of exchange was 23 francs, 75 cents, per pound sterling; and drew for the same amount, when the exchange was 24 francs, 50 cents. Required the difference?

SPAIN.

In Spain, accounts are principally kept in Piastres, Reals, and Maravedies.

There are 4 different kinds of reals, viz. *Vellon*, *New Plate*, *Old Plate*, and *Mexican Plate*.

The Real vellon is the most current coin in Spain. It consists of 8½ quartos, or 34 *maravedies vellon.*

The Real of New Plate (Real de plata Provincial) is worth 17 quartos, or 68 maravedies vellon.

The Real of Old Plate (Real de plata Antigua) is chiefly used in transactions with foreign countries and in exchanges, and is

worth 16 quortos, 34 maravedies of plate, or 64 maravedies vellon.

The Real of Mexican Plate (Real de plata Mexicana) is used in transactions with Spanish America, where accounts are mostly kept in hard Dollars, reals, and quarters, and sometimes in sixteenths. The Mexican Real is $\frac{1}{8}$ of a hard Dollar, and worth 34 Mexican maravedies of plate, or 85 maravedies of vellon.

In Exchanges, as observed above, *plate* only is used, and in drawing bills, it is now customary to write, " payable in *effective,*" to prevent them from being paid in *Vales Reales*, or government paper, which is often at a *discount*.

THE COINS OF SPAIN ARE THE FOLLOWING:

		Reals Vel.	M.
Of Gold	The Doubloon of 8 Escudos, or 4 Pistoles, which passes for	320	0
	The Doubloon of 4 Escudos, or double pistole	160	0
	The Doubloon de Oro, or Pistole	80	0
	The Escudo	40	0
	The Coronilla, or veinten de Oro	20	0
Of Silver	The Dollar, or peso duro	20	0
	The Half dollar, or escudo vellon	10	0
	The Peceta Mexicana, or $\frac{1}{4}$ ditto	5	0
	The Real of Mexican plate	2	17
Of Silver and Copper.	The Peceta provincial	4	0
	The Real of provincial plate	2	0
	The Real vellon	1	0
Of Copper	The Piece of 2 quartos	8	0
	The Quarto	4	0
	The Octavo	2	0

From the year 1730 to 1772, the gold and silver coins were of the same degree of fineness with those of England; but, in the year 1772, both the gold and silver were reduced $\frac{1}{17}$ part, and in 1786, the standard of the gold was again reduced about the same quantity.

The gold coins, above-mentioned, are of the Coinage of 1772. Those of former coinages, which still remain in circulation, bear a premium of about $\frac{1}{2}$ per cent.

The Doubloon, or quadruple Pistole, when assayed at the London mint, some years ago, was found to weigh 17 dwt. 8 grs.

and to be $4\frac{1}{2}$ grains worse than English standard; therefore, its value in English gold coin, is £3. 4s. $0\frac{1}{2}$d.

The Dollar was found to weigh the same as the Doubloon, and to be 8 dwt. worse than English standard; and, therefore, its value in English silver coin is 4s. $3\frac{1}{4}$d.

The value of the gold coins of Spain is to the silver as 16 to 1.

MONIES OF EXCHANGE.

34	Maravedies	=	1 Real.
8	Reals	=	1 Piastre
4	Piastres	=	1 Pistole of exchange, or Doubloon*.
375	Maravedies	=	1 Ducat.

Exchanges between London and Spain are negociated by the *piastre,* which is also called the *pezza, dollar of exchange,* and *piece of eight.*

The Ducato, or Ducado de Cambio, and the Pistole, are likewise used in Exchanges.

COURSE OF EXCHANGE.

			Uncertain Prices.			*Certain Prices.*
London	gives	36	Pence sterling	for	1	Dollar of ex.
Amsterdam	gives	93	Grotes Flem.	for	1	Ducat of ex.
France	gives	15	Francs	for	1	Doubloon plate
Genoa	rec.	121	Dollars plate	for	100	Pezze of $5\frac{3}{4}$ lire
Hamburgh	gives	90	Grotes Flemish	for	1	Ducat of ex.
Leghorn	rec.	130	Dollars of ex.	for	100	Pezze of 8 reals
Naples	rec.	315	Maravedies plate	for	1	Ducat di regno
Portugal	gives	2400	Rees	for	1	Doubloon plate

USANCE.

At Madrid and Seville, the Usance, for bills drawn from England, France, and Italy, is 60 days after date.

From Amsterdam and Hamburgh, 2 months after date.

At Cadiz, the Usance from France is 1 month after date, and from the other parts of Europe, 60 days.

---

* In exchanges with England, *Plate* only is used, which is more valuable than *Vellon,* in the proportion of 32 to 17; that is, 17 reals *plate* are equal to 32 reals *vellon.*

DAYS OF GRACE.

At Madrid, Seville, Bilboa, and Barcelona, 14 days of grace are allowed for bills accepted before they become due, otherwise no days of grace are allowed.

At Cadiz, 6 days of grace are allowed in all cases.

EXAMPLE I.

In 4795 piastres, 5 reals, 16 maravedies, how much sterling money; exchange, $36\frac{1}{2}$d. per piastre?

METHOD I.

Pias.		pias.	re.	m.		d.
1	:	4795	5	16	: :	$36\frac{1}{2}$
8		8				
8		38365				
34		34				
272		153476				
		115095				
		1304426				
		$36\frac{1}{2}$				
		7826556				
		3913278				
		652213				

272)47611549(12 | 175042 2
272
2041
1904
1371
1360
1154
1088
669
544
125
4
500
544 nearly

2,0 | 1458,6 $10\frac{1}{2}$

£729 6 $10\frac{1}{2}$

METHOD II.

		P.		
		4795		
2 s. $= \frac{1}{10}$ of a Pound	=	479	10	0
1 s. $= \frac{1}{2}$	=	239	15	0
$\frac{1}{2}$ d. $= \frac{1}{24}$	=	9	19	$9\frac{1}{2}$
4 r. $= \frac{1}{2}$	=	0	1	$6\frac{1}{4}$
1 r. $= \frac{1}{4}$	=	0	0	$4\frac{1}{2}$
16 m. $= \frac{1}{2}$	=	0	0	$2\frac{1}{4}$
		£729	6	$10\frac{1}{2}$

METHOD III.

$$
\begin{array}{rl}
 & 4795.683825 \text{ piastres} \\
 & \qquad\quad 3\tfrac{1}{24} \text{ shillings} \\
\hline
 & 14387.051475 \\
\tfrac{1}{24} = & 199.820125 \\
\hline
 & 2,0)14586.871600 \\
\hline
 & £729.34358 = £729 \;\; 6 \;\; 10\tfrac{1}{2}
\end{array}
$$

EXAMPLE II.

In 64320 reals vellon, how much sterling money; exchange, $34\frac{1}{2}$d. per piastre of plate?

$$
\begin{array}{l}
8)\underline{64320} \text{ reals} \\
\quad 8040 \text{ piastres vel.}
\end{array}
\qquad
\begin{array}{r}
\text{then } 32 : 8040 :: 17 \\
\text{or } \;\; 4 : 1005 :: 17 \\
17 \\
\hline
4)17085 \\
\hline
4271\tfrac{1}{4} \text{ piastres of plate} \\
3 \text{ shillings} \\
\hline
12813\tfrac{3}{4} \text{ deduct } \tfrac{1}{8} \text{ of 1 shil.} \\
533\tfrac{29}{32} \\
\hline
2,0)1227,9\tfrac{27}{32} \\
\hline
£613 \;\; 19 \;\; 10
\end{array}
$$

$$
\begin{array}{rl}
\text{or} & 4271.25 \text{ piastres} \\
\hline
20\text{d.} = \tfrac{1}{12} = & 355.9375 \\
12\text{d.} = \tfrac{1}{20} = & 213.5625 \\
2\tfrac{1}{2}\text{d.} = \tfrac{1}{8} \text{ of } 20\text{d.} = & 44.4922 \\
\hline
 & £613.9922 \quad = \quad £613 \;\; 19 \;\; 10
\end{array}
$$

EXERCISES.

1. In £587. 16s. 8d. sterling, how many dollars; exchange, 39d. per dollar?

2. In 5420 piastres, 6 reals, how much sterling money; exchange, $37\frac{1}{2}$d. per piastre?

3. In 1860 reals vellon, how much sterling money; exchange, $37\frac{1}{2}$d. per piastre of plate?

4. How much Hambro' banco in 4712 reals, 5 mar. vellon; exchange, $85\frac{1}{7}$ grotes per ducat?

5. How many guilders banco of Holland in 45600 reals plate; exchange, 92gr. Flemish, per ducat?

6. How many current guilders of Holland in 3000 reals, 17 mar. vellon: exchange, 93 grotes per ducat; agio, $3\frac{1}{4}$ per cent?

7. How many reals vellon in 7860 livres; exchange, 14 francs, 25 cents, per pistole?

8. London remitted to Spain £1800, when the Course of Exchange was 35d. per piastre, and drew, to be reimbursed, at $37\frac{1}{2}$d. per piastre. Required the gain or loss on the transaction.

LISBON AND OPORTO.

In Portugal, accounts are kept in Milrees and Rees, and they are separated from each other by a mark thus, 780⊕500; that is, 780 milrees, 500 rees.

MONIES OF EXCHANGE.

400 Rees	=	1 Crusado of Exchange, or Crusade
1000 ditto	=	1 Milree = $2\frac{1}{4}$ Crusados
4800 ditto	=	1 Moidore
6400 ditto	=	1 Joannes, or Meia.

THE OTHER COINS OF PORTUGAL ARE,

Gold	The Dobra	=	12800 Rees
	Half Joannes	=	3200 ditto
	Testoon	=	1600 ditto
	Oito Testoon	=	800 ditto

Silver	New Crusado	=	800 Rees
	Half ditto	=	240 ditto
	Quarter ditto	=	120 ditto
	Eighth ditto	=	60 ditto
	Testoon	=	100 ditto
	Half ditto	=	50 ditto
	Vinten	=	20 ditto

The Copper coins are pieces of 10, 5, 3, and 1½ rees.

The Fineness of the gold coins is about the same as those of England, and has suffered no alteration since the year 1722.

The proportion between gold and silver in Portugal is as 16 to 1.

The Joannes of 6400 rees is worth £1. 15s. 11d. sterling; the Milree 5s. 7¼d., and the Crusado 2s. 3d.

The new silver Crusado is worth about 2s. 9d. sterling, and therefore the Milree, valued in silver, is 5s. 8¼d.

Payments in Portugal have for some years past been made in what is termed *legal* money *(moeda legal)*, which consists of one half in *effective* money or specie, and the other half in Government *paper*, which is very often at a considerable discount.

USANCE.

The usance for bills drawn from London is 30 days after sight, from Spain 15 days after sight, from Germany and Holland 2 months after date, from France 60 days ditto, from Italy 3 months after date.

DAYS OF GRACE.

Foreign bills, which have been accepted, are allowed 6 days of grace; but unaccepted bills must either be paid or protested on the very day they become due.

Bills, drawn from any part of the Portuguese dominions, both in Europe and other parts of the world, are allowed 15 days grace.

COURSE OF EXCHANGE.

		Uncertain Prices.		*Certain Prices.*
London	gives	67 Pence Sterling	for	1 Milree.
Amsterdam	gives	45 Grotes Flem. banco	for	1 Old Crusade.
France	rec.	470 Rees	for	3 Francs.
Genoa	rec.	750 Rees	for	1 Pezza fuori di Banco.
Hamburgh	gives	43 Grotes Flem. banco	for	1 Crusade.
Leghorn	rec.	810 Rees	for	1 Pezza of 8 reals.
Naples	rec.	670 Rees	for	1 Ducat di Regno.
Spain	rec.	2430 Rees	for	1 Pistole of Exchange.
Venice	rec.	66 Rees	for	1 Lira Piccola.
Vienna	rec.	360 Rees	for	1 Florin current.

Rees being decimal parts of a milree, exchanges are most easily calculated by decimals.

EXAMPLE.

In 780 milrees, 475 rees, how much English money, exchange at 55½d. per milree?

```
Rees 780.475
         555
    --------
     3902375
    3902375
   3902375
   ----------
12) 43316.3625
   -----------
20) 3609.696875
   -----------
   £180.48484375 = £180 9s. 8⅜d.
```

```
                    Rees 780.475
                         -------
 4s.  = 1/5. =          156.095
 6d.  = 1/8. =           19.512
 1½d. = 1/4. =            4.878
                        --------
                        £180.485
                              20
                         -------
                           9.700
                              12
                         -------
                           8.400
```

Paper money is reduced to effective, and *vice versa*, by the following

RULE.

As 100 is to 100, minus the discount, so is any sum in paper to its equivalent in effective; and

As 100, minus the discount, is to 100, so is the effective to its equivalent in paper.

EXERCISES.

1. In 1360 milrees, 125 rees, how much sterling money; exchange 64d. per milree?

2. In 7850 crusados, 310 rees, how much sterling money; exchange at 65d. per milree?

3. In £320. 10s. 6d. how many milrees and rees; exchange $62\frac{1}{8}$d. per milree?

4. Londou drew on Lisbon for £1240. 7s. 3d., exchange $60\frac{1}{2}$d. per milree; how much Portuguese money will retire the draft, when the exchange is at 56d.?

5. How many current guilders of Holland in 4230 milrees; exchange 45 grotes Flem. per crusado; agio 3 per cent?

6. A merchant brings 1800 moidores from Portugal to London, which he passes for 27s.; how much does he gain or lose, exchange being $62\frac{1}{2}$d. per milree?

LEGHORN.

At **Leghorn** there are two monies of account; namely, the Lira and Pezza. The former is chiefly used in *domestic* trade, and the latter in *foreign* transactions and exchanges.

There are two different values of the Lira, distinguished by the appellations, *moneta buona*, and *moneta lunga;* the former is $4\frac{8}{23}$ per cent better than the latter, or in the proportion of 24 to 23.

In domestic trade *moneta lunga* is always understood, unless otherwise expressed; but some articles of merchandise are bought and sold, sometimes in the one, and sometimes in the other.

THE COINS OF LEGHORN ARE THE FOLLOWING:

Gold	1	Ruspono	=	40	Lire
	1	Doppia, or Pistole	=	23	ditto
		Half Doppia	=	$11\frac{1}{2}$	ditto
	1	Zecchino gigliato	=	$13\frac{1}{2}$	ditto
Silver	1	Francescono	=	$6\frac{2}{3}$	ditto, or 10 Paoli
	1	Ducato, or Piastra	=	7	ditto
		Pezza della Rosa	=	$5\frac{3}{4}$	ditto
Copper	1	Soldo	=	2	Quatrini
	1	Quatrino	=	4	Denari

The value of these coins, in *moneta buona* and *moneta lunga*, is as follows:

		Moneta buona.		Moneta lunga.
Doppia	=	23 Lire	=	24 Lire
Ruspono	=	40 ditto	=	41 ditto, $9\frac{3}{4}$ soldi
Sequino	=	13 ditto, $6\frac{1}{3}$ soldi	=	13 ditto, 18 ditto
Pezza	=	5 ditto, 15 ditto	=	6 ditto
Testone	=	2 ditto	=	2 ditto, $1\frac{17}{23}$ ditto

The Lira is worth about 8d. sterling; the Pezza, 3s. $10\frac{3}{4}$d.; the Ducato, 4s. $8\frac{1}{2}$d; the Francescono, 4s. 6d.; and the Sequino, 9s. 6d.

The foreign coins, that are in circulation at Leghorn, are Dutch or German Ducats; Roman Sequins; Spanish Pistoles and Dollars; Portuguese Joannes; and French Ecus.

MONIES OF EXCHANGE.

12 Denari di Pezza	=	1	Soldo di Pezza
20 Soldi di Pezza	=	1	Pezza, or Piastre of 8 reals

Also, 12 Denari di Lira	=	1	Soldo di Lira
20 Soldi di Lira	=	1	Lira

USANCE.

The Usance for bills drawn on England and Portugal, is 3 months after date; from Hamburgh, Holland, and Spain, 2

months after date; from France, 30 days after date; and from Genoa, Milan, and Switzerland, 8 days after sight.

DAYS OF GRACE.

There are no DAYS of GRACE, but bills of exchange are paid only on Mondays, Wednesdays, and Fridays.

COURSE OF EXCHANGE.

			Uncertain Prices.		*Certain Prices.*	
London	gives	54	Pence sterling	for	1 Pezza	of 8 Reals
France	gives	5¼	Francs	for	1 Pezza	ditto
Geneva	gives	109	Ecus of 3 Florins	for	100 Pezze	ditto
Genoa	gives	124	Soldi fuori di banco	for	1 Pezza	ditto
Hamburgh	gives	89	Grotes Flem. banco	for	1 Pezza	ditto
Holland	gives	95	Grotes ditto	for	1 Pezza	ditto
Portugal	gives	855	Rees	for	1 Pezza	ditto
Petersburgh	gives	190	Rubles	for	1 Pezza	ditto
Spain	gives	140	Dollars of Exch.	for	100 Pezze	ditto
Venice	gives	10¼	Lire Piccoli	for	1 Pezza	ditto

EXAMPLE I.

In £392. 18s. 4d., how much money of Leghorn; exchange 50d. per pezza?

d.		£	s.	d.		p.
50	:	392	18	4	: :	1
		20				
		7858				
		12				

5,0)9430,0

1886 Pezze

EXAMPLE II.

In 1876 pezze, 12 soldi, 5 denari, how much sterling money; exchange, 50¼d. per pezza?

METHOD I.

P.		P.	s.	d.		d.
1	:	1876	12	5	::	$50\frac{1}{4}$

```
  20
 ------
 37532
    12
 ------
 450389
   50¼
 ------
 22519450
  112597¼
 ---------
24,0)22632047¼
 ---------
12)  94300¼
 ---------
2,0)  785.8,4¼
 ---------
   £392 18 4¼
```

METHOD II.

		P.	s.	d.		d.
		1876	12	5	at	$50\frac{1}{4}$
4s.	$= \frac{1}{5}$ £ =	375	4	0		
2d.	$= \frac{1}{24}$ of 4s. =	15	12	8		
	$\frac{1}{4}$ =	1	19	1		
10s.	$= \frac{1}{2}$ =	0	2	1		
$2\frac{1}{2}$d.	$= \frac{1}{4}$ =	0	0	$6\frac{1}{4}$		
		£392	18	$4\frac{1}{4}$		

EXERCISES.

1. In 1580 piastres, 15 soldi, how much sterling money; exchange at 54d. per piastre?

2. In 859 piastres, 10 soldi, 6 denari, how much sterling; exchange $50\frac{1}{2}$d. per piastre?

3. In £325 18s. 4d. how many piastres of Leghorn; exchange at 50d. per piastre?

4. How many guilders of Holland in 4860 piastres; exchange at 94 grotes Flemish per piastre?

5. In 3725 Leghorn dollars of exchange, how many piastres of Spain; exchange at 128 dollars per 100 piastres?

6. When 50 barrels of Gorgona anchovies sell at Leghorn at

7 lire per barrel, effective money, the charges on which amount to 3 piastres, 7 soldi, 9 denari, at what rate per barrel may they be shipped; exchange being $52\frac{1}{4}$d. per pezza?

GENOA.

At Genoa accounts are kept in Lire, Soldi, and Denari fuori di Banco; but there are several other monies of account, of which the following are the most common:

The Scudo d'Oro, worth 9 Lire, 8 Soldi Permesso, or 10 Lire, $16\frac{1}{2}$ Soldi fuori di Banco.

The Scudo d'oro Marche, which is nearly 1 per cent worse than the above.

The Scudo d'Argento, worth 7 Lire, 12 Soldi Permesso, or 8 Lire, $14\frac{4}{5}$ fuori di Banco.

The Pezza, or Piastre, worth 5 Lire Permesso, or 5 Lire, 15 Soldi fuori di Banco.

Scudo di Cambio, worth 4 Lire Permesso, or 4 Lire, 12 Soldi fuori di Banco.

100 Lire Banco make 115 Lire fuori di Banco; hence 4 Lire Banco is equal to $4\frac{3}{5}$ Lire fuori di Banco.

100 Scudi d'oro Marche di Permesso are equal to $122\frac{2}{3}$ Scudi d'Argento.

These Scudi are all *imaginary* monies, and each of them is divided into 20 Soldi, or 24 Denari, distinguished by the above names of Soldi and Denari d'Oro, d'Oro Marche, d'Argento, and di Cambio.

The Pezza is also *imaginary*, and is divided into 20 Soldi, or 240 Denari di Pezza.

The term *Permesso* was given to bank money after the suppression of the famous Bank of St. George, in the year 1746. This kind of money being 15 per cent better than moneta fuori di banco, in which accounts are kept; it is necessary, on drawing upon Genoa, to insert the words *fuori di banco* in the body of the bill, which would otherwise be paid in moneta di permesso.

The value of the coins of Genoa in Fuori di Banco are as follows:

Of Gold { The Doppie, or Pistole, of 23 Lire, 12 Soldi.
The Scudo d'oro of 11 Lire, 16 Soldi.
The Zecchino of 13 Lire, 10 Soldi.

Of Silver { The Scudo d'Argento, or Genovina of 9 Lire, 10 Soldi, when of full weight.
The Scudo di Cambio, or St. Gianbatista, of 5 Lire
The Giorgino, of 1 Lire, 6 Soldi.

Copper—Pieces of 8, 4, and 2 Denari.

These were the current coins of Genoa previous to the year 1790; but in that year a new coinage took place, which consisted of gold Genovine, at 96 Lire, half, quarter, and eighth ditto Lire, and silver Scudi of 8 Lire; besides half, quarter, and eighth ditto, in the same proportion.

Genoa being united to France in 1804, the *French coins* were then introduced; but the Genoese coins are still allowed to circulate, and future coinages of them are to take place when necessary.

The Pezza of 5¾ Lire, by which the exchange between London and Genoa is regulated, is worth 46d.; and £1 sterling is worth 30 Lire fuori di Banco, in silver; but in gold, 12 Soldi, 10 Denari more.

At Genoa there are certain allowances made on several kinds of goods, but the limits of this work will not admit of mentioning them.

USANCE.

The Usance for Bills drawn on Genoa from London and Lisbon is 3 months after date; from Amsterdam, Hamburgh, Spain, and Sicily, 2 months after date; Leghorn and Milan, 8 days after sight; and from Venice and Rome 15 days after sight.

DAYS OF GRACE.

Thirty days of grace are allowed here; but bills are generally protested in a week after they are due.

COURSE OF EXCHANGE.

			Uncertain Prices.		*Certain Prices.*
London	gives	4	Pence Sterling	for	1 Pezza fuori di Banco
Amsterdam	gives	87	Grotes Flemish	for	1 Pezza do.
France	gives	4	Francs, 75 Cents	for	1 Pezza do.
Hamburgh	rec.	43	Soldi fuori di Banco	for	1 Marc Banco
Leghorn	rec.	125	Soldi fuori di Banco	for	1 Pezza of 8 Reals.
Lisbon	gives	760	Rees	for	1 Pezza fuori di Banco
Naples	rec.	104	Soldi fuori di Banco	for	1 Ducat di Regno
Spain	gives	690	Maravedies of Plate	for	1 Scudo d'oro M. per.
Venice	gives	34	Soldi Picoli	for	1 Lire fuori di Banco
Vienna	rec.	45	Soldi fuori di Banco	for	1 Gulden.

EXAMPLE I.

What is the value of 820 Piastres, 15 Soldi, 6 Denari fuori di Banco, in sterling money; exchange at 54d. per Pezza?

	P.	s.	d.	
	820	15	6	
4s. = $\frac{1}{5}$ =	164	3	$1\frac{1}{5}$	
6d. = $\frac{1}{8}$ =	20	10	$4\frac{3}{5}$	
	£184	13	6	Answer.

EXAMPLE II.

In £184. 15s. 6d. sterling, how many Piastres of Genoa; exchange 45d. per Pezza?

£	s.	d.
184	15	6
20		
3695		
12		

$$45\begin{cases} 9)\,44346 \\ 5)\;\;4927-3 \end{cases}$$

Pias. $985-\frac{11}{45}$ = P.985. 9s. 4d.

EXERCISES.

1. In £239. 11s. 3d. sterling, how many lire, soldi, and denari fuori di banco; exchange at 50d. per pezza?

2. In 4926 lire, 8 soldi, 4 denari fuori di banco, how much sterling money; exchange at 46d. per pezza?

3. In 1215 florins, 10 stivers banco, how many lire fuori di banco; exchange at 85 grotes, Flemish banco, per pezza of 5¾ lire fuori di banco?

4. In 1307 lire, 3 soldi, 4 denari of Genoa, how much money of Leghorn; exchange at 124 soldi fuori di banco, per pezza of 8 reals?

5. The sum of 1480 lire, 8 soldi fuori di banco, is due by a merchant in Genoa, to a house in London; required what sum of sterling money will discharge the debt; exchange at 45½d. per pezza fuori di banco?

6. In 1153 francs, 80 cents, how many lire soldi and denari fuori di banco; exchange at 4 francs, 80 cents, per pezza of 5¾ lire fuori di banco?

VENICE.

At Venice there are three kinds of money; namely, *Valuta* Piccola, Valuta Corrente, and Valuta di Banco.

Valuta Piccola is used in the sale and purchase of merchandise.

Valuta Corrente was the standard of their coin, which was fixed by the Venetian government, in 1686, and continued to be the legal value of these coins, till 1750, when Moneta *Piccola* was introduced.

Valuta di Banco is the money in which the Bank of Venice kept its accounts; and though this establishment does not now exist, in its original form, yet some account of its nature may be useful to mercantile students.

The Bank of Venice was instituted in 1587; its original capital was five millions of Ducats.

The sums that were deposited there, by merchants and traders, were secured on the credit of the state. The proprietors received no interest for their money, but could draw it out at any time, or transfer it in payment, after the manner of the Banks of Amsterdam and Hamburgh, or other banks of deposit. Bills of exchange were mostly paid in Banco, and wholesale bargains of merchandise, which exceeded the value of 300 Ducats, were also paid in the same money.

Before the year 1750, Banco was constantly 20 per cent better than Valuta Corrente; about that time a further agio of 29 per cent was added, to reduce it to Valuta Piccola, and the Ducat Banco was fixed at 9 Lire, 12 Soldi Piccoli; thus, 31 Ducats Banco were worth 48 Soldi Piccoli, which gave an agio in favour of the bank, of 54$\frac{4}{5}$ per cent.

Such was the prosperous and highly respected state of the Bank of Venice, from its commencement to the year 1797, when the French seized upon the city, and ceded it to Austria. At that period, the ruin of the establishment commenced.

In 1805, Venice was incorporated with the kingdom of Italy; and, in 1808, the constitution of the bank was completely changed.

At Venice, accounts are kept in Lire, or Soldi, Denari, and also in Ducats and Grossi.

12 Denari	=	1 Soldi
20 Soldo	=	1 Lira, or Livre
6$\frac{1}{5}$ Lire = 24 Grossi	=	1 Ducat.

These monies, as already stated, have the several denominations of Valuta di Banco, Valuta Corrente, and Valuta Piccola. Valuta di Banco is the money in which the *present Bank* keeps accounts, and has been reckoned 20 per cent better than Valuta Corrente, and Valuta Corrente 20 per cent better than Valuta Piccola.

THE COINS OF THE OLD REPUBLIC OF VENICE, ARE THE FOLLOWING:

Gold { The Doppia, or Pistole of 38 Lire
Zecchino, or Sequin of 22 Lire
The Half and Quarter ditto }

Silver { The Scudo Veneto, or Della Croce of 12 Lire, 8 Soldi
The Half and Quarter ditto, in proportion
The Ducatone, or Giustina of 11 Lire
The Ducate effettivo, of 8 Lire
The Half and Quarter ditto, in proportion.
There are also several base silver pieces of 30, 20, 15, 10, and 5 Soldi. }

Copper.—The Soldo and the Half Soldo, or Bagattino.

These coins are now valued in Moneta Piccola, which is the effective currency of Venice; but it bears a fluctuating agio, which, in February, 1805, was 37 per cent.

The Lira is worth about 5d. sterling.

The Ducat of Account, of $6\frac{1}{4}$ Lire Piccola, is worth $31\frac{1}{2}$d. sterling, nearly.

The silver Ducat is worth about $40\frac{1}{2}$d. sterling.

One pound sterling is, therefore, worth about 48 Lire; but, if the value of the Lire be taken from the coinage introduced by the Austrian government, it will be about $4\frac{1}{4}$d.; and, therefore, £1. sterling is worth 56 Lire, $9\frac{1}{2}$ Soldi Piccoli.

The exchange, between Venice and London, was formerly transacted by giving the Ducat Banco, for an uncertain number of Pence; but, when the French got possession of Italy, they introduced the Italian Livre, which is the same as the French Franc, for the purpose of establishing an uniformity of coins throughout the whole of Italy.

The coins now in circulation, in Lombardy and Venice, and daily issuing from the Milan Mint, consist of the Dollar of 5 Livres (or 5 French Francs) and its parts.

The Austrian government retain this money of account, in the Treasury Registers; but, in the various states, the Old Livres are, from inveterate use, retained in commerce, with the exception of Venice, where the Italian Livre is employed, in quoting the exchange, and also in merchants accounts.

The proportion between the Livre of the States and the Italian Livre, or French Franc, is as follows:

	Livres.		Italian Livre, or, French Franc.
Milan*	27.000	=	20.723
Modena	54.000	=	20.723
Reggio	81.000	=	20.723
Venice	40.500	=	20.723
Valtellini	54.900	=	20.723
Chiavenna	69 065	=	41.446
Parma	757.000	=	183.481
Tuscany	100.	=	84.

The Livre of Bologna is equal to the fifth of the Roman Scudo of 10 Pauls.

COURSE OF EXCHANGE.

London	rec.	25	Francs	for	£1	sterling
Amsterdam	rec.	2	Francs	for	1	Florin Banco
Constantinople	rec.	1	Franc, 75 cents	for	1	Piastre
France	rec.	2	Livres	for	1	Franc
Genoa	rec.	1	Florin	for	1	Lira fuori di Banco
Hamburgh	rec.	2	Florins, 12 cents	for	1	Marc Banco
Leghorn	rec.	5	Francs, 23 cents	for	1	Pezza of 8 reals
Milan	rec.	90	Cents	for	1	Lira Current
Naples	rec.	5	Francs	for	1	Ducat di Regno
Portugal	gives	66	Rees	for	60	Cents
Rome	rec.	7	Francs	for	1	Scudo Romano
Vienna	rec.	2	Francs, 25 cents	for	1	Florin Current

The exchange between London and Venice being now quoted in French francs, and nearly the same number received for £1 as from France, the calculations with Venice are performed in the same manner as with Paris or Bourdeaux.

EXAMPLE.

In 196 pezze, 17 soldi, 6 denari, of Leghorn, how many francs; exchange at 5 francs, 25 cent, per pezza of 8 reals?

---

* Milan gives London 32 livres, 11 sous, or nearly 25 French francs, for £1 sterling; and Leghorn gives 132½ sous for 1 pezza of 8 reals.

METHOD I.

	P.	s.	d.
	196	17	6
	5.25		
	980		
	392		
	980		
	1029.00		
$17\frac{1}{2}$ soldi $=\frac{7}{8}=$	4.60		
Francs	1033.60		

METHOD II.

	$196\frac{7}{8}$
	$5\frac{1}{4}$
	$984\frac{3}{8}$
	$49\frac{7}{32}$
Florins	$1033\frac{19}{32}$

EXERCISES.

1. A merchant in Venice owes £754. 18s. 4d. in London; how many francs will discharge the debt; exchange at 24 francs, 23 cents, per pound sterling?

2. In 6387 lire piccoli, how much sterling money; exchange at 27 francs per pound sterling?

3. In 7587 lire, 13 soldi, 6 denari piccoli, how much French money, exchange at 2 lire, 5 soldi per franc?

4. In 3372 francs, 30 cents, how much Venetian money; exchange at 2 lire, 5 soldi piccoli, per franc?

5. In 2476 florins, 12 stivers, 8 phennings banco, how much Venetian money; exchange at 4 lire, 18 soldi piccoli, per florin, banco?

6. In 5486 marcs, 12 schillings banco, how much Venetian money; exchange at 4 lire, 6 soldi piccoli, per marc, current; agio, $22\frac{1}{2}$ per cent?

NAPLES.

At Naples, accounts are kept in ducats di regno, and exchanges are negociated by the ducato, or ducat.

MONIES OF EXCHANGE.

10 Grains	=	1 Carlin.
10 Carlins, or 100 grains	=	1 Ducat.

CURRENT COINS.

Gold	Pieces of 2, 4, and 6 Ducats
	Double Onza of 60 Carlini
	Onza of 30 Carlini
Silver	Ducato di regno of 10 Carlini
	Half ditto of 5 Carlini
	Scudo of Sicily of 12 Carlini
	Half ditto of 6 Carlini
	The Tarino, or Tarin of 2 ditto
	Pieces of 13, 20, 24, aud 26 Grains

The Ducat of 10 Carlini is worth about 41d. sterling, or £1 is worth about 5 Ducats, 88 Grains.

USANCE.

The Usance for bills drawn from England is 3 months, and from Spain, 2 months.

DAYS OF GRACE.

Three Days of Grace are allowed, except for bills at sight.

COURSE OF EXCHANGE.

London	gives	42	Pence sterling	for	1	Ducat
Amsterdam	rec.	51	Grains	for	1	Florin Banco.
France	gives	4½	Francs	for	1	Ducat
Genoa	gives	108	Soldi fuori di banco	for	1	Ducat
Hamburgh	rec.	44	Grains	for	1	Marc Banco.
Leghorn	rec.	118	Ducats	for	100	Pezzaof8 reals
Portugal	gives	650	Rees	for	1	Ducat di regno
Spain	rec.	86	Grains	for	1	Dollar of ex.
Venice	gives	200	Soldi Piccoli	for	1	Ducat

EXAMPLE

In £800 sterling, how many ducats, &c.; exchange at 42d. sterling?

```
 d.      £        D.
 42  :  800  : :  1
         20
       -----
       16000
       12
       ------
    6 |192000
 42 --|------
    7 | 32000
      ----------
Ducats  4571  4  3 nearly.
```

EXERCISES.

1. In 1600 ducats, how much sterling money; exchange at $38\frac{1}{2}$d. per ducat?

2. In 1487 ducats, 8 carlins, how much sterling money; exchange at $38\frac{3}{4}$d. per ducat?

3. In £795. 18s. 5d. sterling, how many ducats; exchange at $41\frac{1}{3}$d. per ducat?

4. In 4826 ducats, 95 grs, how many guilders of Holland; exchange at $51\frac{1}{2}$ grains per guilder?

MALTA.

At Malta, accounts are kept in Scudi, Tari, and Grani.

20 Grani = 1 Taro, or Tarin
12 Tari = 24 Carlini = 1 Scudo.

These monies have *two* values; the one denominated of *Silver*, and the other of *Copper*, or current. The former being to the latter as 3 to 2.

THE COINS OF MALTA ARE THE FOLLOWING:

Gold { The Double Louis of 20 Scudi, copper, or $13\frac{1}{3}$ silver.
The Louis and Half ditto, in the same proportion.

Silver { The Oncia, or Ounce of 30 Tari
The Half ditto of 15 ditto
The Scudo of 12 Tari, current.
The Half ditto of 6 Tari, current }

Copper.—The Tari and Pieces of 10, 5, 2½, and 1 Grani.

The Scudo current money is worth about 1s. 9⅓d.; the piece of 30 Tari about 53⅓d. and the Louis d'or about 19s. 8d. sterling.

Malta exchanges with London by the Sicilian Dollar of 2½ Scudi, or 30 Tari, which, at present, is quoted at 48d.

EXAMPLE.

In 500 Scudi, 4 Tari, 10 grs., how much sterling money; exchange at 48d. per 30 Tari?

METHOD I.

	S.	T.	
	500	4½	
	12		
3,0)	600,4½		
	200,$\frac{3}{20}$		
4s.=£⅕ =	£40	0	7⅕

METHOD II.

	S.	T.	
	500	4½	
		2	
5)	1000	9	
	200	1⅘	
4s. = £⅕ =	£40	0	7⅕

METHOD III.

	S.		
2½)	500		
	200		
⅕=	£40		
3 Tari=$\frac{1}{10}$=	0	0	4⅘
1½do.=½=	0	0	2⅖
	£40	0	7⅕

EXERCISES.

1. In 458 scudi, 10 tari, 8 grains, how much sterling money; exchange at 47d. per 30 Tari?

2. In £428. 6s. 8d. how many scudi; exchange at 47½d. per 30 Tari?

3. In 784 scudi, 8 tari, 7 grains, how much sterling money; exchange 46¼d. per 30 Tari?

PALERMO.

At Palermo, and throughout all Sicily, accounts are kept in Onzie, Tari, and Grani.

MONIES OF EXCHANGE.

	20 Grani or Grains	=	1 Taro or Tarin
	30 Tari	=	1 Onza or Ounce
Also,	12 Tari	=	1 Scudo or Sicilian Crown
	5 Scudi	=	2 Onzie or Ounces

THE COINS OF SICILY ARE THE FOLLOWING:

Gold	The double Onza, or 6 Ducat piece, of 60 Tari
	The Onza, or 3 Ducat piece, of 30 Tari
Silver	The Scudo of 12 Tari
	Pieces of 6, 4, 3, 2, and 1 Tari
	The Carlino of 10 Grani

The Tari and Carlino of Sicily are only half the value of the coins of the same name in Naples.

USANCE AND DAYS OF GRACE.

The Usance for bills drawn on Leghorn and Genoa, is 1 month after acceptance, or 2 months after date; on Rome, Venice, and

Naples, 8, or 15 days after sight; and on London, 3 months after date.

There are no Days of Grace allowed here.

COURSE OF EXCHANGE.

		Uncertain Prices.			Certain Prices.
London	gives	115	Pence	for	1 Onza
Amsterdam	receives	5½	Tari	for	1 Florin Banco
France	receives	50	Grains	for	1 Franc
Genoa	receives	40	Grains	for	1 Lira fuori di Banco
Leghorn	receives	12½	Tari	for	1 Pezza of 8 Reals
Naples	gives	120	Ducats	for 100	Sicilian Crowns
Portugal	receives	6½	Tari	for	1 Old Crusade
Spain	receives	8½	Tari	for	1 Dollar of exchange

EXAMPLE.

In 1486 Ounces, 2 Tari, 6 Grani, how much sterling money; exchange at 114d. per Ounce?

$$\begin{array}{cccccccc} \text{Oz.} & & \text{Oz.} & \text{T.} & \text{G.} & & \text{d.} & \text{s. d.} \\ 1 & : & 1486 & 2 & 6 & :: & 114 & = 9\ \ 6 \end{array}$$

$$\begin{array}{rr} & 9\frac{1}{2} \\ \hline & 13374 \\ & 743 \\ 2\frac{1}{4}\text{ Tar.} = \frac{1}{13} = & 0\ \ 9\frac{1}{2} \\ \hline 2,0)1411,7 & 9\frac{1}{2} \\ \hline £705\ 17 & 9\frac{1}{2} \\ \hline \end{array}$$

EXERCISES.

1. In £343. 14s. 4d. how much Sicilian money; exchange at 113d. per Ounce?

2. In 748 ounces, 8 tari, 5 grani, how much sterling money; exchange at 112d. per Ounce?

3. In 3941 pezze, 13 soldi, 4 denari, of Leghorn, how much Sicilian money; exchange at 12 tari, per Pezze?

4. In 1316 ounces, 16 tari, 15 grani, how much Spanish money; exchange at $8\frac{2}{3}$ tari, per Dollar of Plate.

VIENNA.

At Vienna accounts are kept in Guldens, Creutzers, and Pfenings; and Exchanges are negociated in Florins and Creutzers, or in Rixdollars and Creutzers.

MONIES OF EXCHANGE.

4	Pfenings	=	1 Creutzer or Kreutzer
60	Creutzers	=	1 Florin or Gulden
$1\frac{1}{2}$	Florins, or 90 Creutzers	=	1 Rixdollar of Account
2	Florins, or 120 Creutzers	=	1 Rixdollar of Specie

CURRENT COINS.

Gold	The double Souverain of $12\frac{2}{3}$ Florins The Souverain of $6\frac{1}{3}$ Florins The Imperial Ducat of $4\frac{1}{2}$, with double and quadruple Ducats in proportion.
Silver	The Specie Rixdollar of 2 Florins The Florin The Half ditto The Kopstick of 20 Creutzers
Copper	The Creutzer, the half Creutzer, the Groschel, and the Pfening.

The value of the Souverain, in English money, is about 13s. $10\frac{1}{3}$d.; the Ducat, about 9s. $4\frac{3}{4}$d.; the Florin, about 2s. $1\frac{1}{2}$d.; and the Rixdollar, which is an *imaginary* coin, about 4s. $2\frac{1}{2}$d.

USANCE AND DAYS OF GRACE.

The Usance is 14 days after acceptance.

Three Days of Grace are allowed, but not for bills at sight.

COURSE OF EXCHANGE.

			Uncertain Prices.			*Certain Prices.*
London	rec.	8	Florins, 48 Creutzers	for	1	Pound sterling
Amsterdam	rec.	144	Rixdollars	for	100	Rixdollars Banco
France	rec.	24	Creutzers	for	1	Franc
Hamburgh	rec.	144	Rixdollars	for	100	Rixdollars Banco
Leghorn	giv.	64	Soldi Moneta Buona	for	1	Florin
Spain	rec.	200	Florins current	for	100	Ducats of Exch.

EXAMPLE.

In 7482 Florins, 45 Creutzers, how much sterling money; exchange at 8 Florins, 48 Creutzers, per pound sterling?

```
F. Creut.     F. Creut.     £
8   48   :   7482   45  ::  1
60           60
----         ----
528          )448965(£850. 6s. 3d.
              4224
              ----
               2656
               2640
               ----
                 165
                  20
                ----
                3300
                3168
                ----
                 132
                  12
                ----
                1584
                1584
                ----
```

EXERCISES.

1. In 3326 florins, 14 creutzers, how much sterling money; exchange at 11 florins, 25 creutzers, per pound sterling?

2. In 2604 florins, 28 creutzers, how much sterling money; exchange at 12 florins, 16 creutzers, per pound sterling?

3. In £748. 12s. 4d. sterling, how many Florins and Creutzers; exchange at 9 Florins, 18 Creutzers, per pound sterling?

4. A remittance, of £824. 10s., was made in bills from London to Vienna; what was the sum, received at Vienna, in Rixdollars of Account, the charges being £5. 4s. 6d.; exchange, 8 Florins, 27 Creutzers, per pound sterling?

TURKEY*.

Accounts are kept throughout Turkey in Piastres, which the Turks call Grouch, and the English Dollars.

MONIES OF EXCHANGE.

3 Aspers	=	1 Para
40 Paras, or 120 Aspers,	=	1 Piastre

The Asper and the Para are real coins, but the Piastre is sometimes divided into 80, and sometimes into 100, imaginary parts, called Aspers and Minas.

A Jux, or Juck, is 100,000 real Aspers; and a Chise, or Purse, is 500 Aspers.

THE COINS OF TURKEY ARE THE FOLLOWING:

Gold:
- The Sultanin, or Sequin Fonducli, of 4 Piastres, but is for 5
- The Sequin Mahbub, or Zermahbub, of 3 Piastres
- The half ditto, Nisbie, of 1 Piastre, 20 Paras
- The Roubbie of 1 Piastre.

These coins vary a little in their value.

* Bills are seldom negociated directly upon London, from any part of Turkey, remittances being usually made by the way of Vienna, Marseilles, or Leghorn.

Silver { The Allmichlec of 60 Paras
The Grouch, or Dollar, of 40 Paras
The Zelotta of 30 Paras
The Roubb of 10 Paras
The Para of 3 Aspers.

USANCE AND DAYS OF GRACE.

Bills between Constantinople and the chief trading towns in Europe are usually drawn at 31 days sight; but from one place of Turkey on another, at 11 days sight.

Many European merchants pay their bills on the very day they become due; others take the same number of days of grace as are allowed in the place they reside.

COURSE OF EXCHANGE.

		Uncertain Prices.			*Certain Prices.*	
London	receives	18	Piastres	for	1	Pound sterling
Amsterdam	receives	64	Paras	for	1	Florin current
France	gives	145	Centimes	for	1	Piastre
Hamburgh	gives	25	Grotes Flem.	for	1	ditto
Genoa	gives	1	Lira	for	24	Paras
Leghorn	receives	145	Paras	for	1	Pezza of 8 reals
Naples	receives	116	Paras	for	1	Ducat di regno
Smyrna	gives	110½	Piastres	for	100	Piastres
Vienna	receives	52	Paras	for	1	Florin current.

EXAMPLE.

In 1460 Piastres, or Dollars, 13 Paras, how much sterling money; exchange at 18 Piastres per pound sterling?

P.		P. Par.		£
18	:	1460 13	: :	1
40		40		
72,0		)5841,3		
		£81 2 7		

EXERCISES.

1. In 1804 Piastres, 12 Paras, how much sterling money; exchange at 17 Piastres, per pound sterling?

2. Reduce £204. 5s. 6d. to Turkish money; exchange at 17½ Piastres per pound sterling?

ST. PETERSBURGH* AND REVEL.

In Russia accounts are kept in Rubles and Copecks.

MONIES OF EXCHANGE.

10 Copecks = 1 Grieve, or Grievener
10 Grieves, or 100 Copecks = 1 Ruble

RUSSIAN COINS.

Gold:
- The Imperial of 10 Rubles
- The half ditto of 5 ditto
- The double Ducat of 4½ ditto
- The Ducat of 2¼ ditto

Silver:
- The Ruble of 100 Copecks
- The half and quarter ditto, in proportion
- Pieces of 15 and 10 Copecks
- The Piat Copie of 5 ditto
- The Altin of 3 ditto

Copper:
- Pieces of 5, 2, and 1 Copecks
- The Denuschka of ½ ditto
- The Poluschka of ¼ ditto

* There is no direct Exchange between St. Petersburgh and London, commercial debts being settled by drawing and remitting bills on other countries.

VALUE OF THESE COINS IN ENGLISH MONEY:

	£	s.	d.
The Imperial is worth	1	12	$9\frac{1}{2}$
The Ducat of 2 Rubles ...	0	9	0
The Ruble (1805)	0	3	8

Exchanges are negociated by the Ruble, which is subject to great fluctuation. In the year 1799 it was 50 per cent below par, and, in 1808, the silver Ruble was worth 2 Rubles of Exchange.

The chief circulating medium, in Russia, is Assignations, or Bank Notes, which are issued by a Bank, called the Assignation Bank. They fluctuate considerably in value, and are mostly at a discount, with respect to gold and silver.

USANCE.

Petersburgh usually draws on London at 3 months date.
On Hamburgh and Amsterdam, at 65 days date.

DAYS OF GRACE.

Bills drawn in Russia, which are payable after date, are allowed 10 Days Grace; but, if payable at sight, 3 days only*.

COURSE OF EXCHANGE.

		Uncertain Prices.	Certain Prices.
London	gives	30 Pence sterling	for 1 Ruble
Amsterdam	gives	27 Stivers	for 1 Ditto
Constantinople	rec.	50 Copecks	for 1 Piastre
France	gives	280 Cents	for 1 Ruble
Hamburgh	gives	26 Schil. Flem. Banco	for 1 Ditto
Vienna	gives	120 Creutzers	for 1 Ditto

EXAMPLE I.

In 964 Rubles, 20 Copecks, how much sterling money; exchange at 30d. per Ruble?

* In Russia, the Julian, or Old Style, is still used; and, therefore, 12 days must be added, to bills drawn in Russia, in order to reduce the date to the Gregorian, or New Style.

R.		R.		D.			R.
1	:	964⅕	: :	30	or thus,		964⅕
		30			30d. = £⅛ =		120 10 0
		12)28926			⅕ of 30d. =		0 0 6
		2,0) 241.0 6					£120 10 6
		£120 10 6					

EXAMPLE II.

In 7863 Guilders, 10 Stivers, 2 Pennings, how much Russian money; exchange at 28½ Stivers per Ruble?

S.		G. S. P.
28½	:	7863 10 2
16		20
456		157270
		16
		456)2516322
	Rubles	5518 25

EXERCISES.

1. In 3446 Rubles, 61 Copecks, how much sterling money; exchange at 29½d. per Ruble?

2. In £748. 4s. 7d. how much Russian money; exchange at 29½d. per Ruble?

3. A merchant in St. Petersburgh shipped 250 Poods of Flax for a merchant in London, at 8 Rubles, 15 Copecks, per Pood, charges, till on board, 9 Rubles; the insurance, freight, and other charges, amounted to £49. 13s. 8½d. Required the prime cost of the Flax, per Pood, in sterling money?

4. A merchant in Hull imported from St. Petersburgh 418 Poods of Tallow, of 40lb. each. Required the number of tons

and value in sterling money, at 5 Rubles, 35 Copecks, per Pood; the charges of freight, &c. being £73. 18s. 7d.; exchange at 32¼d. per Ruble?

RIGA.

At Riga, accounts are kept in Rixdollars, Alberts, and Groschen, or Ferdings, and sometimes in Rubles and Copecks.

MONIES OF EXCHANGE.

90 Grochen = 80 Ferdings
80 Ferdings = 3 Florins = 1 Rixdollar Alberts.

The Rixdollar is valued in two kinds of money; these are, Alberts and Riga currency.

The Currency is generally reckoned 33⅓ per cent worse than Alberts Dollars, but the Agio is sometimes 40 per cent on the Alberts Dollar.

The coins now current at Riga are the Russian coins. (See St. Petersburgh.

Exchanges with Britain are negociated by giving so many Groschen for £1 sterling, at present about (370). With Amsterdam and Hamburgh, by giving from 90 to 100 Rixdollars Alberts for 100 Rixdollars Current of those places.

The Usance, Days of Grace, &c. are the same as at St. Petersburgh.

SWEDEN.

Accounts are kept in Sweden, in Riksdalers, or Rixdollars, each of which consists of 48 Skillings, and each Skilling of 12 Runstycken, or Runstycks.

In Sweden, money is distinguished by Banco and Currency, as at Hamburgh and Amsterdam. Banco is above 50 per cent better than Currency, 2 dollars of the former being worth 3 of the latter.

Paper currency is very common and extensive in this country, Bank Notes being issued as low as $\frac{1}{8}$ of a Dollar, or 7d. sterling.

MONIES OF EXCHANGE.

12 Fenings, or Ore, = 1 Skilling,
48 Skillings = 1 Rixdollar specie.

THE COINS OF SWEDEN ARE THE FOLLOWING:

Gold	The Double Ducat, the Ducat, and Half Ducat. The Ducat is worth about 9s. $2\frac{1}{2}$d. sterling, in English gold, but passes in Sweden for 94 skillings, which are worth only 9s. 1d in English silver.
Silver	The Rixdollar, of 6 silver dahlers, or 48 skillings, and $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{1}{24}$ of the Rixdollar of the same proportionate value.
Copper	Pieces of 12, 9, 6, $4\frac{1}{2}$, 3, $2\frac{1}{4}$, and $1\frac{1}{2}$, Copper Dahler and the Runstycke of 1 Copper ore.

The value of the Specie Rixdollars, as ascertained at the Mint in London, some years ago, is 4s. 6d. in English silver.

The Skilling is worth about $1\frac{1}{4}$d.; the Silver Dahler, $9\frac{1}{4}$d.; the Copper Dahler, 3d. sterling. Hence, £1 sterling is equal in value to 207 Skillings; or, 25 Dahler, 30 ore, silver; or, 77 Dahler, 26 ore, copper.

USANCE AND DAYS OF GRACE.

The Usance is reckoned 1 month after acceptance.

Six Days of Grace are allowed, but not for bills at sight.

COURSE OF EXCHANGE WITH STOCKHOLM.

		Uncertain Prices.			*Certain Prices.*
London	rec.	$4\frac{1}{2}$	Rixdollars	for £1	sterling
Amsterdam	rec.	44	Skillings	for 1	Rixdollar Banco
Copenhagen	rec.	26	Skillings	for 1	Rixdollar
Dantzic	rec.	$9\frac{1}{2}$	Skillings	for 1	Florin current
France	rec.	24	Skillings	for 3	Florins
Hamburgh	rec.	47	Skillings	for 1	Rixdollar Banco
Portugal	rec.	21	Skillings	for 1	Crusade of 400 rees.
Spain	rec.	42	Skillings	for 1	Ducat Plate.

To convert Bank money into Currency, add $\frac{1}{2}$ of the sum to itself; and to convert Current money into Bank money, deduct $\frac{1}{3}$ of the sum from itself.

EXAMPLE I.

In 6433 Rixdollars, 24 Skillings Banco, how many Rixdollars, current?

	R.	S.
	6433	24
$\frac{1}{2}=$	3216	36
Rixd.	9650	12

EXAMPLE II.

In 8496 Rixdollars, 18 Skillings, how much sterling money; exchange at 4 Rixdollars, 42 Skillings, per Pound sterling?

R.	S.		R.	S.		£
4	42	:	8496	18	::	1
8			8			
39	)		67971			
			£1742	16		$11\frac{1}{13}$

Decimally.

R.

4.875)8496.375

£1742.846=£1742 16 11

EXERCISES.

1. In 9292 rixdollars, 19 skillings, how much sterling money; exchange at 4 rixdollars, 30 skillings, per pound sterling?

2. In £328. 17s. 6d. sterling, how many rixdollars and skillings; exchange at $4\frac{1}{2}$ rixdollars per pound sterling?

3. In 587 florins, 17 stivers, 8 pennings Banco, of Amsterdam, how many rixdollars and skillings; exchange at 44 skillings per rixdollar?

4. In 498 rixdollars, 28 skillings, how much Hamburgh money; exchange at $46\frac{1}{4}$ skillings per rixdollar of 3 marcs Banco?

COPENHAGEN.

In Copenhagen, and in most parts of Denmark, accounts are kept in Riksdallers, or Rixdollars, which are subdivided into marcs and skillings.

MONIES OF EXCHANGE.

12 Fenings	=	1 Skilling
16 Skillings	=	1 Marc
6 Marcs, or 96 Skil.	=	1 Rixdollar
6 Marcs Danish	=	3 Marcs Lubs.

The Fening, Skilling, and Marc Danish, are, respectively, double the Fening, Skilling, and Marc Lubs.

THE COINS OF DENMARK ARE THE FOLLOWING:

Gold	The Specie *effective* Ducat, worth about $2\frac{1}{2}$ Rixdollars, or 15 Marcs The Current Ducat of 2 Rixdollars, or 12 Marcs.
Silver	The Specie, or *effective* Rixdollar, of 7 Marcs, 6 Skillings, Danish Currency The Krohn, or Crown, of 4 Marcs, or 64 Skillings Danish; Half Crowns also of 2 Marcs The Ebrœers, or *Justus Judex*, of 14 Skillings The Ryksort at 24 Skillings Pieces of 15, 10, 8, 4, and 2 Skillings, currency.

Copper.—The Skilling, Danish; *half* and *quarter* ditto.

The Current Rixdollar is worth 3s. 8d. sterling; the Crown, about 1s. 4d.; and the Current Ducat, 7s. $5\frac{1}{2}$d. sterling.

USANCE AND DAYS OF GRACE.

There is no established Usance here.

There are 8 Days of Grace allowed, but none for bills at sight.

COURSE OF EXCHANGE.

		Uncertain Prices.		Certain Prices.
London	rec.	6 Rixdollars	for	£1 Sterling
Amsterdam	rec.	144 Ditto current	for	100 Rixdollars Banco
Dantzic	rec.	88 Rixdollars	for	100 Rixdollars
France	rec.	25 Skillings Danish	for	1 Franc
Hamburgh	rec.	148 Rixdollars	for	100 Rixdollars Banco

EXAMPLE.

In 7669 rixdollars, 2 skillings Danish, how much sterling money; exchange at 6 rixdollars, 1 marc, per pound sterling?

R.	M.		R.	M.	S.		£
6	1	:	7669	0	2	::	1
6			6				
37			46014				
16			16				
592			)736226				
			£1243	12	6		

EXERCISES.

1. In £621. 16s. 3d. sterling, how much Danish money; exchange at 6 rixdollars, 1 marc, per pound sterling?

2. In 5000 rixdollars Danish, how much sterling money; exchange at 6 rixdollars, 7 skillings, per pound sterling?

3. In £979. 11s. 10d. sterling, how much Danish money; exchange at 6 rixdollars, 52 skillings, per pound sterling?

4. How much Danish money will be necessary to discharge a debt, in Hamburgh, of 2547 marcs, Hambro' Banco; exchange 142 rixdollars Danish, for 100 rixdollars Banco?

DANTZIC, KONIGSBERG, AND MEMEL.

At these places accounts are kept in Guldens, or Florins, and Groschen.

MONIES OF EXCHANGE.

18 Pfenings = 1 Groschen = 3 Schillings
30 Groschen = 1 Florin, or Gulden
3 Florins = 1 Rixdollar, or *Thaler*
3 Prussian Guldens make 4 Florins of Dantzic.

THE CURRENT COINS ARE THE FOLLOWING:

Gold { The Dutch Ducat, fixed at 12 Florins Dantzic currency.

Silver { The Reichsthaler, worth about 6 Florins
The Florin of 30 Groschen
The Tympfen of 18 ditto; the Sechser of 6, and the Dutgen of 3 Groschen
The Groschen of 3 Schillings.

Copper—The Schilling.

USANCE AND DAYS OF GRACE.

The Usance is 14 days after acceptance.

There are 10 Days of Grace allowed, but none for bills at sight.

COURSE OF EXCHANGE.

		Uncertain Prices.		*Certain Prices.*
London	rec.	18 Florins	for	1 Sterling
Amsterdam	rec.	290 Groschen	for	1 Flemish
France	rec.	108 Rixdollars	for	300 Francs
Hamburgh	rec.	130 Groschen	for	1 Rixdollar Banco

EXAMPLE.

In 14726 florins, 16 groschen, how much sterling money; exchange at $20\frac{1}{4}$ florins, for £1. sterling?

F.		F.	G.		£
$20\frac{1}{4}$	:	14726	16	::	1
4			30		
81		441796			
30			4		
243,0		)176718,4			
		£727	4	$8\frac{1}{2}$	

EXERCISES.

1. In 4964 florins, 21 groschen, Dantzic currency, how much sterling money; exchange at 18 florins per pound sterling?

2. In £206. 17s. 3d. sterling, how much money of Dantzic; exchange at 24 florins per pound sterling?

3. When tallow sells, at Konigsberg, at 14 florins per stone of 33 lb., what is that per cwt. in sterling money; exchange at 18 florins per pound sterling?

4. J. J., of London, imports a cargo of wheat from Konigsberg, the cost of which was 680 florins per last, of 11 quarters each; charges and commission, 102 florins, 21 gr.; freight 18s. per quarter; exchange at 20 florins per pound sterling: what does it cost him per quarter?

SCOTLAND.

Accounts are kept, in Scotland, in Pounds, Shillings, and Pence, as in England; and, since the Union of the two Kingdoms, in 1707, the current coins are the same in both.

Scots money, which was used in Scotland, previous to the Union, and is still used on some occasions, is divided in the same manner as sterling money, and is $\frac{1}{12}$ part of its value.

When bills are drawn upon London, from any part of Scotland, they are always at a small discount, or there is a small premium allowed, in favour of London, such as 1, or $1\frac{1}{2}$ per cent; but the premium is generally allowed in time, by drawing on London at a certain number of days after date, the par being 40 days after date. The premium, however, varies from 40 to 60 days, but it is generally higher for small bills than large ones.—If the premium be 73 days, it is equal to 1 per cent.

This is not only the case with bills drawn on London, from any part of Scotland, but the same practice prevails, in some degree, in drawing bills upon London from the distant parts of England.

EXAMPLE I.

What is the net proceeds of a bill of £500, drawn at Edinburgh, on London, at $\frac{7}{8}$ per cent?

£		s.	d.		£
100	:	17	6	: :	500
1	:	17	6	: :	5
			5		
£4		17	6		
500		0	0		
£495		12	6	Net proceeds.	

EXAMPLE II.

A merchant in Glasgow received a bill on London of £250; what sum did he receive, the bill being discounted the same day, and drawn at a premium of 50 days?

	£250	0	0
Int. for 53 days, including 3 days of grace	1	16	$3\frac{1}{2}$
	£248	3	$8\frac{1}{2}$

EXERCISES.

1. A merchant in London receives a bill from Edinburgh on T. Coutts and Co. bankers, for £780. 8s. at a premium of ¼ per cent; required the amount of the premium?

2. A merchant in London purchased goods in Dundee, amounting to £229. 10s. 6d., for which he gave his acceptance 1st May, payable in London, 4 months after date, which was sold 1st June, at 30 days premium; how much money was received from it?

IRELAND.

In Ireland, accounts are kept in Pounds, Shillings, and Pence, as in England, and the current coins of both countries are the same*, but their *values* are not the same; 1 English shilling being equal to 1s. 1d. Irish, when at par, and therefore £1. English is equal to £1. 1s. 8d. Irish, or £100 English is worth £108. 6s. 8d. Irish; but the course of exchange varies from 6 to 20 per cent.

Ireland exchanges chiefly with England, and English money, when at par, is converted into Irish by adding $\frac{1}{12}$ of the sum to itself, and Irish into English by deducting $\frac{1}{13}$ of the sum from itself.

USANCE.

Bills drawn from Dublin are mostly at 21 days sight, or at 31 days date. If the term of a bill be longer, an advance is made accordingly, on the course of exchange.

Thus, if the course of exchange be 12¼, at 21 days sight, it will be 12½ at 45 days date, and 12¾ at 60 days date; each advance being nearly equal to the interest for the corresponding time.

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* This being the case, it is unnecessary to extend this article.

DAYS OF GRACE.

The Days of Grace, and all other regulations respecting bills of exchange, are the same as in England.

UNITED STATES OF AMERICA.

Accounts in the United States were formerly kept in Pounds, Shillings, and Pence, currency, and this practice is still retained, on many occasions, but the value of the currency is different in different States.

In Maryland, Pensylvania, New Jersey, and Delaware, the ratio of sterling money, to that of currency, is as 5 to 3; hence £1. sterling is worth £1. 13s. 4d. currency; or 12s. sterling is worth £1. currency.

In Virginia, New Hampshire, Massachusetts, Connecticut, and Rhode Island, the ratio is as 4 to 3; or 15s. sterling is worth £1. currency.

In North Carolina and New York, the ratio is as 16 to 9; or 11s. 3d. sterling is worth £1. currency.

In Georgia and South Carolina, the ratio is as 28 to 27; or 19s. 3$\frac{3}{7}$d. sterling is worth £1. currency.

MONIES OF EXCHANGE.

10 Cents = 1 Dime
10 Dimes = 1 Dollar.

The Dollar is also divided into 1000 equal parts, called Mills. The *par* of the Dollar is reckoned 4s. 6d.; hence 40 Dollars = £9 sterling.

COINS.

The coins most common in the United States, are Spanish

Dollars, but most European coins are also current; but the value of these is commonly expressed in Dollars and Hundredth parts of a Dollar, called Cents.

For example, £1 sterling is valued at 4 Dollars, 44 cents; a Franc at $18\frac{1}{2}$ cents; a Dutch Guilder, 39 cents; a Hamburgh Marc Banco, $33\frac{1}{3}$ cents; a Spanish Real of Plate at 10 cents; a Milree of Portugal at $1\frac{1}{4}$ Dollar; and all other denominations of money are in the same proportion, and no foreign coins, except the Spanish Dollars, are deemed a legal tender.

The value of the Dollar in the different States is regulated by the value of the currency in each State. In New England and Virginia, it passes for 6s.; in New York and North Carolina, for 8s.; in Maryland, Pensylvania, Jersey, and Delaware, for 7s. 6d.; and in Georgia and South Carolina, for 4s. 8d.

In the year 1789, a uniform method of keeping accounts was established, in the United States, by an Act of Congress; viz. in Dollors of 10 Dimes, 100 Cents, or 1000 Mills, which is adopted in all public accounts.

At the same time, the American Government established a mint, and ordered money to be coined, of gold, silver and copper, of the following denominations and values; viz.

Gold	The Eagle, of the value of 10 Dollars, or Units, or £2. 3s. 8d. in English gold Half and Quarter ditto, in the same proportion.
Silver	The Dollar, or Unit, of the value of 1 Spanish Milled Dollar, or 4s. $3\frac{3}{4}$d. in English silver Half and Quarter Dollars, Dimes and Half Dimes, in the same proportion.
Copper	The Cent of the value of $\frac{1}{100}$ of a Dollar, or about $1\frac{1}{2}$d. sterling Half Cent, in the same proportion.

At Philadelphia, a public Bank was established in the year 1790, which was called the United States Bank.

The capital of this Bank was fixed at 10 millions of dollars, and divided into 25,000 of 400 dollars each. These shares are transferable, and yield a dividend of 7 or 8 per cent, payable half-yearly. The constitution and government of this bank are nearly on the plan of the Bank of England; it is therefore unnecessary to give a particular description of this establishment.

AMERICAN STOCK, OR PUBLIC FUNDS.

The war, in which the American States engaged with Great Britain, requiring much more money for its support than could be immediately raised, the Congress authorised the emission of bills of credit to the amount of 3,000,000 of Spanish dollars, for the purpose of carrying on the war, and the twelve confederated Colonies were pledged for their redemption. This plan was had recourse to as long as it was found possible to maintain the circulation of the bills; but, at length, the immense quantity of paper in circulation caused it to decline in value very rapidly, which induced the Congress to come to the resolution of limiting their emissions to 200,000,000 of dollars. This sum was so great, that their agents could not obtain the necessary supplies on credit, and in the year 1780, it became necessary to attempt other means of carrying on the war. Other means were then resorted to; such as, requisitions on the several states, regular accounts of which were to be kept and afterwards paid by the general government in Spanish milled dollars.

In 1783, Congress pledged the honour of the United States for the payment of all the public creditors, and, in 1787, passed an ordinance for establishing a board of Commissioners for settling all accounts between the United States and the individual States; but it was not till the year 1790, that the depreciated paper currency of the war was actually funded, by an act of Congress, for making provision for the debt of the United States.

After this, several other acts were passed, at different periods, relating to the methods of providing for the expenses of the government, and of paying the interest of its debts.

The whole amount of the public debt of the United States, on the 1st January, 1805, was

	Dollars.
Foreign debt	4,065,000
Deferred Stock, 6 per cent	13,672,966
3 per cent Stock	19,094,231
Old 6 per cent Stock	28,190,862
5½ per cent Stock	1,847,500
4½ per cent Stock	176,000
6 per cent, 1796	80,000
6 per cent Navy	711,700
8 per cent Stock	6,462,300
Bank of United States loan	700,000
Louisiana Stock, 1803	11,250,000
	86,250,559

The above amount is exclusive of the sums redeemed by the Commissioners of the Sinking Fund, which, at the above period, amounted to 11,689,366 dollars.

The *interest*, or dividends, on the above funds, becomes due quarterly, and the days of payment are the 31st March, 30th June, 30th September, and 31st December, and are regularly paid, either at the Treasury of the United States, or at the loan offices, in the different States, as may suit the conveniency of the holder.

The dividends may also be received in Great Britain, or Holland, by authorising the Bank of the United States to receive them, who will remit them to their agents in London, or Amsterdam, at the current rate of exchange.

The dividends payable in London are remitted to Messrs. Baring, Brothers, and Co. by whom they are paid on any Monday, Wednesday, Thursday, or Saturday, between the hours of ten and two.

The purchaser of American stock receives a certificate, stating, that the United States are indebted to him, or his assigns, the amount therein specified, and the assignment of these certificates is the mode of making transfers in these funds. The holder of such certificates may, at any time, have them registered in his name in the books of the Treasury at Washington, or at the Loan Office of any particular state, and receive a new certificate in his own name. This transaction is, however, attended with some expense to persons residing in Great Britain, as it is usual to employ a Notary.

The affairs of America having undergone little alteration, in a financial point of view, and the difficulty of procuring accurate information of these affairs considerable, it is presumed the above statement will convey a tolerable idea of the nature of the American public funds.

Shares in the bank of the United States are bought and sold in London, in the same manner as American stock, and this stock is preferred by many persons, not only on account of the probability of an increase of dividend, but because they may receive their dividends in London, on signing a requisition to that effect, without any risk of deduction for bad bills.

The capital of the bank is 10,000,000 of dollars, and divided

into 25,000 shares, of 400 dollars each, and has hitherto yielded considerable profits.

The Brokerage on all descriptions of American Stock, bought and sold in London, is 5s. *per cent.*

EXAMPLE I.

Required the value of 10670 dollars, 55 cents, American 3 per Cent stock, at 55 per cent.

Dol.	c.		£	s.	d.
10670	55,	at 4s. 6d. per dol. =	2400	17	6

	£2400	17	6
			11
	26409	12	6
			5
100)	1320,48	2	6
	£1320	9	7½ Answer.

EXAMPLE II.

How much American Old 6 per Cent Stock may be purchased for £379. 8s. 6d. sterling, at 118 per cent?

£		£	s.	d.		D.
9	:	379	8	6	::	40
				10		
		3794	5			
			4			
9)		15177				

1686,33¼ Dollars, at 118 per cent
100

118)168633

D. 1429.09 Stock.

EXERCISES.

1. What is the value, in sterling money, of 2725 dollars, 95 cents, 3 per Cent American Stock, at 63¼ per cent?

2. How much American 6 per Cent Navy Stock may be purchased for £865. 9s. 8d. sterling, at 123 per cent?

USANCE AND DAYS OF GRACE.

There is no established Usance with the United States; foreign bills being mostly drawn at a certain number of days after sight.

Three days are allowed for retiring a bill, after it is due, as in England.

COURSE OF EXCHANGE.

			Uncertain Prices.		*Certain Prices.*
London	rec.	440	Dollars, more or less,	for	£100. sterling; or so much Currency p. ct. above or below par.
Amsterdam	rec.	39	Cents	for	1 Guilder current
Hamburgh	rec.	33¼	Cents	for	1 Marc Banco
France	giv.	5	Francs, 34 Centimes	for	1 Dollar.

EXAMPLE I.

In 1869 dollars, 40 cents, how much sterling money; exchange at par?

METHOD I.

D.		D.	c.		s.	d.
1	:	1869	40	::	4	6

$$\begin{array}{rrr}
 & 4\frac{1}{2} & \\
\hline
 & 7476 & \\
 & 934 & 6 \\
\frac{2}{5}\ 4s.6d.= & 1 & 9\frac{3}{5} \\
\hline
2,0)841,2 & & 3\frac{3}{5} \\
\hline
£420 & 12 & 3\frac{3}{5}
\end{array}$$

METHOD II.

$$\begin{array}{rl}
 & \text{D.} \\
 & 1869.40 \\
\hline
4s. = \frac{1}{5} = & 373.88 \\
6d. = \frac{1}{8} = & 46.735 \\
\hline
 & £420.615 = £420\ 12\ \ 3\frac{3}{5}
\end{array}$$

METHOD III.

D.		D. c.		£
40	:	1869 40	::	9
		9		
		4,0)1682,4.60		
		£420.615 = £420 12 $3\frac{1}{4}$		

EXAMPLE II.

In 1467 dollars, 25 cents, how much sterling money; exchange at 2 per cent above par?

D.		D.		D. c.
100	:	102	::	1467 25
40		9		918
4000		918		1173800
				146725
				1320525
				4,000)1346,935.50
				£336.733875 = £336 14 8

EXAMPLE III.

In £786. 17s. 6d. sterling, how much American money; exchange, 2 per cent under par?

METHOD I.

£		£ s. d.		D.
9	:	786 17 6	::	40
		10		
		7868 15 0		
		4		
		9)31475 0 0		

D.		D.		
100	:	102	::	$3497.2\dot{2}$
				102
				699444
				3497222
				100)$3567166\dot{6}$
				D. c.
				$3567.166\dot{6}$ = 3567 $16\frac{2}{3}$ Answer.

METHOD II.

£		£		D.
9	:	786.875	: :	40
100		4080		102
900		62950000		4080
		3147500		

9,00)32104500,00

D. 3567.16⅔

EXERCISES.

1. In 891 dollars, 80 cents, how much sterling money ; exchange at par ?

2. In 4282 dollars, 50 cents, how much sterling money; exchange at 2¼ per cent above par ?

3. In £509. 8s. 6d. sterling, how many dollars ; exchange at 4½ per cent above par ?

4. A merchant in London owes 7824 dollars, 45 cents, to a house in New York; what sum, in sterling money, is necessary to discharge the debt, when exchange is at 3¼ p. ct. under par ?

5. For what sum, in sterling money, should a bill be drawn to remit 5000 dollars, when exchange is 5¼ per cent above par ?

6. In 2482 dollars, 26 cents, how much Dutch money ; exchange at 36 cents per guilder ?

7. In 2296 marcs, 10 schillings, 8 pennings Banco, of Hamburgh, how many dollars ; exchange at 33 cents per marc banco ?

8. In 5348 francs, 35 cents, how many dollars ; exchange at 5 francs, 30 cents, per dollar ?

WEST INDIES.

Accounts are kept in the British islands in Pounds, Shillings, and Pence, currency. The Pound being divided in the same manner as the Pound sterling.

The currency of the West Indies is an imaginary money, and varies in value in different islands, £100 sterling being equal in value to £140 currency in some islands, and £200 currency in others.

The Spanish Dollar is the principal circulating coin in the West Indies, and the standard by which the value of all other monies is regulated.

Although the value of the currency has been attempted to be fixed by law in several of the islands, yet it is chiefly regulated by the course of exchange with London, which is liable to great variation, and even affects the value of the *coins*.

In Jamaica, the value of currency is fixed, £140 currency for £100 sterling; and, instead of exchange, there is a premium on London bills, of from 10 to 20 per cent.

THE CURRENT COINS IN THE BRITISH ISLANDS ARE THE FOLLOWING:

	GOLD.	dwt.	gr.	Value in cur.		
Spanish	The Doubloon weight	17	8	£5	0	0
	The Two Pistole piece ...	8	16	2	10	0
	The Pistole	4	8	1	5	0
	The Half ditto	2	4	0	12	6
Portuguese	The Johanese, or Joe	18	12	5	10	0
	The Half ditto	9	6	2	15	0
	The Quarter ditto	4	15	1	7	6
	The Moidore	6	22	2	0	0
	The Half ditto	3	11	1	0	0
English	The Guinea	5	8	1	12	6
	The Half ditto	2	16	0	16	3
	The Third of a Guinea ...	1	19	0	10	10

			Currency.		
Silver	The Dollar........................	worth	£0	6	8
	The Half ditto	——	0	3	4
	The Quarter ditto	——	0	1	8
	The Eighth	——	0	0	10
	The Sixteenth	——	0	0	5
	The Bit, or Bitt	——	0	0	7½

In some of the islands there are also small copper coins, called Dogs and Half Dogs. The Dog is worth 1½d. currency.

The intrinsic par of the currency of the above coins, in English gold, is £100=£154. 15s. currency.

In the French islands, the French inhabitants keep their accounts in Francs, or Livres, Soles, and Deniers.

In some of the Dutch colonies, accounts are kept in the same coins as in Holland, and others in Pieces of Eight, that is, Piastres current of 8 Reals, or Schillings; each Real being divided into 6 Stivers.

In the Danish islands, accounts are kept in Piastres, or Rixdollars current, also called Pieces of Eight.

In the Spanish islands and in all Spanish America, accounts are kept in Pesos, or Dollars of 8 Reals, which are subdivided into 16 parts, and also into 34 Maravedies Mexican Plate.

Bills drawn in the West Indies, on London, and not duly honoured, are returned to the drawer with certain damages, or charges, which are generally from 8 to 10 per cent.

Currency is converted into sterling money, and *vice versâ*, by simple stating of Proportion.

EXAMPLE.

In £879. 8s. 6d. how much sterling money; exchange at £165 per £100 sterling?

£		£	s.	d.		£
165	:	879	8	8	: :	100
				10		
		8794	6	8		
				10		
165)		87943	6	8		
		£532	19	$9\frac{1}{2}$		

EXERCISES.

1. In £987. 12s. 6d. sterling, how much Jamaica currency; exchange at £140. currency, per £100. sterling?

2. In £7842. 8s. 9d. Barbadoes currency, how much sterling money; exchange £145. per £100. sterling?

3. In £810. 5s. 3d. Trinidad currency, how much sterling money; exchange at £$192\frac{1}{8}$ per £100. sterling?

4. In 7537 dollars, 50 cents, how much sterling money; exchange at 463 dollars per £100. sterling?

5. In £500. sterling, how many dollars; exchange at $51\frac{4}{5}$d. per dollar?

ARBITRATION OF EXCHANGE.

ARBITRATION OF EXCHANGE is a comparison between the Courses of Exchange, of several places, in order to discover the most advantageous method of drawing and remitting bills.

Aribitration is of two kinds, *Simple* and *Compound*.

Simple Arbitration comprehends all exchanges where *three*

places are concerned; and Compound Arbitration all exchanges where *more* than *three* places are concerned.

SIMPLE ARBITRATION.

The use of Simple Arbitration is, to discover whether it be more favourable to exchange *directly* with any place, or through the medium of a *third* place; or, in the case of a *remittance*, whether it would be more advantageous to purchase a bill on the spot, upon an intermediate place, to be sent for negociation to the place which is the ultimate object of the operation; or, in the case of a *draft*, whether it would be more advantageous to make a remittance, from such a place to an intermediate place, upon which the amount might then be valued. The object of these operations may in general be attained, by observing the following directions:

1. If the place where a merchant resides give the *certain* price to that with which he negociates, *in remitting*, he should make use of that intermediate place by which the *highest* exchange may be obtained; but the contrary *in drawing*.

2. But if the place where he resides give the *uncertain* price, it is best to remit through that place which establishes the *lowest* exchange; and *vice versâ, in drawing*.

EXAMPLE.

If the exchange between London and Lisbon be 68d. sterling per milree, and that of Amsterdam on Lisbon 48 grotes Flemish, per old Crusade, what is the arbitrated rate of exchange between London and Amsterdam.

R.		R.		D.
400	:	1000	: :	48
68	:	240		
272,00		2400,00		
		48		
		19200		
		9600		
272)115200				

Sch. gr.

Grotes $423\frac{9}{17}$ = 35 $3\frac{9}{17}$ for £1. sterling.

If the actual or *direct* exchange, between London and Amsterdam, be *higher* than 35 sch. $3\frac{1}{17}$ gr., it is evident London would lose by drawing *directly* on Amsterdam; it would, therefore, be more advantageous for London to *draw indirectly* on Amsterdam through Lisbon; but if London has to *remit* to Amsterdam, it would be most advantageous to do it directly, when the direct course of exchange is higher than the indirect course*.

Simple Arbitration should be well understood by the student, previous to entering upon the study of Compound Arbitration, as it is not only the foundation of Compound Arbitration, but is of more general application, and of greater use in real business, as it seldom happens that exchange transactions are extended to more than three places in one operation.

EXERCISES.

1. If the exchange between London and Madrid be 42d. sterling per dollar of plate, and between Genoa and Madrid, 618 maravedies of plate per scudo d'oro; what is the proportionate or arbitrated rate of exchange between London and Genoa?

2. If the exchange between London and Paris be 24 francs, 85 cents, per pound sterling, and between Paris and Amsterdam 54 grotes for 3 francs, what is the arbitrated rate of exchange between London and Amsterdam?

3. If the exchange between Amsterdam and Paris be $54\frac{1}{2}$ grotes for 3 francs, and between Amsterdam and London 33s. 9 gr. Flemish, per pound sterling, what is the arbitrated rate of exchange between London and Paris?

4. If the exchange between London and Amsterdam be 35s. 5 gr. Flemish, and between Amsterdam and Hamburgh $33\frac{14}{41}$ stivers banco, for 2 marcs banco, what is the arbitrated rate of exchange between London and Hamburgh?

5. If London must remit to Paris at 25 francs, 24 cents, per

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* In observing whether the prices be favourable or unfavourable for any place, the advantage of that place must not be confounded with the advantage of all the places concerned, for it must be recollected, that the place drawn upon and the place remitted to, or the drawer and remitter of a bill, have opposite interests.

pound sterling, and must draw for the value upon Amsterdam at 36s. 9 gr. per pound sterling; but, when the order arrived, the exchange between London and Paris was 25 francs, 85 cents; what ought to be the rate of exchange, between London and Amsterdam, to make up for the advance that would attend the remittance in this case?

6. If a merchant in London has orders to remit to Genoa, when the rate of exchange is $52\frac{1}{4}$d. per pezza, and to draw upon Spain at 41d. per piastre; but before the order was fulfilled the rate of exchange between London and Genoa was at $53\frac{1}{2}$d. per pezza; at what price must London draw upon Spain, to make the remittance and draft upon an equality?

7. A merchant in London was ordered to remit 14250 francs to Venice, at 27 francs per pound sterling, and to draw for the value upon Cadiz at 40d. per piastre; but, when the order arrived, exchange with Venice was at 27 francs, 92 cents, per pound sterling; at what rate of exchange must London draw upon Cadiz, in order that the draft may be equal in value to the remittance?

COMPOUND ARBITRATION.

Compound Arbitration is the comparison of the rates of exchange between more than three places, in order to discover the most advantageous method of drawing or remitting bills of exchange, through the various places concerned in the comparison.

This species of Arbitration is merely an extension of Simple Arbitration, two or more operations of this kind being joined, by which a series, or chain, of Simple Proportions are connected with one another, each of which may be solved separately, or the whole by one operation.

Besides the advantages resulting to commerce, in transactions of this nature, in discharging debts in foreign places, and transferring property from one country to another, considerable profits are sometimes made, by buying and selling bills, and negociating them through different places.

It is, therefore, indispensibly necessary to all persons engag-

ed in extensive foreign commerce, to be thoroughly acquainted with the method of performing calculations of this description.—Foreign merchants are extremely expert in the application of this rule to commercial calculations; which is, perhaps, the reason that they so far excel the English in the science of exchange, although inferior in almost every other.

Previous to entering on the study of this branch of exchange, it is absolutely necessary, that Vulgar and Decimal Fractions, and Simple and Compound Proportion, be well undertood; and it would greatly abridge the labour of calculation, if the accountant knew the manner of using Logarithms.

As these subjects have already been explained, in the former part of this work, it is unnecessary to make any observations on them in this place; but the calculations being mostly performed at one operation, by a combination of Simple Proportions, called the Chain Rule, the following directions, for arranging the terms and performing calculations by that rule, will be found of very easy application.

CHAIN RULE*.

1. Find what terms are antecedents and what are consequents, which may be done as follows:

2. Place that term, or sum, upon which the demand lies, towards the right hand, as the *first* consequent; to the left of this term, and one line lower down, place the first antecedent, which must be of the same name, or kind, as the first consequent, or term of demand, and of the same value as the annexed consequent.

3. In the same manner, the second antecedent must be of the same name as the second consequent, and of the same value as the *third* consequent, and so on for any given number of terms.

4. The terms being all arranged, multiply all the consequents

* Any question, that can be performed by this rule, consists of a number of terms which bear a certain proportion to each other; these terms are divided into two classes, called *Antecedents* and *Consequents*, from being the Antecedents and Consequents of the different proportions.

into each other, and also all the antecedents into each other; then divide the product of the consequents by the product of the antecedents, and the quotient will be the answer, in the same denomination as the last consequent.

It may be of use to remark, that, if the antecedents and consequents are properly arranged, each article, or species of term, is twice entered, except that term which is of the same name with the answer, and which is called the *odd term*.

It may also be remarked, that no two articles of one denomination can be in the same column, because the terms are arranged in the form of an equation, and that quantities, that are equal to each other, are neglected, and, therefore, the answer will be of the same denomination with the last consequent, or odd term.

Some authors place the term of demand at the *bottom* of the column of consequents, instead of placing it at the *top* of the column, as here directed. The result is the same in both cases, but, by placing it at the top, it serves as a direction to the first antecedent, which must be of the same denomination.

The chain rule may be *proved* by reversing the arrangement, that is, by considering the answer as the term of demand, and making the last consequent the first antecedent, and all other consequents, antecedents. The result will then be the original term of demand, if the work be right.

EXAMPLE I.

Suppose the exchange between London and Amsterdam to be 36 Schillings Flemish per £1 sterling; between Amsterdam and Lisbon, 42 Grotes Flemish for 1 Old Crusade; and between Lisbon and Paris, 490 Rees for 3 Francs; what is the arbitrated rate of exchange between London and Paris*?

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* This question may be resolved by three simple proportions, as follows:

Sch.		Sch.		£		s. d.	
36	:	$3\frac{1}{2}$	: :	1	:	1 $11\frac{1}{3}$	
R.		R.		s. d.		s. d.	
400	:	490	: :	1 $11\frac{1}{3}$	:	2 $4\frac{7}{12}$	
S. d.		£		F.		F. C.	
2 $4\frac{7}{12}$	:	1	: :	3	:	25 19	nearly.

		1	Pound sterling
1	Pound sterling	=	36 Schillings Flem.
$3\frac{1}{2}$	Schillings Flem	=	1 Old Crusade
1	Old Crusade	=	400 Rees
490	Rees	=	3 Francs.

Then $\frac{35\times400\times3}{490\times3\frac{1}{2}}=\frac{43200}{1715}=25$ francs, 19 cts. nearly.

Operations of this kind may often be abridged by striking out such numbers, or multiples of numbers, as occur in both columns, or by using Logarithms in performing the operation.

EXAMPLE II.

A merchant in London having to remit £500 to Madrid, how many Piastres will it amount to, if remitted to Amsterdam, at 35 sch. per Pound sterling, from thence to Paris at 58 Grotes per Ecu, from thence to Venice at 100 Ecus per 60 Ducats banco, and from Venice to Madrid at 360 Maravedies per ducat banco; and what would be the gain or loss by this transaction, the direct exchange betwixt London and Madrid being $42\frac{1}{2}$d. per Piastre?

Antecedents.		Consequents.
		£500 Sterling
£1	=	35 Sch. Flem.
1 Sch. Flem.	=	12 Grotes
58 Grotes	=	1 Ecu
100 Ecus	=	60 Ducats Venice
1 Ducat	=	360 Maravedies
272 Maravedies	=	1 Piastre

$$\frac{500\times35\times12\times60\times360}{58\times100\times272}=\frac{4536000000}{1577600}=\overset{P.}{2875}\ \overset{r.}{2}\ \overset{m.}{1}$$

TO FIND THE NUMBER OF PIASTRES AT THE DIRECT EXCHANGE.

d.	£	P.		P.	R.	M.	
$42\frac{1}{2}$:	500 : :	1		2875	2	1	
2	20			2823	5	6	
85	10000		Piastres	51	4	29	Gain by the indirect exchange.
	12						
	120000						
	2						
85)240,000							

r. m.
P. 2823 5 6 by the direct exchange.

The labour of calculation, in questions of this kind, may very often be considerably abridged, by striking out such numbers as occur among the antecedents and consequents, or by dividing by any number, or numbers, that will divide any antecedent and consequent without a remainder.

The foregoing example, when treated in this manner, will stand thus:

Ant.	Cons.
	500 *a.*
1	35
1	12 *b. d.*
b. 58	1
a. 100	60
1	360 *c.*
c. d. 272	1

$$\frac{5 \times 35 \times 3 \times 60 \times 45}{29 \times 17} =$$

$$\frac{1417500}{493} = \overset{\text{P.}}{2875}\ \overset{\text{R.}}{2}\ \overset{\text{M.}}{1}$$

Those numbers that are marked with the letter *a*, are divided by 100; those with *b*, by 2; those with *c*, by 8; and those with *d*, by 2, after having been previously abridged.

Operations of this kind may often be greatly facilitated by the application of logarithms; for, if the sum of the logarithms of the antecedents be subtracted from the sum of the logarithms of the consequents, the remainder will be the logarithm of the answer*.

The above example, wrought in this manner, would stand thus:

Antecedents.		Log.	Consequents.		Log.
58	=	1.763428	500	=	2.698970
100	=	2.000000	35	=	1.544069
272	=	2.434569	12	=	1.079181
			60	=	1.778151
		6.197997	360	=	2.556303

Sum of Log. Consequents	9.656674
Ditto of Antecedents	6.197997
The same as before, 2875.25 =	3.458677

When the numbers are great, or consist of fractional parts, this mode of calculation is much shorter than any other.

* For the manner of using logarithms, see page 136.

EXERCISES.

1. Suppose a merchant in London has a sum of money to receive in Madrid, and the exchange between these two places to be 40d. per piastre, but, instead of drawing directly on Madrid, he draws on Amsterdam, and orders his agent there to draw on Paris, and Paris to draw on Madrid; the exchange between London and Amsterdam being 35 sch. Flem. per pound sterling, between Amsterdam and Paris 53 grotes Flem. per ecu of 3 francs, and between Paris and Madrid, 15 francs, 50 cents, per doubloon of plate; required the *arbitrated* price between London and Madrid.

2. A merchant in Amsterdam has £800. Flemish to remit to London, the exchange being 36 sch. 10 gr. per pound sterling; but, instead of remitting directly, remits to Paris, and desires his agent there to remit to Venice, with orders for Venice to remit to Hamburgh, Hamburgh to remit to Lisbon, and, lastly, Lisbon to remit to London; the exchange between these places being as follows: between Amsterdam and Paris, 56 grotes per ecu of 3 francs, between Paris and Venice, 100 ecus per 60 ducats, between Venice and Hamburgh, 100 grotes per ducat, between Hamburgh and Lisbon, 50 grotes Flem. per crusade of 400 rees, and between Lisbon and London, 64d. sterling per milree: required the gain or loss, by the circular mode of remitting.

3. A merchant in London has a sum to receive in Lisbon, when the exchange is at 66d. sterling per milree; he draws on Lisbon, but remits his bill to Hamburgh to be negociated, and orders the returns to be made to him in bills on Leghorn; the exchange between Hamburgh and Lisbon being 44 grotes Flem. per old crusade, between Hamburgh and Leghorn, 84 grotes Flem. per pezza, and between Leghorn and London, 54d. sterling per pezza; what is the *arbitrated* price between London and Lisbon?

4. In the following example it is required to find the most favourable exchange, from different combinations of the same exchanges.

QUOTATIONS OF EXCHANGE.

London ... on	Amsterdam	36	Schillings Flemish per Pound sterling
————	Cadiz	40	Pence sterling per Piastre of plate
————	Paris	24	Francs per Pound sterling
Amsterdam on	London ...	35	Schillings Flemish per Pound sterling
————	Paris	52	Grotes per 3 Francs
————	Cadiz	92	Grotes per Ducat of plate
Paris ...... on	London ...	23	Francs per Pound sterling
————	Cadiz	16	Francs per Doubloon of plate
————	Amsterdam	53	Grotes per 3 Francs
Cadiz ...... on	London ...	38	Pence sterling per Pound sterling
————	Amsterdam	92	Grotes per Ducat of plate
————	Paris	17	Francs per Doubloon of plate

Then, suppose London has a sum of money to receive in Cadiz, what would be the most advantageous mode of operating from the six following combinations:

1. Suppose London to *draw* on Amsterdam, ordering Amsterdam to draw on Paris, and Cadiz to *remit* to Paris.

2. Suppose London to draw on Paris, ordering Paris to draw on Amsterdam, and Madrid to remit to Amsterdam.

3. Suppose London to draw on Madrid, and to remit the bill to Paris to be negociated, and let the returns be made in a bill on Amsterdam.

4. Suppose London to draw on Madrid, and to remit the bill to be negociated in Amsterdam, and order the returns to be made in a bill on Paris.

5. Suppose London to draw on Amsterdam, ordering Amsterdam to draw on Paris, and Paris on Madrid.

6. Suppose Madrid to remit to Paris, ordering Paris to remit to Amsterdam, and Amsterdam on London.

TO INCLUDE THE CHARGES IN THE CALCULATION.

In the foregoing exercises, there is no notice taken of the charges attendant upon transactions, such as Commission, Brokerage, &c. which, it is evident, ought to be allowed. The charges are generally so much per cent, which are allowed for, as follows:

RULE.

If the money is to be *paid*, *add* the per centage to 100; but, if the money is to be *received*, *subtract* it from 100; then place the *first* term of the proportion among the *antecedents*, and the *second* among the *consequents*.

EXAMPLE.

Suppose London takes a bill on Hamburgh, at 35 sch. 4 gr. Flem. banco, per pound sterling, and remits it to Amsterdam, to be negociated at 34 stivers banco, per 2 marcs banco; what is the arbitrated rate of exchange between London and Amsterdam, 1 *per cent* being charged, at Amsterdam, for commission, brokerage, and postages?

			1	Pound sterling
1	Pound sterling	=	$35\frac{1}{3}$	Schil. Flem.
8	Schil. Flem.	=	3	Marcs banco
2	Marcs banco	=	34	Stivers banco
6	Stivers	=	1	Schil. Flem.
100		=	99	

BY LOGARITHMS.

Antecedents.		Log.	Consequents.		Log.
8	=	0.903090	$35\frac{1}{3}$	=	1.548181
2	=	0.301030	3	=	0.477121
6	=	0.778151	34	=	1.531479
100	=	2.000000	99	=	1.995635
		3.982271	Sum Log. Cons.		5.552416
			Sum Log. Antec.		3.982271
			Answer, 37.166	=	1.570145

Therefore, the *arbitrated* exchange, between London and Amsterdam, is 37 sch. 2 gr. per pound sterling*.

* The intrinsic Par of the monies of different countries, and also questions respecting the arbitration of bullion and other articles of merchandise, may be determined in a similar manner.

EXERCISES.

1. London draws upon Lisbon at 62d. sterling per milree, and orders Lisbon to draw on Madrid, at 2250 rees, per pistole of exchange, charging 1 *per cent* for commission, brokerage, and postages; what course of exchange will those prices establish, between London and Madrid?

2. A merchant in London has occasion to remit 12,000 livres to Paris, but remits to Hamburgh at 35 sch. Flem. per pound sterling, with orders to transmit an equivalent sum to Paris, at 24 sch. banco for 3 francs, deducting $\frac{1}{4}$ per cent for charges; what sum does London disburse by this mode of remittance?

METHOD OF STANDARDING COINS AND BULLION, &c.

In England, *gold* and *silver* are generally bought and sold at so much per ounce *standard*, and this standard must be calculated from the Assay Master's report of weight and fineness.

The common method of determining the value of small quantities of gold and silver is, to allow 4s., per ounce of gold, for every carat *better* or *worse* than standard; and 6d. per ounce, of silver*.

English standard gold is 22 carats fine, and silver, 11 oz. 2 dwt. (See page 354.)

TO STANDARD GOLD.

RULE.

As 22 carats are to the *gross* weight, so is the assay, or real fineness, to the quantity to be added or subtracted from the gross

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* If gold or silver be finer than standard, the difference is termed *Betterness*, by the trade; and, if worse than standard, *Worseness*.—(*B. W.*)

weight, in order to reduce it to the standard fineness. If the report of fineness be better than standard, the additional quantity, or *betterness*, is to be *added*; but if worse, the difference, or *worseness*, is to be *subtracted*.

EXAMPLE I.

What is the quantity of standard gold, in an ingot of the following report, 1 carat, 3 gr. B.; the weight of the ingot being 63 oz. 16 dwt. 8 gr.?

```
car.     oz. dwt. gr.     Betterness
 22  :   63  16   8  ::   1  3
  4               7       4
----     ----------     -----
 88     446  14   8       7

                     oz. dwt. gr.
   88)446  14  8(    5   1  13  betterness nearly
      440           63  16   8
      ---           ----------
        6      oz.  68  17  21  Standard
       20           ----------
      ---
      134
       88
      ---
       46
       24
      ---
      192
      92
      ---
     1112
       88
     ----
      232
      264
     ----
```

EXAMPLE II.

What is the quantity of standard gold, in an ingot of the following report, 2 carats, 1½ gr. W.; the weight of the ingot being 48 oz. 10 dwt. 7 gr.?

	car.		oz.	dwt.	gr.		Worseness
	22	:	48	10	7	::	2 1¼
	4				9¼		4
	88		436	12	15		9¼
			24	5	3½		
88	8		460	17	18½		
	11		57	12	$5\frac{5}{16}$		
Worseness		oz.	5	4	18 nearly		
Gross weight			48	10	7		
Standard		oz.	43	5	13		

EXERCISES.

1. Required the quantity of standard gold, in a piece weighing 27 oz. 17 dwt. 5 gr., being 1 carat, 3 gr. better than standard.

2. Required the quantity of standard guineas, in a piece of gold, weighing 2 lb. 8 oz. 15 dwt. 3¼ gr.; the fineness of which is 2 carats, 2¼ gr. worse than standard.

TO STANDARD SILVER.

RULE.

As 11 oz. 2 dwt. are to the gross weight, so is the assay, or report of fineness, to the quantity to be added or subtracted.

EXAMPLE.

In a bar of silver, weighing 156 oz. 8 dwt., how much standard; the *worseness* being 9¼ dwt.?

	oz. dwt.		oz. dwt.			dwt. W.	
	11 2	:	156	8	::	$9\frac{1}{2}$	
	20			$9\frac{1}{2}$			
	222		1407	12			
			78	4			
		222)	1485	16			
Worseness		oz.	6	13	$20\frac{20}{37}$		
Gross weight			156	8	0		
Standard		oz.	149	14	$3\frac{17}{37}$		

EXERCISES.

1. In a piece of silver 12 dwts. better than standard, and weighing 248 oz. 7 dwts., how much standard?

2. In a piece of silver, of 11½ dwts. W., and weighing 256 oz. 18 dwt. how much standard?

TO FIND THE VALUE OF GOLD AND SILVER SPECIE AND BULLION.

Before determining the value of any coin, or quantity of bullion, it must be reduced to *standard fineness* by the *preceding* rules; the value may then be found as follows:

RULE.

As 1 oz. of standard gold or silver is to its standard, or current value, so is any given weight, of the same metal, to its standard, or current value*.

EXAMPLE I.

What is the value of the double Louis d'or, in sterling money, weighing 9 dwt. 16 gr. English standard?

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* The standard value of English standard gold is £3 17s. 10½d.; and of standard silver, 5s. 2d. (See page 354.)

oz. dwt. gr. £ s. d.
1 : 9 16 : : 3 17 $10\frac{1}{4}$
20 $9\frac{2}{3}$

20 35 0 $10\frac{1}{4}$
$\frac{2}{3}$ = 2 11 $11\frac{1}{6}$

2,0)3,7 12 $9\frac{5}{6}$

£1 17 $7\frac{83}{120}$ value in sterling

EXAMPLE II.

What is the value of a Spanish dollar, the fineness being 5 dwt. 12 gr. worse than standard, and its weight 17 dwt. 9 gr.?

oz. dwt.
11 2 standard fineness
0 $8\frac{1}{2}$ w.

oz. dwt. dwt. gr.
11 2 : 17 9 : : 10 $13\frac{1}{2}$
20 20

222 213.5

17.375.
213.5

86875
52125
17375
34750

dwt. dwt. gr.
222)3709.5625(16.7097 = 16 17 standard silver
222

1489
1332

1575
1554

2162
1998

1645
1554

91

TO FIND THE VALUE.

oz.		dwt.	gr.		s.	d.
1	:	16	17	: :	5	2
20		24				
20		81				
24		32				
480		401				

401
$5\frac{1}{6}$

2005
67 nearly

48,0)207,2

£0 4 $3\frac{3}{4}$ value

When allowance is to be made for charges, it may be done as directed at page 442.

EXAMPLE III.

Required the price, at London, of an ounce of Spanish dollars, bought at Lisbon, at 859 rees per dollar; the course of exchange between Lisbon and London being 63d. sterling per milree, and the aggregate charges 3 per cent?

BY THE CHAIN RULE.

Antecedents.		Consequents.
		1 ounce
865 ounces	=	1000 dollars
1 dollar	=	859 rees
1000 rees	=	63 pence
100	=	103

$$\frac{859 \times 63 \times 103}{865 \times 100} = 64\tfrac{7}{16}\text{d. per ounce}$$

The course of exchange may be determined by reversing the process; or the course of exchange may be determined, when the value of any quantity of bullion, or coin, is known.

EXERCISES.

1. Required the price, in London, of an ounce of standard gold in bars, bought at Paris at 106 livres, 15 sols tournois, per ounce fine; the course of exchange being at $24\frac{1}{2}$ livres?

2. Required the course of exchange between London and Paris, resulting from the sale, in London at £4. 1s. 3d. sterling, of an ounce of standard gold in bars, bought at Paris at 105 livres, 17 sols, 6 deniers tournois, per ounce fine?

3. Required the price, in London, of an ounce of standard silver in bars, bought at Amsterdam, at 25 guilders, 3 stivers current, per marc; the course of exchange between Amsterdam and London being at 36 schillings, 4 gr. and the agio on the bank, $4\frac{3}{4}$ per cent?

4. Required the price, in London, of an ounce of standard silver in bars, bought at Lisbon at 990 rees per ounce; the course of exchange between Lisbon and London being $62\frac{1}{2}$d. per milree?

5. Required the price, in London, of an ounce of standard silver in bars, bought at Cadiz at 105 reals of plate, per marc; the course of exchange between London and Cadiz being $36\frac{1}{4}$d.?

6. Required the course of exchange between London and Cadiz, resulting from the sale, in London, of 1 ounce of new doubloons of English standard fineness, at £3. 18s. 6d.?

FORMS OF MERCHANTS' ACCOUNTS, INVOICES, ACCOUNT SALES, &c.

I.

London, 25th *March*, 1817.

MR. JOHN BARNES,

Bought of Thomas Stalker and Co.

WP. 5 Bales of Cotton

No.		cwt.	qr.	lb.		
1000,	weighing	2	2	26		
5440	....	2	3	11		
8060	....	2	2	27		
4050	....	2	3	17		
9160	....	2	3	18		
	cwt.	14	0	15		
	draft		3	15		
		13	1	0		
	tare		2	7		
	Net cwt.	12	2	21	at 2s. 6d. per lb.	£177 12 6

II.

London, 25th *March*, 1817.

MR. THOMAS DAINTRY,

Bought of Thomas Stalker and Co.

AB 2 Casks Madder, viz.

No.		lb.		lb.		
1,	weighing	1262	tare	66		
2,	....	1198	......	70		
		2460		136		
	tare	136				
	Net lb.	2324	at 3s. 6d. per lb. is			£406 14 0

I.

ACCOUNT SALES *of 260 qrs. of Oats, received from Amsterdam per the Nelson, Captain John Williams, and sold here, by order and on account of Messrs.* WILLIAM WOODSTOCK, *and Co. of Embden.*

BO	260 qrs. of Oats, at 23s. 9d. per qr.			£308 15 0
	CHARGES.			
Duty and Fees		£13 12 6		
Freight, Entry, and Primage		18 14 6		
Brokerage, 1s. per quarter		13 0 0		
Small Charges		1 16 6		
Commission and Guarantee of debts, 4 per cent		12 7 0		
				59 10 6
			Net proceeds	£249 4 6

II.

ACCOUNT SALES *of 56 hhds. of Clayed Sugar, received per the Venus, Capt. W. Cooper, and sold here, by order and on account of Messrs.* JAMES *and Co. of Jamaica.*

EF 56 hhds. of Sugar, weighing

	cwt.				
gross cwt.	530	2	14		
tare	7	3	18		
Net cwt.	522	2	24	at 75s. p. cwt.	£1960 3 7

	CHARGES.		
Freight	£97 16 0		
Landing, weighing, and delivering	10 14 5		
Dock rate	5 10 6		
Brokerage, ½ per cent	9 16 0		
Warehouse rent	1 4 10		
Duty, &c.	260 14 9		
Commission, 2½ per cent	49 0 0		
			£434 16 6
		Net proceeds	£1525 7 1

INVOICE *of 6 hhds. of Tobacco, shipped on board the Success, Captain John Dalby, bound for Rotterdam, by order and on account and risk of Messrs.* **WOOD, LAUGA,** *and Co. there.*

E.F. 6 hhds. Tobacco, viz.

	cwt.	qr.	lb.		cwt.	qr.	lb.			
No. 1,	18	1	10	Tare	1	2	9			
— 2,	19	2	12		1	3	5			
— 3,	18	1	10		1	2	6			
— 4,	18	1	14		1	1	6			
— 5,	12	3	26		1	2	21			
— 6,	12	3	5		1	1	15			
Gross cwt.	100	1	21	cwt.	9	1	6			
Tare	9	1	6		s.	d.				
Net cwt.	91	0	15	at	51	4	per cwt.	£233	18	3

CHARGES.

Bond and Custom House Fees	£0	10	6			
Cost of empty Hogsheads	4	16	0			
Lighterage and small Charges	1	4	0			
Bills of Lading	0	5	6			
Commission, 2½ per cent...............	5	17	0			
				12	13	0
Insurance on £257, at 3 guineas p. ct.	£8	1	11			
Policy, 5s. per cent......................	0	15	0			
Commission, ½ per cent	1	5	8			
				10	2	7
				£255	13	10

Dr. *Messrs.* Baumgarten *and* Meyer *of Hamburgh,*

1817.					
March 10,	To Invoice of 25 hhds. of Sugar, per the Enterprise, Stephens, at 2 months credit		£873	6	3
April 14,	To Cash paid Capt. Hermanson, on their account		23	10	0
May 3,	To Account of Insurance on Goods, per the Urania, Volkerts, from Riga to Amsterdam		32	8	1
June 11,	To Invoice of Coffee, per the Thames, Croft,		450	3	8
Aug. 1,	To Amount of their Draft, on Bruce and Co., paid under protest for their honour		478	15	4
Dec. 31,	To Commission on £502. 5s. 4d. at ½ per cent	£2 10 3			
	Protest	0 7 6			
	Postages	0 17 8			
	Brokerage of Bills	1 11 3			
	Stamps	1 7 6			
			6	14	2
			£1864	17	6

Account of Interest on the above

£	s.	d.	From	Days		£	s.	d.
873	6	3	May 10,	235,	at 5 p. ct. p. ann.	28	2	0
23	10	0	April 14,	261,		0	16	9
32	8	1	Dec. 31,	—		0	0	0
450	3	8	Aug. 11,	142,		8	15	1
478	15	4	Aug. 1,	152,		9	19	5
502	5	4	Dec. 31,	—		0	0	0
					Balance	0	9	1
						£48	2	4

their Account Current with Lewis, Ward & Co. of London. Cr.

1817.								
March 24,	By their remittance, viz. 20th March, at 2 mdt. on Philips and Free.	£360	0	0				
—— 16,	Ditto, at 2½ Usance, on Carruthers and Co.	251	10	9				
					£611	10	9	
April 9,	By our dft. on them at 2 usance, to the order of James and Co.				250	0	0	
May 10,	By our dft. at one month, to Sheriff and Co.				40	12	2	
Aug. 1,	By our dft. at 2 usance, to Dubarry and Loughman				960	0	0	
Dec. 31,	By Balance of Interest due to them				0	9	1	
	By Balance due to us..................				2	5	6	
					£1864	17	6	

to the 31st *December,* 1817.

£	s.	d.	From	Days		£	s.	d.
360	0	0	May 23,	222,	at 5 p. ct. p. ann.	10	19	0
251	10	0	June 3,	211,	....	7	5	5
250	0	0	April 12,	263,	....	9	0	2
40	12	2	May 13,	232,	....	1	5	10
960	0	0	Aug. 4,	149,	....	19	11	11
						£48	2	4

Errors excepted,
London, 31st December, 1817
LEWIS, WARD and Co.

SIMPLE INTEREST.

The most common calculations that occur in business, respecting Simple Interest, having been already explained at considerable length, it is therefore intended, in this place, to exhibit a general view of the subject, in the form of Algebraic Theorems.

Let p represent the principal, or sum put out to interest; t, the time it bears interest; r, the rate, or interest, of £1 for one year; i, the interest for the time; and a, the amount, or sum, of principal and interest.

The interest of £1 for a year being r, it is evident, the interest of £1 for the time, t, must be tr; the interest of p, pounds, ptr; and the amount of principal and interest, $p+ptr$. Hence the following *Theorems:*

1. $i = ptr$. 2. $p = \frac{i}{tr}$ 3. $t = \frac{i}{pr}$

4. $r = \frac{i}{pt}$ 5. $a = ptr+p$ 6. $p = \frac{a}{tr+1}$

7. $t = \frac{a-p}{pr}$ 8. $r = \frac{a-p}{pt}$ 9. $i = \frac{atr}{tr+1}$

EXAMPLE I.

What is the interest of £432, at $3\frac{3}{4}$ per cent, for 4 years?

Here $p=432$, $t=4$, and $r=.0375$, to find i, which is done by Theorem 1st.

$$432\times4\times.0375=64.80=£64.\ 16s.$$

EXAMPLE II.

At what rate of interest will £235. 10s. amount to £282. 12s. in 4 years?

Here $p=235.5$, $a=282.6$, and $t=4$, to find r, therefore by Theorem 8th,

$$r=\frac{282.6-235.5}{235.5\times 4}=\frac{47.1}{942}=.05, \text{ or 5 per cent.}$$

EXERCISES.

1. What sum, put out to interest at 4 per cent, will gain £47. 2s. in 4 years?

2. In what time will £432. 5s. gain £64. 16s. 9d. at $3\frac{3}{4}$ per cent.

3. At what rate of interest will £145 gain £1. 18s. $1\frac{1}{2}$d. in 96 days?

4. What will £173 amount to in 64 days, at 5 per cent?

5. What sum, lent out at $3\frac{3}{4}$ per cent, will amount to £497. 1s. 9d. in 4 years?

6. In what time will £432. 5s. amount to £497. 1s. 9d. at $3\frac{3}{4}$ per cent?

7. At what rate of interest will £432. 5s. amount to £497 1s. 9d. in 4 years?

COMPOUND INTEREST.

COMPOUND INTEREST is an allowance made by the borrower to the lender, not only for the use of the principal, but, likewise, for the use of the interest, after it falls due; so that the interest, being continually added to the principal, at the end of the year, the sum, or amount, becomes a new principal for the succeeding year, and, in this case, the principal and interest are always increasing*. For, although the law does not allow money to be put out at compound interest, yet the interest may be demanded, when it is due, and lent out again; by which means, all the purposes of compound interest may be answered to the lender; and, in purchasing Annuities, Pensions, or Leases in reversion, it is customary to allow the purchaser compound interest for ready money; and, therefore, this branch of arithmetic ought to be well understood by those concerned in such transactions.

Calculations in Compound Interest may be performed by common arithmetic, by using tables already calculated, or by Logarithms.

The first of these methods being extremely laborious, and the second requiring the assistance of a number of tables, the method by Logarithms will be chiefly employed in giving examples of the manner of performing the following exercises, and the various theorems, suited to their application.

Let the letters i, p, t, and a, represent the interest, principal, time, and amount, as in Simple Interest, and let $P = 1 + r$, or the amount of £1 at the end of one year; R, the principal at interest the second year; then, $R + rR = R \times (1 + r) = R^2$, will

---

* The method of finding the amount of any principal in any given time, is to find the amount for the time of the first payment by Simple Interest; then to consider this amount as the principal for the second payment, the amount of which is to be calculated as before, and so on for all the payments to the last; always reckoning the last amount as the principal for the next payment.

be its amount at the end of the second year; in like manner, R^3 is its amount at the end of the third year; R^4 at the end of the fourth year; and R^t at the end of t years; and for the sum p, it will be pR^t; that is, $a=pR^t$; and from this equation are obtained the following Theorems:

1. $a=pR^t$ or Log. a = Log. $(R \times t)$ + Log. p.

2. $p=\frac{a}{R^t}$ or Log. p = Log. a — Log. $(R \times t)$

3. $R=\sqrt[t]{\frac{a}{p}}$ or Log. $R = \frac{\text{Log. } a - \text{Log. } p}{t}$

4. $R^t=\frac{a}{p}$ therefore Log. $t = \frac{\text{Log } a - \text{Log. } p}{\text{Log } R}$

The same things may be obtained from the tables as follows:

1. To find (a), the *amount*, multiply the number corresponding to the rate and time (in Table I.) by the principal, the product is the amount.

2. To find (p) the *principal*, divide the amount by the tabular amount of £1, for the rate and time, the quotient is the principal.

3. To find the *rate* (r) divide the amount by the principal, the quotient is the amount of £1, which find in the table, on the line with the time, and the rate will be found on the top of the same column.

4. To find the *time*, divide the amount by the principal, the quotient is the amount of £1; which find under the rate, and in the left hand column, on a line with it, is the time.

5. If the given number of days, years, &c. is not to be found in the table, divide it into two such numbers as are to be found in the table; then multiply the tabular numbers, thus found, together, and their product is the amount of £1 for the given time.

6. If the time consist of years and days, their respective tabular numbers must be *multiplied* together, to obtain the amount of their sum.

EXAMPLE I.

What will £480 amount to, in 7 years, at 4 per cent compound interest?

Here $p = 480$, $R = 1.04$, and $t =$ to 7, to find a.

BY THEOREM I.

$$
\begin{array}{rcr}
\text{Log. } 1.04 & = & 0.017033 \\
 & & 7 \\
\hline
 & & 0.119231 \\
\text{Log. } £480 & = & 2.681241 \\
\hline
£631.647 & = & 2.800472
\end{array}
$$

£631. 12s. 11¼d., amount in 7 years.

BY TABLE I.

$$
\begin{array}{lr}
\text{Amount of } £1\text{, in 7 years, at 4 per cent,} & 1.315932 \\
 & 480 \\
\hline
 & £631.647 \\
\hline
\end{array}
$$

EXAMPLE II.

What will £500 amount to, in 55 years, 50 days, at 5 per cent, compound interest?

Here $p = 500$, $R = 1.05$, and $t = 55\frac{10}{73}$, to find a.

$$
\begin{array}{rcr}
\text{Log. } 1.05 & = & 0.021189 \\
 & & 55\frac{10}{73} \\
\hline
 & & 105945 \\
 & & 105945 \\
 & & 2903 \\
\hline
 & & 1.168298 \\
\text{Log. } 500 & = & 2.698970 \\
\hline
£7366.888 & = & 3.867268
\end{array}
$$

£7366. 17s. 9d. Amount.

BY THE TABLE.

Amount of £1, in 50 years	=	11.4674
Ditto, ... in 5 years	=	1.27628
Their product is the amount for 55 years	=	14.635631
Amount of £1, in 50 days	=	1.006706
Their product	=	14.7337767
		500
£7366. 17s. 9d.	=	7366.8883500

EXERCISES.

1. What will £1000 amount to, in 10 years, at 4 per cent, compound interest?

2. What is the present worth of £500, due 7 years hence, allowing discount at the rate of 4 per cent, compound interest?

3. At what rate of compound interest will £1250 amount to £1645, in 7 years?

4. In what time will £500 amount to £1000, or any sum double itself, at 5 per cent, compound interest?

5. Required the amount of 1 penny, lent out at 5 per cent, compound interest, for 1000 years.

6. Required the amount of £100, for 100 years, 120 days, at 5 per cent.

ANNUITIES.

An Annuity is a term employed to denote any periodical income, payable *annually*, or at *other* intervals of time.

ANNUITIES IN ARREARS.

Let m = the *arrear* of any annuity (a) unpaid for the time (t) and compound interest reckoned on each payment, from the time it should have been paid; then the most common cases that occur, in transactions of this kind, may be resolved by the following Theorems:

1. $m = a \times \frac{R^t - 1}{r}$ or Log. m = Log. $(R^t - 1)$ − Log. r + Log. a.

2. $a = \frac{mr}{R^t - 1}$ or Log. a = Log. m + Log. r − Log. $(R^t - 1)$

3. $R^t = \frac{mr + a}{a}$ therefore, $t = \frac{\text{Log. } (mr + a) - \text{Log. } a}{\text{Log. } R.}$

4. $\frac{m}{a} R - R^t = \frac{m - a}{a}$

When the annuity is payable at any other interval than a year, and t is equal the number of times that payment is made, R is equal the amount of £1, for 1 of those times, and a equal the sum paid each time.

The sum which would yield a perpetual annuity of £1, at the given rate of interest, is called the *Perpetuity*.

EXAMPLE.

What will an annuity of £50, payable annually, amount to, in 30 years, at $4\frac{1}{2}$ per cent?

This example is resolved by Theorem 1st, where $a = 50$, $r = .045$, $R = 1.045$, and $t = 30$, to find m.

BY LOGARITHMS.

Log. 1.045		=	0.019116
			30
R^t	= 3.7452	=	0.573480
	1		
$R^t - 1$	= 2.7452	=	0.438574
r	= .045	=	−2.653212
			1.785362
	= 50	=	1.698970
	£3050.35	=	3.484332

BY TABLE IV.

Amount of £1, annuity for 30 years	=	61.00707
		50
£3050. 7s.	=	3050.35350

EXERCISES.

1. Required the amount of an annuity of £100, payable yearly, in 15 years, 5 per cent, compound interest?

2. What annuity will amount to £1000, in 15 years, at 5 per cent, per annum, compound interest?

3. In what time will an annuity of £20, payable yearly, amount to £890, at 5 per cent?

4. At what rate will an annuity of £50, payable yearly, for 30 years, amount to £3050. 7s.?

5. What will an annuity of £17. 10s., payable quarterly, amount to, in 5 years, at 5 per cent?

6. What is the present value of 60 years possession of an estate of £100, to commence at the expiration of 40 years, interest at 3 per cent?

PRESENT WORTH OF ANNUITIES.

The present worth of any given annuity may be found by the following theorems, where P represents its present worth, or purchase money, and the other letters the same things as in the preceding theorems.

1. $p=a\times\frac{R^t-1}{rR^t}$ or $\text{Log.}p=\text{Log.}(R^t-1)-\text{Log.}rR^t+a$

2. $a=\frac{prR^t}{R^t-1}$ or $\text{Log}a=\log R^t+\log.p+\log.r-\log.(R^t-1)$

3. $R^t=\frac{a}{a-rp}$ therefore $t=\frac{\log.a-\log.(a-rp)}{\log.R}$

If the annuity be to continue for ever, then R^t and R^t-1 may be considered as the same, and therefore

4. $p=\frac{a}{r}$ 5. $a=pr$ 6. $r=\frac{a}{p}$

EXAMPLE.

What is the present worth of an annuity of £120, payable yearly, to continue 50 years, at 4 per cent, compound interest?

Here $a=120$, $t=50$, and $R=1.04$, to find p.

BY LOGARITHMS.

R	$=$	1.04	$=$	0.017033
				50
R^t	$=$	7.10668	$=$	0. 851650
		1		
R^t-1	$=$	6.10668	$=$	0. 785805
R^t	$=$	0.851667		
r	$=$	-2.602060		
				-1.453727
(R^t-1)		$-rR^t$	$=$	1.332078
		Log. 120	$=$	2.079181
£2577.862			$=$	3.410259

BY TABLE V.

Present value of £1 annuity for 50 years	21.482182
	120
Answer £2577 17s. $2\frac{1}{4}$d. =	£2577.861840

EXERCISES.

1. Required the present worth of an annuity of £10, for 7 years, payable yearly, at 4 per cent, compound interest.

2. Required the present worth of an annuity of £100, payable yearly, to continue for 20 years, at 4 per cent, compound interest.

3. Required the present worth of an annuity of £40, payable quarterly, to continue 50 years, at 4 per cent, compound interest.

4. What annuity, payable yearly, to continue 20 years, may be purchased for £260. 3s. 9¼d., at 4½ per cent, compound interest?

5. An annuity of £25, payable yearly, is purchased for £416. 11s. 6½d.; how many years ought it to continue, interest 4 per cent?

6. Required the value of a freehold estate, of £250 per annum, allowing the purchaser 5 per cent for his money.

7. A farmer, on obtaining a lease of a farm for 25 years, pays a fine, or grassum, of £1000; how much additional rent would have been equivalent to the same; interest at 4 per cent?

REVERSIONARY ANNUITIES.

Let n represent the time after which the annuity is to commence; then the present worth, &c. of any annuity in reversion may be found by the following Theorems:

1. $p=a\times\frac{R^t-1}{rR^{t+n}}$ or $\text{Log}.p=\log.(R^t-1)-\log.rR^{t+n}+\log.a$.

2. $a=p\times\frac{rR^{t+n}}{R^t-1}$ or $\log.a=\log.rR^{t+n}-\log.(R^t-1)+\log.p$.

3. $t=\frac{\log.a-\log.(a-rR^np)}{\log.R}$

4. $n=\frac{\log.(a-\frac{a}{R^t})-\log.rp}{\log.R}$

If the annuity be to continue for ever, after its commencement, R^t is in that case the same as R^t-1: hence

5. $p=\frac{a}{rR^n}$ 6. $a=prR^n$ 7. $rR^n=\frac{a}{p}$

8. $n=\frac{\log.a-\log.pr}{\log.R.}$

EXAMPLE.

Required the present worth of an annuity of £80, payable yearly, for 24 years, to commence 8 years hence, reckoning compound interest at 5 per cent?

Here $a = 80$, $t = 24$, $n = 8$, and $R = 1.05$, to find p.

BY LOGARITHMS.

$$
\begin{array}{rcr}
\text{Log. } R = 1.05 & = & 0.021189 \\
 & & 24 \\
\hline
 & & 84756 \\
 & & 42378 \\
\hline
R^t = 3.22505 & = & 0.508536 \\
1 & & \\
\hline
R^t - 1 = 2.22505 & = & 0.347340 \\
\hline
R = 0.021189 & & \\
t + n = \quad 32 & & \\
\hline
R^{t+n} = 0.678048 & & \\
r = -2.698970 & & \\
rR^{t+n} \text{———} & = & -1.377018 \\
\hline
(R^t - 1) - (rR^{t+n}) & = & 0.970322 \\
a & = & 1.903090 \\
\hline
£747.1572 & = & 2.873412 \\
\hline
\end{array}
$$

BY TABLE V.

Present worth of £1, for (24+8) = 32 years = 15.802676
Deduct for 8 years .... = 6.463212

$$
\begin{array}{rcr}
 & & 9.339464 \\
 & & 80 \\
\hline
£747.\ 3s.\ 1\tfrac{1}{2}d. & = & 747.157120 \\
\hline
\end{array}
$$

EXERCISES.

1. Required the present value of an annuity of £100, payable yearly, for 60 years, to commence 40 years hence, reckoning interest at 3 per cent.

2. What annuity, payable yearly, for 15 years, to commence 5 years hence, may be purchased for £487. 19s. 3¼d., the purchaser being allowed 5 per cent?

3. If an annuity of £87. 10s., to commence at the expiration of 9 years, can be purchased for £500, required the time of its continuance, interest at 5 per cent.

4. A reversionary annuity of £20, to continue 12 years, may be purchased for £145. 16s. 8¾d.; when will it commence, reckoning interest at 5 per cent?

5. What annuity, payable half-yearly, for 21 years, and to commence 4 years hence, may be purchased for £500, reckoning interest at 4 per cent?

MISCELLANEOUS EXERCISES.

1. A person, on obtaining the lease of a farm, for 21 years, pays a fine, or grassum, of £500; what additional annual rent would have been equivalent to that sum, reckoning interest at 4½ per cent?

2. Whether is the reversion of an estate, to continue for ever, after the expiration of 20 years, or a lease of the same estate for 20 years, most advantageous, supposing the annual rent of the estate £600, and compound interest at 4 per cent?

3. A person obtained a lease of a farm for 21 years, 14 of which are to run; what ready money ought he to pay, in order to have 7 years added to the lease, the yearly rent being £300, and reckoning compound interest at 5 per cent?

4. A lease of an estate, to continue 19 years, is offered for £50 per annum, and £400 ready money; but it is proposed

to give an additional rent, instead of advancing the £400, what ought that addition to be?

5. In what time will a sinking fund of 4 millions extinguish a debt of 600 millions of pounds sterling, reckoning interest at $3\frac{1}{2}$ per cent?

6. How much of the national debt will a sinking fund of 5 millions of pounds sterling extinguish in 25 years; and what will the fund itself amount to, at the end of that period?

ANNUITIES ON LIVES.

LIFE ANNUITIES are payments made, at regular periods, during the life of one or more persons.

The value of these for any proposed life, or lives, depends on two circumstances; the *interest* of *money*, and the *probability* of the *duration* of the proposed life, or lives.

As the sale and purchase of Life Annuities have now become a considerable branch of business, it is hoped the following practical rules, for performing calculations of this description, will be found to include the most common cases that occur, and prove of considerable use to those who may be concerned in transactions of this kind*.

In order to facilitate computations, respecting Annuities on Lives, tables have been formed, for exhibiting the rate of mortality, at every age; and, consequently, the probability of *living* to any proposed age.

* Those who wish to be better informed on this subject, may consult Dr. Price on Annuities, and the Practical Calculator, by John Davidson, A.M.

There are two kinds of data, from which these tables may be formed. *One* is furnished by registers of mortality, showing the numbers that die, at all ages. The *other* is by the proportion of deaths, at all ages, to the number of living at those ages, discovered by surveys, or enumerations. (See Davidson's Practical Calculator, page 271.)

PROBLEM I.

TO FIND THE PROBABILITY THAT A PERSON, OF A GIVEN AGE, SHALL REACH ANY PROPOSED AGE.

This probability is expressed by a vulgar fraction, the numerator of which is the number of the living, in the table of observations opposite to the proposed age, and its denominator opposite to the present age of the given life. This fraction, subtracted from 1, leaves a fraction which expresses the probability that the given life shall *not* reach the proposed age.

EXAMPLE.

Required the probability that a man 36 years of age, shall reach 60, or live 24 years; and also the probability that he shall die, before he reach that age.

In Table VII. and opposite to 60, is 2701, and opposite to 36 is 1987; therefore, $\frac{2701}{4688}$ is the probability that a man 36, shall live 24 years, and $\frac{1987}{4688}$ the probability that he shall die in that time.

EXERCISES.

1. Required the probability which a man, 50 years of age, has of living 20 years; and also the probability that he shall die in that time.

2. Required the probability that a person, 40 years old, shall live to 60, and also the probability that the life shall not last so long.

3. Required the probability that a man, 35 years of age, has of living to 80.

PROBLEM II.

TO FIND THE PROBABILITY THAT TWO OR MORE PERSONS, WHOSE AGES ARE GIVEN, HAVE OF LIVING A PROPOSED NUMBER OF YEARS.

RULE.

The probability that any *number* of persons shall *all* live any proposed time, is found by multiplying together the probabilities that each of them shall live that time.

EXAMPLE.

Required the probability that a man 35, and a woman 24, shall both live 20 years?

$$\underset{\text{Man.}}{\frac{3191}{4748}} \times \underset{\text{Woman.}}{\frac{4441}{5636}} = \frac{1417}{2676} = .5295 \text{ Answer.}$$

EXERCISES.

1. Required the probability that a man, 28 years of age, and a woman 23, shall both live 30 years.

2. Required the probability that a man, 47 years of age, and a woman 37, shall both live 7 years.

3. Required the probability that a man, 75 years of age, and a woman 65, shall both live 7 years.

PROBLEM III.

TO FIND THE EXPECTATION OF LIFE*.

* The expectation of life is the number of years which mankind, taken one with another, enjoy, either from birth or any proposed age; the excess in the life of those who survive it, being exactly equal to the deficiency in the life of those who do not reach it.

RULE.

Divide the sum of all the living, in the table of observations, at the given age and upwards, by the number of the living at that age, and from the quotient subtract half a year, the remainder is the expectation of life.

EXAMPLE.

Required the expectation of a male life, aged 70.

$$12485 \div 1541 = 8.108 - .5 = 7.6 \text{ Expectation.}$$

The expectation of life may also be found, in Table VII. by inspection.

EXERCISES.

1. Required the expectation of a male life, aged 38.

2. Required the expectation of a female life, aged 20.

3. Required the expectation of a male life, just born.

PROBLEM IV.

TO FIND THE VALUE OF AN ANNUITY OF £1, ON A GIVEN LIFE.

RULE.

Look in Table VIII. for the given age, and opposite to it will be found the value of the given life.

EXAMPLE.

Required the value of a life of 36 years, interest at 4 per ct.

Opposite to 36, in Table VIII. is 14.939, the value of the given age.

EXERCISES.

1. Required the value of a male life of 60 years, interest at 4 per cent.

2. Required the value of a female life of 60 years, interest at 4 per cent.

3. Required the value of a male life of 70, interest at 4 per cent.

4. Required the value of a life of 40 years, interest at 4 per cent.

PROBLEM V.

TO FIND THE VALUE OF TWO JOINT LIVES*.

RULE.

If the two given lives are to be found in the table, the value of their joint continuance will be found opposite to the given ages.

When the given ages are not to be found in the table, proceed as follows: if one of the ages be found in the table, take out the value of it, and the one in the table next greater than the other given age, and also the one that is next less; subtract the one from the other, then increase or diminish the value of the two nearest ages, found in the table, by a proportional part of this difference, according as the value of the given ages ought to be more or less valuable.

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* To find the value of two joint lives correctly, being a very laborious operation, and to accomplish this, for all ages, by means of tables, requiring a very extensive table, the one inserted in this work only shows the value when the ages are *equal*, or when their difference is 6 years, which, it is hoped, will include a considerable number of cases that frequently occur.

EXAMPLE.

Required the value of two joint lives, whose respective ages are 23 and 26.

Value of 2 joint lives, 23 and 23	=	14.1946
Value of 2 ditto, ... 23 and 29	=	13.635
Difference... 6	=	.5596

26 — 23 = 3 ÷ 6 = ½ of .5596	=	.2793
Value of 2 lives of 23 and 29	=	13.635
Value of 2 joint lives of 23 and 26	=	13.9143

EXERCISES.

1. Required the value of 2 joint lives, whose ages are 26 and 30.

2. Required the value of 2 joint lives, whose ages are 36 and 44.

3. Required the value of 2 joint lives, whose ages are 48 and 54.

PROBLEM VI.

TO FIND THE VALUE OF AN ANNUITY OF £1, ON THE LONGEST OF ANY TWO LIVES.

RULE.

From the sum of the values of the single lives, subtract the value of their joint continuance, and the remainder will be the value of the longest of the two lives.

EXAMPLE.

Required the value of the longest of two lives, aged 10 and 16 years, interest at 4 per cent.

Value of a life of 10 years, by Table VIII,	=	18.891
Ditto, ... of 16 years, ...	=	18.191
Sum,.........	=	37.082
Value of 2 joint lives, aged 10 and 16...	=	15.729
Value required...	=	21.353

EXERCISES.

1. Required the value of the longest of two lives, aged 32 and 38.

2. Required the value of the longest of two lives, aged 30 and 40.

3. Required the value of the longest of two lives, aged 17 and 29.

PROBLEM VII.

TO FIND WHAT SUM IN A SINGLE PRESENT PAYMENT, OR IN ANNUAL PAYMENTS, DURING THE CONTINUANCE OF MARRIAGE, A HUSBAND OUGHT TO PAY IN ORDER TO ENTITLE HIS WIDOW TO A GIVEN ANNUITY FOR HER LIFE.

RULE.

Subtract the value of the joint lives from the value of the wife's life, and the remainder is the value of what may remain of her life, after her husband's death, which, multiplied by the given annuity, gives the single present payment required.

The single payment, divided by the value of the joint lives, increased by unity, gives the annual payment*.

EXAMPLE.

What sum of money, in a single payment, or in annual pay-

* If the annual payments are to continue during the husband's *life*, the value in a single payment should be divided by the value of the husband's life, increased by unity, to find the annual payment.

ments, during marriage, ought a husband, aged 26, to pay, to entitle his wife, aged 23, to an annuity of £50, for what may happen to remain of her life, after his death; interest at 4 per cent?

Value of the wife's life	=	17.150
Value of 2 joint lives, 23 and 26	=	13.914
Value of what remains of wife's life		3.236
		50

Single payment... £161.800, which, divided by 14.914 = £10. 16s. 11¼, annual payment.

EXERCISES.

1. What yearly payment, to continue during his life, ought a husband to pay, to entitle his widow to an annuity of £60. for her life; his age, as well as that of his wife's, being 29 years?

2. What sum ought a husband to pay, to entitle his widow to an annuity of £40. per annum, for her life; the husband's age being 30, and his wife's 24?

PROBLEM VIII.

TO FIND THE VALUE IN READY MONEY, AND ALSO IN ANNUAL PAYMENTS, TILL A PERSON ATTAINS A GIVEN AGE, OF AN ANNUITY, FOR WHAT MAY REMAIN OF HIS LIFE AFTER THAT AGE.

RULE.

Multiply the value of the life, when the annuity is to commence, by the present value of £1, to be received at the end of a number of years, equal to the difference between the person's present age and the age at which the annuity commences; and also by the probability that such life will continue so long, and the product is the number of years purchase that ought to be paid for the annuity, which, multiplied by the given annuity, is its value in ready money. This value, divided by the value of

the life, for one year less than the proposed term, increased by unity, gives the annual payment*.

EXAMPLE.

A man, 34 years of age, wishes to purchase an annuity of £40, for what may remain of his life after 55 years of age; required the value of that annuity, in *ready money*, and also in annual payments, till he attain that age.

Value of a life of 55 years.............................. $=$ 10.320
Present value of £1, payable 21 years hence... $=$ .438833
Probability that a person, 34 years, will live 21 years $= \frac{1595}{2403}$
therefore, $10.32 \times .438833 \times \frac{1595}{2403} = 3.00567$ years purchase; and $3.00567 \times 40 =$ £120.2268 $=$ £120. 4s. 6¼d., value in ready money.

TO FIND THE ANNUAL PAYMENTS.

Value of a life of 54 years .... $=$ 10.603
Present value of £1, payable 20 years hence... $=$ .456387
Probability that a person, 34 yrs. will live 20 yrs. $= \frac{3286}{4806} = \frac{1643}{2403}$
therefore, $10.603 \times .453687 \times \frac{1643}{2403} = 3.311815.$

Value of a life of 34 years..................	15.321
Ditto, after a term of 20 years	3.311815
Ditto, for a term of 14 years...............	12.009185

$120.2268 \div 13.009185 =$ £9.2456 $=$ £9. 4s. 10d. nearly.

EXERCISES.

1. A husband wishes to purchase an annuity of £20 for his wife, aged 25 years, to commence when she is 55 years of age; how much ready money ought he to pay for the annuity?

2. What sum ought a man, 20 years of age, to pay weekly, till he is 65 years old, to entitle him to a weekly payment of 7s. for life, after he attains to that age?

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* The value of a life, for a term of years, is obtained by deducting its value, *after* the *term*, from its value at the *commencement* of the *term*.

PROBLEM IX.

TO FIND WHAT SUM IN READY MONEY, OR IN ANNUAL PAYMENTS, DURING LIFE, A PERSON OF A GIVEN AGE OUGHT TO PAY, FOR AN ASSURANCE OF ANY GIVEN SUM ON HIS LIFE; OR, WHICH IS THE SAME THING, FOR A SUM OF MONEY TO BE PAID TO HIS HEIRS AT HIS DEATH.

RULE.

Subtract the value of the life from the perpetuity (which is 100, divided by the rate per cent) multiply the remainder by the product of the given sum, into the interest of £100 for a year; and this last product, divided by £100, increased by its interest for a year, gives the single present payment, which, divided by the value of the life, increased by unity, gives the annual payment*.

EXAMPLE.

What sum in ready money, or in annual payments, during life, ought a man, 40 years of age, to pay for an assurance of £1000 on his life, interest 4 per cent?

$\frac{100}{4} = \frac{25 - 13.669 \times 4000}{104} = 435.8877 = £435.\ 16s.\ 1\frac{1}{4}d.$ single payment; and £435. 16s. $1\frac{1}{4}$d. ÷ 14.669 = £29. 14s. $2\frac{1}{4}$d. annual payment.

If the reversion is an *annuity* for ever, or an estate in *fee simple*, to be entered upon after a given life, its present value, in a *single* payment, will be the value of the life subtracted from the perpetuity, and the remainder multiplied by the annual rent of the estate; and the value, in *annual* payments will be, as before, the single payment, divided by the value of the life, increased by unity.

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* If the payments are to be made *half-yearly*, the single payment *must* be divided by the value of the life, increased by *seven-tenths*.

Universally, the value of a reversionary *estate*, after any given life, or lives, is to the value of a corresponding reversionary *sum;* as £100 *plus* its interest for a year, is to £100.

EXERCISES.

1. What sum, in ready money, or in annual payments during life, ought a man, 30 years of age, to pay for an assurance of £500 on his life?

2. What sum, in ready money, or in annual payments, ought to be given for an estate of £100 a year; to be entered upon after the life of a man, 30 years of age?

PROBLEM X.

TO FIND THE PREMIUM OF ASSURANCE FOR A GIVEN SUM, FOR A TERM OF YEARS, ON A GIVEN LIFE.

RULE.

Divide the value of the life for one year less than the given term increased by unity, by £1 increased by its interest for a year; from the quotient subtract the value of the life for the given term, and the remainder, multiplied by the given sum, will be the value required.

If the single premium be divided by the value of the life increased by unity, for one year less than the given term, the quotient will be the equivalent annual payment*.

EXAMPLE.

Required the value of an assurance of £50 in a single payment, and also in annual payments, for a term of 7 years, on the life of a person, 40 years of age; interest 4 per cent.

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* The premium for a given *sum*, multiplied by £1 increased by its interest for a year, gives the premium for an equivalent annuity for ever.

Value of a life of 40 years	14.034
Ditto, after 7 years = 12.472 × [illegible] × .7599178	8.333
Value of ditto for 7 years	5.7005
Value of ditto, after 6 years = 12.724 × [illegible] × .790314	9.0225
Value of ditto for 6 years	5.0125

$$1 + 5.012 = \frac{6.0125}{1.04} = 5.703 - 5.7005 = .0798$$

$$\begin{array}{r} 50 \\ \hline 3.9900 \end{array} =$$

$$£3\ 19\ 9\tfrac{1}{2} \div 6.0125 = .664 = 13s.\ 3\tfrac{1}{2}d.$$

EXERCISES.

1. Required the value of an assurance of £500 in a single and in annual payments, for a term of 5 years, on the life of a person 16 years of age.

2. Required the value of £50 per annum for ever; to be entered upon at the death of a person 30 years of age, should that happen in 15 years.

3. Required the value of an assurance of £100 in a single and also in annual payments, for a term of 10 years, on the life of a person 50 years of age.

TABLE I.

Showing the Amount of £1 for Years.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.03	1.035	1.04	1.045	1.05
2	1.0609	1.071225	1.0816	1.092025	1.1025
3	1.092727	1.108717	1.124864	1.141166	1.157625
4	1.125508	1.147523	1.169858	1.192518	1.215506
5	1.159274	1.187686	1.216652	1.246181	1.276281
6	1.194052	1.229255	1.265319	1.302260	1.340095
7	1.229873	1.272279	1.315931	1.360861	1.407103
8	1.266770	1.316808	1.368569	1.422100	1.477455
9	1.304773	1.362897	1.423211	1.486095	1.551328
10	1.343916	1.410598	1.480244	1.552969	1.628894
11	1.384233	1.459969	1.539454	1.622853	1.710339
12	1.425760	1.511068	1.601032	1.695881	1.795856
13	1.468533	1.563956	1.665073	1.772196	1.885649
14	1.512589	1.618694	1.731676	1.851944	1.979931
15	1.557967	1.675348	1.800943	1.935282	2.078928
16	1.604706	1.733986	1.872981	2.022370	2.182874
17	1.652847	1.794675	1.947900	2.113376	2.292018
18	1.702433	1.857489	2.025816	2.208478	2.406619
19	1.753506	1.922501	2.106849	2.307860	2.526950
20	1.806111	1.989788	2.191123	2.411714	2.653297
21	1.860294	2.059431	2.278768	2.520241	2.785962
22	1.916103	2.131511	2.369918	2.633652	2.925260
23	1.973586	2.206114	2.464715	2.752166	3.071523
24	2.032794	2.283328	2.563304	2.876013	3.22510
25	2.093777	2.363244	2.665836	3.005434	3.386354
26	2.156591	2.445958	2.772469	3.140679	3.555672
27	2.221289	2.531567	2.883368	3.282009	3.733456
28	2.287927	2.620171	2.998703	3.429699	3.920129
29	2.356565	2.711877	3.118651	3.584036	4.116135
30	2.427262	2.806793	3.243397	3.745318	4.321942
31	2.500080	2.905031	3.373133	3.913857	4.538039
32	2.575082	3.006707	3.508058	4 089981	4.764941
33	2.652335	3.111942	3.648381	4.274030	5.003188
34	2.731905	3.220860	3.794316	4.466361	5.253348
35	2.813862	3.333590	3.946088	4.667347	5.516015

TABLE I. *Continued.*

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
36	2.898278	3.450266	4.013932	4.877378	5.791816
37	2.985226	3.571025	4.268089	5.096860	6.081406
38	3.074783	3.696011	4.438813	5.326219	6.385477
39	3.167036	3.825371	4.616365	5.565899	6.704751
40	3.262037	3.959259	4.801020	5.816364	7.039988
41	3.359898	4.097833	4.993061	6.078100	7.391988
42	3.460695	4.241257	5.192783	6.351615	7.761587
43	3.564516	4.389702	5.400495	6.637438	8.149666
44	3.671452	4.543341	5.616515	6.936193	8.557150
45	3.781595	4.702358	5.841175	7.248248	8.985007
46	3.895043	4.866941	6.074822	7.574419	9.434258
47	4.011895	5.037284	6.317815	7.915268	9.905971
48	4.132251	5.213588	6.570528	8.271455	10.401269
49	4.256219	5.396064	6.833349	8.643671	10.921333
50	4.383906	5.584926	7.106683	9.032636	11.46740

TABLE II.

Showing the Amount of £1 for Days.

Dys.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.000080	1.000094	1.000107	1.000120	1.000133
2	.000161	.000188	.000214	.000241	.000267
3	.000242	.000282	.000322	.000361	.000401
4	.000324	.000377	.000429	.000482	.000534
5	.000405	.000471	.000537	.000603	.000668
6	1.000486	1.000565	1.000644	1.000723	1.000802
7	.000567	.000660	.000752	.000844	.000936
8	.000648	.000754	.000860	.000965	.001069
9	.000729	.000848	.000967	.001085	.001203
10	.000810	.000942	.001075	.001206	.001337
20	1.001620	1.001886	1.002151	1.002414	1.002677
30	.002432	.002831	.003228	.003624	.004018
40	.003244	.003777	.004307	.004835	.005361
50	.004057	.004723	.005387	.006047	.006705
60	.004870	.005671	.006468	.007261	.008052

TABLE II. *Continued.*

Dys.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
70	1.005684	1.006619	1.007550	1.008477	1.009400
80	.006499	.007568	.008633	.009694	.010751
90	.007315	.008518	.009717	.010912	.012103
100	.008131	.009469	.010803	.012132	.013456
120	.009765	.011374	.012977	.014576	.016169
140	1.011402	1.013282	1.015157	1.017026	1.018890
160	.013041	.015194	.017341	.019482	.021617
180	.014683	.017109	.019529	.021944	.024352
200	.016328	.019028	.021723	.024412	.027094
220	.017975	.020951	.023921	.026885	.029844
240	1.019626	1.022877	1.026124	1.029365	1.032601
260	.021278	.024807	.028331	.031851	.035365
280	.022934	.026741	.030544	.034342	.038137
300	.024592	.028638	.032761	.036840	.040916
320	.026253	.030619	.034983	.039344	.043702
340	.027916	.032564	.037209	.041854	.046496
360	.029583	.034512	.039441	.044370	.049298
365	.03	.035	.04	.045	.05

TABLE III.

Showing the present Worth of £1 for Years.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	.970873	.966183	.961538	.956937	.952381
2	.942595	.933510	.924556	.915729	.907029
3	.915141	.901942	.888996	.876296	.863837
4	.888487	.871442	.854804	.838561	.822702
5	.862608	.841973	.821927	.802451	.783526
6	.837484	.813500	.790314	.767895	.746215
7	.813091	.785991	.759917	.734828	.710681
8	.789409	.759411	.730690	.703185	.676839
9	.766416	.733731	.702586	.672904	.644608
10	.744093	.708918	.675564	.643927	.613913
11	.722421	.684945	.649580	.616198	.584679
12	.701379	.661783	.624597	.589663	.556837
13	.680951	.639404	.600574	.564271	.530321
14	.661117	.617781	.577475	.539972	.505067
15	.641861	.596190	.555264	.516720	.481017

TABLE III. *Continued.*

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
16	.623166	.576705	.533908	.494469	.458111
17	.605016	.557203	.513373	.473176	.436296
18	.587394	.538361	.493628	.452800	.415520
19	.570286	.520155	.474642	.433301	.395734
20	.553675	.502565	.456387	.413642	.376889
21	.537549	.485570	.438833	.396787	.358942
22	.521892	.469150	.421955	.379700	.341849
23	.506691	.453285	.405726	.363350	.325571
24	.491933	.437957	.390121	.347703	.310067
25	.477605	.423147	.375116	.332730	.295302
26	.463694	.408837	.360689	.318402	.281240
27	.450189	.395012	.346816	.304691	.267848
28	.437076	.381654	.333477	.291570	.255093
29	.424346	.368748	.320651	.279015	.242946
30	.411986	.356278	.308318	.267000	.231377
31	.399987	.344230	.296460	.255502	.220359
32	.388337	.332589	.285057	.244499	.209866
33	.377026	.321342	.274094	.233971	.199872
34	.366044	.310476	.263552	.223895	.190354
35	.355383	.299976	.253415	.214254	.181290
36	.345032	.289832	.243668	.205028	.172657
37	.334982	.280031	.234296	.196199	.164435
38	.325226	.270561	.225285	.187750	.156605
39	.315753	.261412	.216620	.179665	.149147
40	.306556	.252572	.208289	.171928	.142045
41	.297628	.244031	.200277	.164525	.135281
42	.288959	.235779	.192574	.157440	.128839
43	.280542	.227805	.185168	.150660	.122704
44	.272371	.220102	.178046	.144172	.116861
45	.264438	.212659	.171198	.137964	.111296
46	.256736	.205467	.164613	.132023	.105996
47	.249258	.198519	.158282	.126338	.100949
48	.241998	.191806	.152194	.120897	.096142
49	.234950	.185320	.146341	.115691	.091563
50	.228107	.179053	.140712	.110709	.087203

TABLE IV.

Showing the Amount of £1 Annuity for Years.

Yrs.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.	1.	1.	1.
2	2.035	2.04	2.045	2.05
3	3.106225	3.1216	3.137025	3.1525
4	4.214942	4.246464	4.278191	4.310125
5	5.362465	5.416322	5.470709	5.525631
6	6.550152	6.632975	6.716891	6.801912
7	7.779407	7.898294	8.019151	8.142008
8	9.051686	9.214226	9.380013	9.549108
9	10.368495	10.582795	10.802114	11.026564
10	11.731393	12.006107	12.288209	12.577892
11	13.141991	13.486351	13.841178	14.206787
12	14.601961	15.025805	15.464031	15.917126
13	16.113030	16.626837	17.159913	17.712982
14	17.676986	18.291911	18.932109	19.598032
15	19.295680	20.023587	20.784054	21.578563
16	20.971029	21.824531	22.719336	23.657491
17	22.705015	23.697512	24.741706	25.840366
18	24.499691	25.645412	26.855083	28.132384
19	26.357180	27.671229	29.063562	30.539003
20	28.279681	29.778078	31.371422	33.065954
21	30.269470	31.969201	33.783136	35.719251
22	32.328902	34.247969	36.303377	38.505214
23	34.460413	36.617888	38.937029	41.430475
24	36.666528	39.082604	41.689196	44.501998
25	38.949856	41.645908	44.565210	47.727098
26	41.313101	44.311744	47.570644	51.113453
27	43.759060	47.084214	50.711323	54.669126
28	46.290627	49.967583	53.993332	58.402582
29	48.910799	52.966286	57.423033	62.322711
30	51.622677	56.084937	61.007069	66.438847
31	54.429471	59.328335	64.752387	70.760789
32	57.334502	62.701468	68.666245	75.298829
33	60.341210	66.209527	72.756226	80.063770
34	63.453152	69.857908	77.030256	85.066959
35	66.674012	73.652224	81.496618	90.320307

TABLE IV. *Continued.*

Yrs.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
36	70.007603	77.598313	86.163965	95.836322
37	73.457869	81.702246	91.041344	101.628138
38	77.028894	85.970336	96.138204	107.709545
39	80.724906	90.409149	101.464424	114.095023
40	84.550277	95.025515	107.030323	120.799774
41	88.509537	99.826536	112.846687	127.839762
42	92.607371	104.819597	118.924788	135.231751
43	96.848629	110.012381	125.276404	142.993338
44	101.238331	115.412876	131.913842	151.143005
45	105.781672	121.029392	138.849965	159.700155
46	110.484031	126.870567	146.098213	168.685163
47	115.350972	132.945390	153.672633	178.119421
48	120.388256	139.263206	161.587901	188.025392
49	125.601845	145.833734	169.859357	198.426662
50	130.997910	152.667083	178.503028	209.347995

TABLE V.

Showing the present Worth of £1 Annuity for Years.

Yrs.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	0.966183	0.961538	0.956937	0.952380
2	1.899694	1.886094	1.872667	1.859410
3	2.801637	2.775091	2.748964	2.723248
4	3.673079	3.629895	3.587525	3.545950
5	4.515052	4.451822	4.389976	4.329476
6	5.328553	5.242136	5.157872	5.075692
7	6.114543	6.002054	5.892700	5.786373
8	6.873955	6.732744	6.595886	6.463212
9	7.607686	7.435331	7.268790	7.107821
10	8.316605	8.110895	7.912718	7.721734
11	9.001551	8.760476	8.528916	8.306414
12	9.663334	9.385073	9.118580	8.863251
13	10.302738	9.985647	9.682852	9.393573
14	10.920520	10.563122	10.222825	9.898640
15	11.517410	11.118386	10.739545	10.379658

TABLE V. *Continued.*

Yrs.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
16	12.094116	11.652294	11.234015	10.837769
17	12.651320	12.165668	11.707191	11.274066
18	13.189681	12.659296	12.159991	11.689586
19	13.709837	13.133938	12.593293	12.085310
20	14.212403	13.590325	13.007936	12.462210
21	14.697974	14.029158	13.404723	12.821152
22	15.167124	14.451114	13.784424	13.163002
23	15.620410	14.856840	14.147774	13.488573
24	16.058367	15.246961	14.495478	13.798641
25	16.481514	15.622078	14.828208	14.093944
26	16.890352	15.982767	15.146611	14.375185
27	17.285364	16.329584	15.451302	14.643033
28	17.667018	16.663061	15.742873	14.898127
29	18.035767	16.983713	16.021888	15.141073
30	18.392045	17.292031	16.288888	15.372451
31	18.736275	17.588492	16.544390	15.592810
32	19.068865	17.87355	16.788890	15.802676
33	19.390208	18.147644	17.022862	16.002549
34	19.700684	18.411196	17.246758	16.192903
35	20.000661	18 664611	17.461012	16.374194
36	20.290493	18.908280	17.666040	16.546851
37	20.570525	19.142577	17.862239	16.711287
38	20.841087	19.367862	18.049990	16.867892
39	21.102499	19.584483	18.229655	17.017040
40	21.355072	19.792772	18.401584	17.159086
41	21.599103	19.99305	18.566109	17.294367
42	21.834882	20.185625	18.723549	17.423207
43	22.062688	20.370793	18.874210	17.545911
44	22.282791	20.548839	19.018383	17.662773
45	22.495450	20.720037	19.156347	17.774069
46	22.700918	20.884651	19.288370	17.880066
47	22.899437	21.042934	19.414708	17.981015
48	23.091244	21.195128	19.535606	18.077157
49	23.276564	21.34147	19.651298	18.168721
50	23.455617	21.482182	19.762007	18.255925

TABLE VI.

Showing the Annuity which £1 will purchase.

Yrs.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.
1	1.03	1.035	1.04	1.045	1.05
2	.522610	.526400	.530196	.533997	.537804
3	.353530	.356934	.360348	.363773	.367208
4	.269027	.272251	.275490	.278743	.282011
5	.218354	.221481	.224627	.227791	.230974
6	.184597	.187668	.190761	.193878	.197015
7	.160506	.163544	.166609	.169701	.172819
8	.142456	.145476	.148527	.151609	.154721
9	.128433	.131446	.134493	.137574	.140690
10	.117230	.120241	.123290	.126378	.129504
11	.108077	.111092	.114149	.117248	.120389
12	.100462	.103484	.106552	.109666	.112825
13	.094029	.097061	.100143	.103275	.106455
14	.088526	.091570	.094669	.097820	.101024
15	.083766	.086825	.089941	.093113	.096342
16	.079610	.082684	.085820	.089015	.092269
17	.075952	.079043	.082198	.085417	.088699
18	.072708	.075816	.078993	.082236	.085546
19	.069813	.072940	.076138	.079407	.082745
20	.067215	.070361	.073581	.076876	.080243
21	.064871	.068036	.071280	.074600	.077996
22	.062747	.065932	.069198	.072545	.075970
23	.060813	.064018	.067309	.070682	.074136
24	.059047	.062272	.065586	.068987	.072470
25	.057427	.060674	.064012	.067439	.070952
26	.055938	.059205	.062567	.066021	.069564
27	.054564	.057852	.061238	.064719	.068291
28	.053293	.056602	.060013	.063520	.067122
29	.052114	.055445	.058879	.062414	.066045
30	.051019	.054371	.057830	.061391	.065051
31	.049998	.053372	.056855	.060443	.064132
32	.049046	.052441	.055948	.059563	.063280
33	.048156	.051572	.055103	.058744	.062490
34	.047322	.050759	.054314	.057981	.061755
35	.046539	.049998	.053577	.057270	.061071

TABLE VI. *Continued.*

Yrs.	3 per cent	3½ per cent	4 per cent	4½ per cent	5 per cent.
36	.045803	.049284	.052886	.056605	.060434
37	.045111	.048613	.052239	.055984	.059839
38	.044459	.047982	.051631	.055401	.059284
39	.043843	.047387	.051060	.054855	.058764
40	.043262	.046827	.030523	.054343	.058278
41	.042712	.046298	.050017	.053861	.057822
42	.042191	.045798	.049540	.053408	.057394
43	.041698	.045325	.049089	.052982	.056993
44	.041229	.044877	.048664	.052580	.056616
45	.040785	.044453	.048262	.052202	.056261
46	.040362	.044051	.047882	.051844	.055928
47	.039960	.043669	.047521	.051507	.055614
48	.039577	.043306	.047180	.051188	.055318
49	.039213	.042961	.046857	.050887	.055039
50	.038865	.042633	.046550	.050602	.054776

TABLE VII.

Showing the probabilities of the duration of Human Life, among Males and Females; deduced from observations of the proportions of the living, to the numbers who have died at all ages, for 21 years, from 1755, to 1776, in the kingdom of Sweden.

	MALES. Born 10282—282 born dead.			FEMALES. Born 10217—217 born dead.		
Ages.	Living.	Decr.	Expec.	Living.	Decr.	Expec.
Born alive	10000	2300	33.2	10000	2090	35.7
1	7700	500	42.45	7910	518	44.
2	7200	337	43.83	7392	350	46.05
3	6863	240	44.96	7042	250	47.31
4	6623	150	45.57	6792	135	48.04
5	6473	125	45.62	6657	120	48.
6	6348	105	45.5	6537	105	47.87

TABLE VII. *Continued.*

	MALES.			FEMALES.		
Ages.	Living.	Decr.	Expec.	Living.	Decr.	Expec.
7	6243	90	45.26	6432	85	47.64
8	6153	75	44.91	6347	70	47.28
9	6078	65	44.46	6277	60	46.8
10	6013	55	43.94	6217	52	46.25
11	5958	45	43.26	6165	46	45.55
12	5913	45	42.58	6119	40	44.85
13	5868	40	41.91	6079	35	44.15
14	5828	40	41.24	6044	35	43.46
15	5788	39	40.56	6009	35	42.76
16	5749	39	39.83	5974	40	42.04
17	5710	39	39.11	5934	40	41.31
18	5671	44	38.39	5894	42	40.59
19	5627	44	37.67	5852	43	39.87
20	5583	50	36.95	5809	43	39.15
21	5533	50	36.28	5766	43	38.43
22	5483	50	35.62	5723	43	37.72
23	5433	55	34.96	5680	44	37.01
24	5378	55	34.3	5636	45	36.29
25	5323	55	33.63	5591	45	35.58
26	5268	55	32.98	5546	50	34.9
27	5213	55	32.32	5496	52	34.21
28	5158	55	31.66	5444	55	33.53
29	5103	56	31.	5389	55	32.85
30	5047	59	30.34	5334	60	32.17
31	4988	60	29.69	5274	60	31.54
32	4928	60	29.04	5214	65	30.91
33	4868	60	28.39	5149	65	30.28
34	4808	60	27.74	5084	65	29.66
35	4748	60	27.09	5019	60	29.03
36	4688	60	26.43	4959	56	28.26
37	4628	60	25.76	4903	56	27.50
38	4568	60	25.09	4847	56	26.74
39	4508	60	25.42	4791	58	25.97
40	4448	65	23.75	4733	65	25.21
41	4383	72	23.15	4668	75	24.68
42	4311	80	22.54	4593	76	24.75
43	4231	80	21.93	4517	76	23.62
44	4151	80	21.32	4441	75	23.1
45	4071	80	20.71	4366	72	22.57
46	3991	80	20.12	4294	67	21.91
47	3911	80	19.52	4227	65	21.24
48	3831	80	18.92	4162	65	20.58

TABLE VII. *Continued.*

	MALES.			FEMALES.		
Ages.	Living.	Decr.	Expect.	Living.	Decr.	Expect.
49	3751	85	18.32	4097	70	19.92
50	3666	95	17.72	4027	75	19.26
51	3571	95	17.17	3952	80	18.64
52	3476	95	16.63	3872	85	18.01
53	3381	95	16.08	3787	85	17.39
54	3286	95	15.53	3702	85	16.77
55	3191	95	14.98	3617	85	16.15
56	3096	95	14.43	3532	85	15.53
57	3001	100	13.87	3447	90	14.92
58	2901	100	13.33	3357	90	14.31
59	2801	100	12.79	3267	100	13.69
60	2701	105	12.24	3167	110	13.08
61	2596	110	11.72	3057	118	12.56
62	2486	115	11.21	2939	120	12.04
63	2371	115	10.73	2819	120	11.52
64	2256	115	10.26	2699	120	11.01
65	2141	115	9.78	2579	120	10.49
66	2026	115	9.3	2459	120	9.97
67	1911	120	8.84	2339	120	9.46
68	1791	125	8.4	2219	120	8.94
69	1666	125	7.99	2099	120	8.42
70	1541	125	7.6	1979	130	7.91
71	1416	125	7.22	1849	140	7.53
72	1291	120	6.87	1709	150	7.16
73	1171	120	6.53	1559	160	6.73
74	1051	110	6.22	1399	150	6.4
75	941	105	5.89	1249	140	6.03
76	836	100	5.56	1109	130	5.73
77	736	90	5.25	979	120	5.43
78	646	85	4.92	859	110	5.11
79	561	80	4.59	749	100	4.79
80	481	75	4.27	649	95	4.46

TABLE VIII.

Showing the values of Annuities on SINGLE LIVES, *according to the probabilities of the duration of life in the kingdom of Sweden.—Interest 4 per cent.*

Ages.	Lives in Gen.	Ages.	Lives in Gen.	Ages.	Lives in Gen.
1	16.661	3	18.139	5	18.715
2	17.537	4	18.554	6	18.833

TABLE VIII. *Continued.*

Ages.	Lives in Gen.	Ages.	Lives in Gen.	Ages.	Lives in Gen.
7	18.912	32	15.668	57	9.718
8	18.943	33	15.497	58	9.416
9	18.933	34	15.321	59	9.101
10	18.891	35	15.138	60	8.789
11	18.820	36	14.939	61	8.490
12	18.721	37	14.726	62	8.201
13	18 609	38	14.504	63	7.917
14	18.476	39	14.272	64	7.626
15	18.336	40	14.034	65	7.328
16	18.191	41	13.805	66	7.022
17	18.046	42	13.595	67	6.709
18	17.897	43	13.391	68	6.398
19	17.752	44	13.157	69	6.093
20	17.603	45	12.929	70	5.783
21	17.458	46	12.749	71	5.491
22	17.307	47	12.472	72	5.220
23	17 150	48	12.217	73	4.969
24	16.997	49	11.930	74	4.758
25	16.839	50	11.658	75	4.534
26	16.675	51	11.399	76	4.310
27	16.512	52	11.138	77	4.084
28	16.346	53	10.875	78	3.840
29	16.178	54	10 603	79	3.590
30	16.006	55	10.320	80	3.331
31	15.839	56	10.025	81	3.081

TABLE IX.

Showing the Values of Annuities on Two Joint Lives, *according to the probabilities of the duration of life, in Table* VIII.—*Interest at 4 per cent.*

Difference of Age 0.

Ages.	Values.	Ages.	Values.	Ages.	Values.
1, 1	12.2533	11, 11	16.0878	21, 21	14.5254
2, 2	13.5835	12, 12	15 9826	22, 22	14.3610
3, 3	14.5595	13, 13	15.8567	23, 23	14.1946
4, 4	15.2683	14, 14	15.7025	24, 24	14.0209
5, 5	15.5785	15, 15	15.5361	25, 25	13.8502
6, 6	15.8211	16, 16	15.3622	26, 26	13.6718
7, 7	16.0037	17, 17	15.1969	27, 27	13.4961
8, 8	16.1104	18, 18	15.0243	28, 28	13.3234
9, 9	16.1532	19, 19	14.8547	29, 29	13.1484
10, 10	16.1417	20, 20	14.6830	30, 30	12.7965

TABLE IX. *Continued.*

Ages.	Values.	Ages.	Values.	Ages.	Values.
31, 31	12.7961	48, 48	9.2363	65, 65	4.8813
32, 32	12.6247	49, 49	8.9665	66, 66	4.6265
33, 33	12.4566	50, 50	8.7072	67, 67	4.3626
34, 34	12.2870	51, 51	8.4695	68, 68	4.1036
35, 35	12.1102	52, 52	8.2303	69, 69	3.8515
36, 36	11.9045	53, 53	7.9948	70, 70	3.5937
37, 37	11.6838	54, 54	7.7485	71, 71	3.3456
38, 38	11.4523	55, 55	7.4952	72, 72	3.1288
39, 39	11.2094	56, 56	7.2292	73, 73	2.9351
40, 40	10.9644	57, 57	6.9541	74, 74	2.7972
41, 41	10.7329	58, 58	6.6787	75, 75	2.6486
42, 42	10.5312	59, 59	6.3882	76, 76	2.4903
43, 43	10.3464	60, 60	6.1050	77, 77	2.3400
44, 44	10.1545	61, 61	5.8445	78, 78	2.1700
45, 45	9.9548	62, 62	5.6008	79, 79	1.9671
46, 46	9.7363	63, 63	5.3677	80, 80	1.7589
47, 47	9.4974	64, 64	5.1281	81, 81	1.6005

Difference of Age Six Years.

Ages.	Values.	Ages.	Values.	Ages.	Values.
1, 7	13.989	28, 34	12.763	55, 61	6.555
2, 8	14.780	29, 35	12.586	56, 62	6.299
3, 9	15.323	30, 36	12.390	57, 63	6.045
4, 10	15.685	31, 37	12.192	58, 64	5.788
5, 11	15.817	32, 38	11.988	59, 65	5.519
6, 12	15.887	33, 39	11.779	60, 66	5.249
7, 13	15.914	34, 40	11.568	61, 67	4.984
8, 14	15.888	35, 41	11.361	62, 68	4.729
9, 15	15.824	36, 42	11.156	63, 69	4.482
10, 16	15.729	37, 43	10.953	64, 70	4.231
11, 17	15.617	38, 44	10.741	65, 71	3.982
12, 18	15.477	39, 45	10.519	66, 72	3.750
13, 19	15.327	40, 46	10.286	67, 73	3.527
14, 20	15.164	41, 47	10.049	68, 74	3.340
15, 21	15.001	42, 48	9.813	69, 75	3.147
16, 22	14.832	43, 49	9.581	70, 76	2.946
17, 23	14.665	44, 50	9.351	71, 77	2.752
18, 24	14.491	45, 51	9.129	72, 78	2.558
19, 25	14.320	46, 52	8.897	73, 79	2.355
20, 26	14.144	47, 53	8.658	74, 80	2.172
21, 27	13.976	48, 54	8.402	75, 81	2.017
22, 28	13.807	49, 55	8.139	76, 82	1.877
23, 29	13.635	50, 56	7.874	77, 83	1.756
24, 30	13.455	51, 57	7.613	78, 84	1.639
25, 31	13.284	52, 58	7.351	79, 85	1.524
26, 32	13.108	53, 59	7.083	80, 86	1.416
27, 33	12.935	54, 60	6.814	81, 87	1.320

A

TABLE OF LOGARITHMS,

OF NATURAL NUMBERS,

FROM 1 TO 10,000.

LOGARITHMS.

N.	Logar.	N.	Logar.	N.	Logar.
1	0.000000	34	1.531479	67	1.826075
2	301030	35	544068	68	832509
3	477121	36	556302	69	838849
4	602060	37	568202	70	845098
5	698970	38	579784	71	851258
6	778151	39	591065	72	857332
7	0.845098	40	1.602060	73	1.863323
8	903090	41	612784	74	869232
9	954242	42	623249	75	875061
10	1.000000	43	633468	76	880814
11	041393	44	643453	77	886491
12	079181	45	653212	78	892095
13	1.113943	46	1.662758	79	1.897627
14	146128	47	672098	80	903090
15	176091	48	681241	81	908485
16	204120	49	690196	82	913814
17	230449	50	698970	83	919078
18	255273	51	707570	84	924279
19	278754	52	716003	85	929419
20	1.301030	53	1.724276	86	1.934498
21	322219	54	732394	87	939519
22	342423	55	740363	88	944483
23	361728	56	748188	89	949390
24	380211	57	755875	90	954242
25	397940	58	763428	91	959041
26	414973	59	770852	92	963788
27	1.431364	60	1.778151	93	1.968483
28	447158	61	785330	94	973128
29	462398	62	792392	95	977724
30	477121	63	799340	96	982271
31	491362	64	806180	97	986772
32	505150	65	812913	98	991226
33	518514	66	819544	99	995635

Num.	0	1	2	3	4
100	2.000000	2.000434	2.000868	2.001301	2.001734
101	004321	004751	005180	005609	006038
102	008600	009026	009451	009876	010300
103	012837	013259	013680	014100	014520
104	017033	017451	017868	018284	018700
105	021189	021603	022016	022428	022841
106	025306	025715	026124	026533	026942
107	029384	029789	030195	030600	031004
108	033424	033826	034227	034628	035029
109	037426	037825	038223	038620	039017
110	2.041393	2.041787	2.042182	2.042575	2.042969
111	045323	045714	046105	046495	046885
112	049218	049606	049993	050380	050766
113	053078	053463	053846	054230	054613
114	056905	057286	057666	058046	058426
115	060698	061075	061452	061829	062206
116	064458	064832	065206	065580	065953
117	068186	068557	068928	069298	069668
118	071882	072250	072617	072985	073352
119	075547	075912	076276	076640	077004
120	079181	079543	079904	080266	080626
121	2.082785	2.083144	2.083503	2.083861	2.084219
122	086360	086716	087071	087426	087781
123	089905	090258	090611	090963	091315
124	093422	093772	094122	094471	094820
125	096910	097257	097604	097951	098297
126	100370	100715	101059	101403	101747
127	103804	104145	104487	104828	105169
128	107210	107549	107888	108227	108565
129	110590	110926	111262	111598	111934
130	113943	114277	114611	114944	115278
131	117271	117603	117934	118265	118595
132	2.120574	2.120903	2.121231	2.121560	2.121888
133	123852	124178	124504	124830	125156
134	127105	127429	127752	128076	128399
135	130334	130655	130977	131298	131619
136	133539	133858	134177	134496	134814
137	136721	137037	137354	137670	137987
138	139879	140194	140508	140822	141136
139	143015	143327	143639	143951	144263
140	146128	146438	146748	147058	147367
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019116	019532	019947	020361	020775	416
023252	023664	024075	024486	024896	412
027350	027757	028164	028571	028978	408
031408	031812	032216	032619	033021	404
035430	035830	036229	036629	037028	400
039414	039811	040207	040602	040998	396
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047275	047664	048053	048442	048830	389
051152	051538	051924	052309	052694	386
054996	055378	055760	056142	056524	382
058805	059185	059563	059942	060320	378
062582	062958	063333	063709	064083	376
066326	066698	067071	067443	067814	372
070038	070407	070776	071145	071514	369
073718	074085	074451	074816	075182	366
077368	077731	078094	078457	078819	363
080987	081347	081707	082067	082426	360
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091667	092018	092370	092721	093071	351
095169	095518	095866	096215	096562	349
098644	098990	099335	099681	100026	346
102090	102434	102777	103119	103462	343
105510	105851	106191	106531	106870	340
108903	109241	109578	109916	110253	338
112270	112605	112940	113275	113609	335
115610	115943	116276	116608	116940	333
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128722	129045	129368	129690	130012	323
131939	132260	132580	132900	133219	320
135133	135451	135768	136086	136403	318
138303	138618	138934	139249	139564	315
141450	141763	142076	142389	142702	314
144574	144885	145196	145507	145818	311
147676	147985	148294	148603	148911	309
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153815	154119	154424	154728	155032	304

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146	164353	164650	164947	165244	165541
147	167317	167613	167908	168203	168497
148	170262	170555	170848	171141	171434
149	173186	173478	173768	174060	174351
150	176091	176381	176670	176959	177248
151	178977	179264	179552	179839	180126
152	181844	182129	182415	182700	182985
153	2.184691	2.184975	2.185259	2.185542	2.185825
154	187521	187803	188084	188366	188647
155	190332	190612	190892	191171	191451
156	193125	193403	193681	193959	194237
157	195900	196176	196452	196729	197005
158	198657	198932	199206	199481	199755
159	201397	201670	201943	202216	202488
160	204120	204391	204662	204933	205204
161	206826	207095	207365	207634	207903
162	209515	209783	210051	210318	210586
163	212188	212454	212720	212986	213252
164	2.214844	2.215109	2.215373	2.215638	2.215902
165	217484	217747	218010	218273	218535
166	220108	220370	220631	220892	221153
167	222716	222976	223236	223496	223755
168	225309	225568	225826	226084	226342
169	227887	228144	228400	228657	228913
170	230449	230704	230960	231215	231470
171	232996	233250	233504	233757	234011
172	235528	235781	236033	236285	236537
173	238046	238297	238548	238799	239049
174	240549	240799	241048	241297	241546
175	2.243038	2.243286	2.243534	2.243782	2.244030
176	245513	245759	246006	246252	246499
177	247973	248219	248464	248709	248954
178	250420	250664	250908	251151	251395
179	252853	253096	253338	253580	253822
180	255272	255514	255755	255996	256236
181	257679	257918	258158	258398	258637
182	260071	260310	260548	260787	261025
183	262451	262688	262925	263162	263399
184	264818	265054	265290	265525	265761
185	267172	267406	267641	267875	268110

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168792	169086	169380	169674	169968	295
171726	172019	172311	172603	172895	292
174641	174932	175222	175512	175802	291
177536	177825	178113	178401	178689	289
180413	180699	180986	181272	181558	287
183270	183554	183839	184123	184407	285
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191730	192010	192289	192567	192846	279
194514	194792	195069	195346	195623	278
197281	197556	197832	198107	198382	276
200029	200303	200577	200850	201124	274
202761	203033	203305	203577	203848	272
205475	205745	206016	206286	206556	271
208172	208441	208710	208978	209247	269
210853	211120	211388	211654	211921	267
213518	213783	214049	214314	214579	266
2.216166	2.216430	2.216694	2.216957	2.217221	264
218798	219060	219322	219584	219846	262
221414	221675	221936	222196	222456	261
224015	224274	224533	224792	225051	259
226600	226858	227115	227372	227630	258
229170	229426	229682	229938	230193	256
231724	231979	232233	232488	232742	254
234264	234517	234770	235023	235276	253
236789	237041	237292	237544	237795	251
239299	239550	239800	240050	240300	250
241795	242044	242293	242541	242790	249
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246745	246991	247236	247482	247728	246
249198	249443	249687	249932	250176	245
251638	251881	252125	252367	252610	243
254064	254306	254548	254790	255031	242
256477	256718	256958	257198	257439	241
258877	259116	259355	259594	259833	239
261263	261501	261788	261976	262214	238
263636	263873	264109	264345	264582	237
265996	266232	266467	266702	266937	235
268344	268578	268812	269046	269279	234

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188	274158	274389	274620	274850	275081
189	276462	276691	276921	277151	277380
190	278754	278982	279210	279439	279667
191	281033	281261	281488	281715	281942
192	283301	283527	283753	283979	284205
193	285557	285782	286007	286232	286456
194	287802	288025	288249	288473	288696
195	290035	290257	290480	290702	290925
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197	294466	294687	294907	295127	295347
198	296665	296884	297104	297323	297542
199	298853	299071	299289	299507	299725
200	301030	301247	301464	301681	301898
201	303196	303412	303628	303844	304059
202	305351	305566	305781	305996	306210
203	307496	307710	307924	308137	308351
204	309630	309843	310056	310268	310481
205	311754	311966	312177	312389	312600
206	313867	314078	314289	314499	314710
207	2.315970	2.316180	2.316390	2.316599	2.316809
208	318063	318272	318481	318689	318898
209	320146	320354	320562	320769	320977
210	322219	322426	322633	322839	323046
211	324282	324488	324694	324899	325105
212	326336	326541	326745	326950	327154
213	328380	328583	328787	328991	329194
214	330414	330617	330819	331022	331225
215	332438	332640	332842	333044	333246
216	334454	334655	334856	335056	335257
217	336460	336660	336860	337060	337260
218	2 338456	2.338656	2.338855	2.339054	2.339253
219	340444	340642	340840	341039	341237
220	342423	342620	342817	343014	343212
221	344392	344589	344785	344981	345178
222	346353	346549	346744	346939	347135
223	348305	348500	348694	348889	349083
224	350248	350442	350636	350829	351023
225	352182	352375	352568	352761	352954
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227	356026	356217	356408	356599	356790
228	357935	358125	358316	358506	358696

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277609	277838	278067	278296	278525	229
279895	280123	280351	280578	280806	228
282169	282395	282622	282849	283075	227
284431	284656	284882	285107	285332	226
286681	286905	287130	287354	287579	225
288920	289143	289366	289589	289812	223
291147	291369	291591	291813	292034	222
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295567	295787	296007	296226	296446	220
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304275	304490	304706	304921	305136	216
306425	306639	306854	307068	307282	215
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312812	313023	313234	313445	313656	211
314920	315130	315340	315550	315766	210
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325310	325516	325721	325926	326131	205
327359	327563	327767	327972	328176	204
329398	329601	329804	330008	330211	203
331427	331630	331832	332034	332236	202
333447	333649	333850	334051	334253	202
335458	335658	335859	336059	336259	201
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341434	341632	341830	342028	342225	198
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345374	345570	345766	345961	346157	196
347330	347525	347720	347915	348110	195
349277	349472	349666	349860	350054	194
351216	351410	351603	351796	351989	193
353146	353339	353532	353724	353916	193
350068	355260	355451	355643	355834	192
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358886	359076	359266	359456	359646	190

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232	365488	365675	365862	366049	366236
233	367356	367542	367728	367915	368101
234	369216	369401	369587	369772	369958
235	371068	371253	371437	371622	371806
236	372912	373096	373280	373464	373647
237	374748	374932	375115	375298	375481
238	376577	376759	376942	377124	377306
239	2.378398	2.378580	2.378761	2.378943	2.379124
240	380211	380392	380573	380754	380934
241	382017	382197	382377	382557	382737
242	383815	383995	384174	384353	384533
243	385606	385785	385964	386142	386321
244	387390	387568	387746	387923	388101
245	389166	389343	389520	389697	389874
246	390935	391111	391288	391464	391641
247	392697	392873	393048	393224	393400
248	394452	394627	394802	394977	395152
249	396199	396374	396548	396722	396896
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252	401400	401573	401745	401917	402089
253	403120	403292	403464	403635	403807
254	404834	405005	405175	405346	405517
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256	408240	408409	408579	408749	408918
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259	413300	413467	413635	413802	413970
260	414973	415140	415307	415474	415641
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262	418301	418467	418633	418798	418964
263	419956	420121	420286	420451	420616
264	421604	421768	421933	422097	422261
265	423246	423410	423573	423737	423901
266	424882	425045	425208	425371	425534
267	426511	426674	426836	426999	427161
268	428135	428297	428459	428621	428782
269	429752	429914	430075	430236	430398
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271	432969	433129	433290	433450	433610

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362671	362859	363048	363236	363424	188
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366423	366610	366796	366983	367169	187
368287	368473	368659	368844	369030	186
370143	370328	370513	370698	370883	185
371991	372175	372360	372544	372728	184
373831	374015	374198	374382	374565	184
375664	375846	376029	376212	376394	183
377488	377670	377852	378034	378216	182
2.379305	2.379487	2.379668	2.379849	2.380030	181
381115	381296	381476	381656	381837	181
382917	383097	383277	383456	383636	180
384712	384891	385070	385249	385427	179
386499	386677	386855	387034	387212	178
388279	388456	388634	388811	388989	178
390051	390228	390405	390582	390758	177
391817	391993	392169	392345	392521	176
393575	393751	393926	394101	394276	176
395326	395501	395676	395850	396025	175
397070	397245	397418	397592	397766	174
2.398808	2.398981	2.399154	2.399327	2.399501	173
400538	400711	400883	401056	401228	173
402261	402433	402605	402777	402949	172
403978	404149	404320	404492	404663	171
405688	405858	406029	406199	406370	171
407391	407561	407731	407900	408070	170
409087	409257	409426	409595	409764	169
410777	410946	411114	411283	411451	169
412460	412628	412796	412964	413132	168
414137	414305	414472	414639	414806	167
415808	415974	416141	416308	416474	167
2.417472	2.417638	2.417804	2.417970	2.418135	166
419129	419295	419460	419625	419191	165
420781	420945	421110	421275	421439	165
422426	422590	422754	422918	423082	164
424064	424228	424392	424555	424718	163
425697	425860	426023	426186	426349	163
427324	427486	427648	427811	427973	162
428944	429106	429268	429429	429591	162
430559	430720	430881	431042	431203	161
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433770	433930	434090	434249	434409	160

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275	439333	439491	439648	439806	439964
276	440909	441066	441224	441381	441538
277	442480	442636	442793	442950	443106
278	444045	444201	444357	444513	444669
279	445604	445760	445915	446071	446226
280	447158	447313	447468	447623	447778
281	448706	448861	449015	449170	449324
282	2.450249	2.450403	2.450557	2.450711	2.450865
283	451786	451940	452093	452247	452400
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285	454845	454997	455149	455302	455454
286	456366	456518	456670	456821	456973
287	457882	458033	458184	458336	458487
288	459392	459543	459694	459845	459995
289	460898	461048	461198	461348	461498
290	462398	462548	462697	462847	462997
291	463893	464042	464191	464340	464489
292	465383	465532	465680	465829	465977
293	2.466868	2.467016	2.467164	2.467312	2.467460
294	468347	468495	468643	468790	468938
295	469822	469969	470116	470263	470410
296	471292	471438	471585	471732	471878
297	472756	472903	473049	473195	473341
298	474216	474362	474508	474653	474799
299	475671	475816	475962	476107	476252
300	477121	477266	477411	477555	477700
301	478566	478711	478855	478999	479143
302	480007	480151	480294	480438	480582
303	481443	481586	481729	481872	482016
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306	485721	485863	486005	486147	486289
307	487138	487280	487421	487563	487704
308	488551	488692	488833	488973	489114
309	489958	490099	490239	490380	490520
310	491362	491502	491642	491782	491922
311	492760	492900	493040	493179	493319
312	494155	494294	494433	494572	494711
313	495544	495683	495822	495960	496099
314	496930	497068	497206	497344	497482

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438542	438700	438859	439017	439175	158
440122	440279	440437	440594	440752	158
441695	441852	442009	442166	442323	157
443263	443419	443576	443732	443888	157
444825	444981	445137	445293	445448	156
446382	446537	446692	446848	447003	155
447933	448088	448242	448397	448552	155
449478	449633	449787	449941	450095	154
2.451018	2.451172	2.451326	2.451479	2.451633	154
452553	452706	452859	453012	453165	153
454082	454235	454387	454540	454692	153
455606	455758	455910	456062	456214	152
457125	457276	457428	457579	457730	152
458638	458789	458940	459091	459242	151
460146	460296	460447	460597	460747	151
461649	461799	461948	462098	462248	150
463146	463296	463445	463594	463744	150
464639	464787	464936	465085	465234	149
466126	466274	466423	466571	466719	149
2.467608	2.467756	2.467904	2.468052	2.468200	148
469085	469233	469380	469527	469675	147
470557	470704	470851	470998	471145	147
472025	472171	472317	472464	472610	146
473487	473633	473779	473925	474070	146
474944	475090	475235	475381	475526	146
476397	476542	476687	476832	476976	145
477844	477989	478133	478278	478422	145
479287	479431	479575	479719	479863	144
480725	480869	481012	481156	481299	144
482159	482302	482445	482588	482731	143
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485011	485153	485295	485437	485579	142
486430	486572	486714	486855	486997	142
487845	487986	488127	488269	488410	141
489255	489396	489537	489677	489818	141
490661	490801	490941	491081	491222	140
492062	492201	492341	492481	492621	140
493458	493597	493737	493876	494015	139
494850	494989	495128	495267	495406	139
496237	496376	496514	496653	496791	139
497621	497759	497897	498035	498173	138

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317	501059	501196	501333	501470	501607
318	502427	502564	502700	502837	502973
319	503791	503927	504063	504199	504335
320	505150	505286	505421	505557	505693
321	506505	506640	506775	506911	507046
322	507856	507991	508125	508260	508395
323	509202	509337	509471	509606	509740
324	510545	510679	510813	510947	511081
325	2.511883	2.512017	2.512150	2.512284	2.512417
326	513218	513351	513484	513617	513750
327	514548	514680	514813	514946	515079
328	515874	516006	516139	516271	516403
329	517196	517328	517460	517592	517724
330	518514	518645	518777	518909	519040
331	519828	519959	520090	520221	520352
332	521138	521269	521400	521530	521661
333	522444	522575	522705	522835	522966
334	523746	523876	524006	524136	524266
335	525045	525174	525304	525433	525563
336	2.526339	2.526468	2.526598	2.526727	2.526856
337	527630	527759	527888	528016	528145
338	528917	529045	529174	529302	529430
339	530200	530328	530456	530584	530712
340	531479	531607	531734	531862	531989
341	532754	532882	533009	533136	533263
342	534026	534153	534280	534407	534534
343	535294	535421	535546	535674	535800
344	536558	536685	536811	536937	537063
345	537819	537945	538071	538197	538322
346	539076	539202	539327	539452	539578
347	2.540329	2.540455	2.540580	2.540705	2.540830
348	541579	541704	541829	541953	542078
349	542825	542950	543074	543199	543323
350	544068	544192	544316	544440	544564
351	545307	545431	545554	545678	545802
352	546543	546666	546789	546913	547036
353	547775	547898	548021	548144	548266
354	549003	549126	549249	549371	549494
355	550228	550351	550473	550595	550717
356	551450	551572	551694	551816	551938
357	552668	552790	552911	553033	553154

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504471	504607	504743	504878	505014	136
505828	505963	506099	506234	506370	136
507181	507316	507451	507586	507721	135
508530	508664	508799	508933	509068	135
509874	510008	510143	510277	510411	134
511215	511348	511482	511616	511750	134
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516535	516668	516800	516932	517064	132
517855	517987	518119	518251	518382	132
519171	519303	519434	519565	519697	132
520483	520614	520745	520876	521007	131
521792	521922	522053	522183	522314	131
523096	523226	523356	523486	523616	130
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525692	525822	525951	526081	526210	129
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535927	536053	536179	536306	536432	126
537189	537315	537441	537567	537692	126
538448	538574	538699	538825	538951	126
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542203	542327	542452	542576	542701	125
543447	543571	543696	543820	543944	124
544688	544812	544936	545060	545183	124
545925	546049	546172	546296	546419	124
547159	547282	547405	547529	547652	123
548389	548512	548635	548758	548881	123
549616	549739	549861	549984	550106	123
550840	550962	551084	551206	551328	122
552059	552181	552303	552425	552546	122
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361	557507	557627	557748	557868	557988
362	558709	558828	558948	559068	559188
363	559907	560026	560146	560265	560385
364	561101	561221	561340	561459	561578
365	562293	562412	562531	562650	562768
366	563481	563600	563718	563837	563955
367	564666	564784	564903	565021	565139
368	2.565848	2.565966	2.566084	2.566202	2.566320
369	567026	567144	567262	567379	567497
370	568202	568319	568436	568554	568671
371	569374	569491	569608	569725	569842
372	570543	570660	570776	570893	571010
373	571709	571825	571942	572058	572174
374	572872	572988	573104	573220	573336
375	574031	574147	574263	574379	574494
376	575188	575303	575419	575534	575650
377	576341	576456	576572	576687	576802
378	577492	577607	577721	577836	577951
379	2.578639	2.578754	2.578868	2.578983	2.579097
380	579784	579898	580012	580126	580240
381	580925	581039	581153	581267	581381
382	582063	582177	582291	582404	582518
383	583199	583312	583425	583539	583652
384	584331	584444	584557	584670	584783
385	585461	585573	585686	585799	585912
386	586587	586700	586812	586925	587037
387	587711	587823	587935	588047	588100
388	588832	588944	589055	589167	589279
389	589950	590061	590173	590284	590396
390	2.591065	2.591176	2.591287	2.591398	2.591510
391	592177	592288	592399	592510	592621
392	593286	593397	593508	593618	593729
393	594392	594503	594613	594724	594834
394	595496	595606	595717	595827	595937
395	596597	596707	596817	596927	597037
396	597695	597805	597914	598024	598134
397	598790	598900	599009	599119	599228
398	599883	599992	600101	600210	600319
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564074	564192	564311	564429	564548	119
565257	565375	565494	565612	565730	118
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567614	567732	567849	567967	568084	118
568788	568905	569023	569140	569257	117
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571126	571243	571359	571476	571592	117
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574610	574726	574841	574957	575072	116
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576917	577032	577147	577262	577377	115
578066	578181	578295	578410	578525	115
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582631	582745	582858	582972	583085	114
583765	583879	583992	584105	584218	113
584896	585009	585122	585235	585348	113
586024	586137	586250	586362	586475	113
587149	587262	587374	587486	587599	112
588272	588384	588496	588608	588720	112
589391	589503	589614	589726	589838	112
590507	590619	590730	590842	590953	112
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592732	592843	592954	593064	593175	111
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404	606381	606489	606596	606704	606811
405	607455	607562	607669	607777	607884
406	608526	608633	608740	608847	608954
407	609594	609701	609808	609914	610021
408	610660	610767	610873	610979	611086
409	611723	611829	611936	612042	612148
410	612784	612890	612996	613102	613207
411	2.613842	2.613947	2.614053	2.614159	2.614264
412	614897	615003	615108	615213	615319
413	615950	616055	616160	616265	616370
414	617000	617105	617210	617315	617420
415	618048	618153	618257	618362	618466
416	619093	619198	619302	619406	619511
417	620136	620240	620344	620448	620552
418	621176	621280	621384	621488	621592
419	622214	622318	622421	622525	622628
420	623249	623353	623456	623559	623663
421	624282	624385	624488	624591	624694
422	2.625312	2.625415	2.625518	2.625621	2.625724
423	626340	626443	626546	626648	626751
424	627366	627468	627571	627673	627775
425	628389	628491	628593	628695	628797
426	629410	629511	629613	629715	629817
427	630428	630529	630631	630733	630834
428	631444	631545	631647	631748	631849
429	632457	632558	632660	632761	632862
430	633468	633569	633670	633771	633872
431	634477	634578	634679	634779	634880
432	635484	635584	635685	635785	635886
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434	637490	637590	637690	637790	637890
435	638489	638589	638689	638789	638888
436	639486	639586	639686	639785	639885
437	640481	640581	640680	640779	640879
438	641474	641573	641672	641771	641870
439	642464	642563	642662	642761	642860
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441	644439	644537	644635	644734	644832
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612254	612360	612466	612572	612678	106
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616475	616580	616685	616790	616895	105
617524	617629	617734	617839	617943	105
618571	618675	618780	618884	618989	105
619615	619719	619823	619928	620032	104
620656	620760	620864	620968	621072	104
621695	621799	621903	622007	622110	104
622732	622835	622939	623042	623146	104
623766	623869	623972	624076	624179	103
624798	624901	625004	625107	625209	103
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628900	629002	629104	629206	629308	102
629919	630021	630123	630224	630326	102
630936	631038	631139	631241	631342	102
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633973	634074	634175	634276	634376	100
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447	650307	650405	650502	650599	650696
448	651278	651375	651472	651569	651666
449	652246	652343	652440	652536	652633
450	653212	653309	653405	653502	653598
451	654176	654273	654369	654465	654562
452	655138	655234	655331	655427	655523
453	656098	656194	656290	656386	656481
454	2.657056	2.657151	2.657247	2.657343	2.657438
455	658011	658107	658202	658298	658393
456	658965	659060	659155	659250	659346
457	659916	660011	660106	660201	660296
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459	661813	661907	662002	662096	662191
460	662758	662852	662947	663041	663135
461	663701	663795	663889	663983	664078
462	664642	664736	664830	664924	665018
463	665581	665675	665768	665862	665956
464	666518	666612	666705	666799	666892
465	2.667453	2.667546	2.667640	2.667733	2.667826
466	668386	668479	668572	668665	668758
467	669317	669410	669503	669596	669689
468	670246	670339	670431	670524	670617
469	671173	671265	671358	671451	671543
470	672098	672190	672283	672375	672467
471	673021	673113	673205	673297	673390
472	673942	674034	674126	674218	674310
473	674861	674953	675045	675136	675228
474	675778	675870	675961	676053	676145
475	676694	676785	676876	676968	677059
476	2.677607	2.677698	2.677789	2.677881	2.677972
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480	681241	681332	681422	681513	681603
481	682145	682235	682326	682416	682506
482	683047	683137	683227	683317	683407
483	683947	684037	684127	684217	684307
484	684845	684935	685025	685114	685204
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486	686636	686726	686815	686904	686994

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659441	659536	659631	659726	659821	95
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664172	664266	664360	664454	664548	94
665112	665206	665299	665393	665487	94
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666986	667079	667173	667266	667359	94
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668852	668945	669038	669131	669224	93
669782	669874	669967	670060	670153	93
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671636	671728	671821	671913	672005	93
672560	672652	672744	672836	672929	92
673482	673574	673666	673758	673850	92
674402	674494	674586	674677	674769	92
675320	675412	675503	675595	675687	92
676236	676328	676419	676511	676602	92
677150	677242	677333	677424	677516	91
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679882	679973	680063	680154	680245	91
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681693	681784	681874	681964	682055	90
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490	690196	690285	690373	690462	690550
491	691081	691170	691258	691347	691435
492	691965	692053	692142	692230	692318
493	692847	692935	693023	693111	693199
494	693727	693815	693903	693991	694078
495	694605	694693	694781	694868	694956
496	695482	695569	695657	695744	695832
497	2.696356	2.696444	2.696531	2.696618	2.696706
498	697229	697316	697404	697491	697578
499	698100	698188	698275	698362	698448
500	698970	699057	699144	699230	699317
501	699838	699924	700011	700098	700184
502	700704	700790	700877	700963	701050
503	701568	701654	701741	701827	701913
504	702430	702517	702603	702689	702775
505	703291	703377	703463	703549	703635
506	704150	704236	704322	704408	704494
507	705008	705094	705179	705265	705350
508	2.705864	2.705949	2.706035	2.706120	2.706205
509	706718	706803	706888	706974	707059
510	707570	707655	707740	707826	707911
511	708421	708506	708591	708676	708761
512	709270	709355	709440	709524	709609
513	710117	710202	710287	710371	710456
514	710963	711048	711132	711216	711301
515	711807	711891	711976	712060	712144
516	712650	712734	712818	712902	712986
517	713490	713574	713658	713742	713826
518	714330	714414	714497	714581	714665
519	2.715167	2.715251	2.715335	2.715418	2.715502
520	716003	716087	716170	716254	716337
521	716838	716921	717004	717088	717171
522	717670	717754	717837	717920	718003
523	718502	718585	718668	718751	718834
524	719331	719414	719497	719580	719663
525	720159	720242	720325	720407	720490
526	720986	721068	721151	721233	721316
527	721811	721893	721975	722058	722140
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529	723456	723538	723620	723702	723784

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717254	717338	717421	717504	717587	83
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718917	719000	719083	719165	719248	83
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534	727541	727623	727704	727785	727866
535	728354	728435	728516	728597	728678
536	729165	729246	729327	729408	729489
537	729974	730055	730136	730217	730298
538	730783	730863	730944	731024	731105
539	731589	731669	731750	731830	731911
540	2.732394	2.732474	.732555	2.732635	2.732715
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542	733999	734079	734159	734240	734320
543	734800	734880	734960	735040	735120
544	735599	735679	735758	735838	735918
545	736396	736476	736556	736635	736715
546	737193	737272	737352	737431	737511
547	737987	738067	738146	738225	738305
548	738781	738860	738939	739018	739097
549	739572	739651	739730	739810	739889
550	740363	740442	740521	740599	740678
551	2.741152	2.741230	2.741309	2.741388	2.741467
552	741939	742018	742096	742175	742[illegible]
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555	744293	744371	744449	744528	744606
556	745075	745153	745231	745309	745387
557	745855	745933	746011	746089	746167
558	746634	746712	746790	746868	746945
559	747412	747489	747567	747645	747722
560	748188	748266	748343	748421	748498
561	748963	749040	749118	749195	749272
562	2.749736	2.749814	2.749891	2.749968	2.750045
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564	751279	751356	751433	751510	751587
565	752048	752125	752202	752279	752356
566	752816	752893	752970	753047	753123
567	753583	753660	753736	753813	753889
568	754348	754425	754501	754578	754654
569	755112	755189	755265	755341	755417
570	755875	755951	756027	756103	756180
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572	757396	757472	757548	757624	757700

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730378	730459	730540	730621	730702	81
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731991	732072	732152	732233	732313	81
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735998	736078	736167	736237	736317	80
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738384	738463	738543	738622	738701	79
739177	739256	739335	739414	739493	79
739968	740047	740126	740205	740284	79
740757	740836	740915	740994	741073	79
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746245	746323	746401	746479	746556	78
747023	747101	747179	747256	747334	78
747800	747878	747955	748033	748110	78
748576	748653	748731	748808	748885	77
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754730	754807	754883	754960	755036	76
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757775	757851	757927	758003	758079	76

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576	760422	760498	760573	760649	760724
577	761176	761251	761326	761402	761477
578	761928	762003	762078	762153	762228
579	762679	762754	762829	762904	762978
580	763428	763503	763578	763653	763727
581	764176	764251	764326	764400	764475
582	764923	764998	765072	765147	765221
583	2.765669	2.765743	2.765818	2.765892	2.765966
584	766413	766487	766562	766636	766710
585	767156	767230	767304	767379	767453
586	767898	767972	768046	768120	768194
587	768638	768712	768786	768860	768934
588	769377	769451	769525	769599	769673
589	770115	770189	770263	770336	770410
590	770852	770926	770999	771073	771146
591	771587	771661	771734	771808	771881
592	772322	772395	772468	772542	772615
593	773055	773128	773201	773274	773348
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595	774517	774590	774663	774736	774809
596	775246	775319	775392	775465	775538
597	775974	776047	776120	776193	776265
598	776701	776774	776846	776919	776992
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601	778874	778947	779019	779091	779163
602	779596	779669	779741	779813	779885
603	780317	780389	780461	780533	780605
604	781037	781109	781181	781253	781324
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608	783904	783975	784046	784118	784189
609	784617	784689	784760	784831	784902
610	785330	785401	785472	785543	785615
611	786041	786112	786183	786254	786325
612	786751	786822	786893	786964	787035
613	787460	787531	787602	787673	787744
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763802	763877	763952	764027	764101	75
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768268	768342	768416	768490	768564	74
769008	769082	769156	769230	769303	74
769746	769820	769894	769968	770042	74
770484	770557	770631	770705	770778	74
771220	771293	771367	771440	771514	74
771955	772028	772102	772175	772248	73
772688	772762	772835	772908	772981	73
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776338	776411	776483	776556	776629	73
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778513	778585	778658	778730	778802	72
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781396	781468	781540	781612	781684	72
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783546	783618	783689	783761	783832	71
784261	784332	784403	784475	784546	71
784974	785045	785116	785187	785259	71
785686	785757	785828	785899	785970	71
786396	786467	786538	786609	786680	71
787106	787177	787248	787319	787390	71
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620	792392	792462	792532	792602	792672
621	793092	793162	793231	793301	793371
622	793790	793860	793930	794000	794070
623	794488	794558	794627	794697	794767
624	795185	795254	795324	795393	795463
625	795880	795949	796019	796088	796158
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627	797268	797337	797406	797475	797545
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629	798651	798720	798789	798858	798927
630	799341	799409	799478	799547	799616
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632	800717	800786	800854	800923	800992
633	801404	801472	801541	801609	801678
634	802089	802158	802226	802295	802363
635	802774	802842	802910	802979	803047
636	803457	803525	803594	803662	803730
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640	806180	806248	806316	806384	806451
641	806858	806926	806994	807061	807129
642	807535	807603	807670	807738	807806
643	808211	808279	808346	808414	808481
644	808886	808953	809021	809088	809155
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654	815578	815644	815710	815777	815843
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657	817565	817631	817698	817764	817830
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793441	793511	793581	793651	793721	70
794139	794209	794279	794349	794418	70
794836	794906	794976	795045	795115	70
795532	795602	795671	795741	795810	69
796227	796297	796366	796436	796505	69
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798305	798374	798443	798512	798582	69
798996	799065	799134	799203	799272	69
799685	799754	799823	799892	799960	69
800373	800442	800511	800580	800648	69
801060	801129	801198	801267	801335	69
801747	801815	801884	801952	802021	69
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803116	803184	803252	803321	803389	68
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805161	805229	805297	805365	805433	68
805840	805908	805976	806044	806112	68
806519	806587	806655	806723	806790	68
807197	807264	807332	807400	807467	68
807873	807941	808008	808076	808143	68
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809223	809290	809358	809425	809492	67
809896	809963	810031	810098	810165	67
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811240	811307	811374	811441	811508	67
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813247	813314	813381	813447	813514	67
813914	813981	814048	814114	814181	67
814580	814647	814714	814780	814847	67
815246	815312	815378	815445	815511	66
815910	815976	816042	816109	816175	66
816573	816639	816705	816771	816838	66
817235	817301	817367	817433	817499	66
817896	817962	818028	818094	818160	66
818556	818622	818688	818754	818819	66

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661	820201	820267	820333	820398	820464
662	820858	820924	820989	821055	821120
663	821513	821579	821644	821710	821775
664	822168	822233	822299	822364	822430
665	822822	822887	822952	823017	823083
666	823474	823539	823605	823670	823735
667	824126	824191	824256	824321	824386
668	824776	824841	824906	824971	825036
669	2.825426	2.825491	2.825556	2.825621	2.825686
670	826075	826140	826204	826269	826334
671	826722	826787	826852	826917	826981
672	827369	827434	827498	827563	827628
673	828015	828080	828144	828209	828273
674	828660	828724	828789	828853	828918
675	829304	829368	829432	829497	829561
676	829947	830011	830075	830139	830204
677	830589	830653	830717	830781	830845
678	831230	831294	831358	831422	831486
679	831870	831934	831998	832062	832125
680	2.832509	2.832573	2.832637	2.832700	2.832764
681	833147	833211	833275	833338	833402
682	833784	833848	833912	833975	834039
683	834421	834484	834548	834611	834675
684	835056	835120	834183	835246	835310
685	835691	835754	835817	835881	835944
686	836324	836387	836451	836514	836577
687	836957	837020	837083	837146	837209
688	837588	837652	837715	837778	837841
689	838219	838282	838345	838408	838471
690	838849	838912	838975	839038	839101
691	2.839478	2.839541	2.839604	2.839667	2.839729
692	840106	840169	840232	840294	840357
693	840733	840796	840859	840921	840984
694	841359	841422	841485	841547	841610
695	841985	842047	842110	842172	842235
696	842609	842672	842734	842796	842859
697	843233	843295	843357	843420	843482
698	843855	843918	843980	844042	844104
699	844477	844539	844601	844663	844726
700	845098	845160	845222	845284	845346
701	845718	845780	845842	845904	845966

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821186	821251	821317	821382	821448	66
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828338	828402	828466	828531	828595	64
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829625	829690	829754	829818	829882	64
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830909	830973	831037	831102	831166	64
831550	831614	831678	831742	831806	64
832189	832253	832317	832381	832445	64
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834738	834802	834866	834929	834993	64
835373	835437	835500	835564	835627	63
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841672	841735	841797	841860	841922	63
842297	842360	842422	842484	842547	62
842921	842983	843046	843108	843170	62
843544	843606	843669	843731	843793	62
844166	844229	844291	844353	844415	62
844788	844850	844912	844974	845036	62
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705	848189	848251	848312	848374	848435
706	848805	848866	848928	848989	849051
707	849419	849481	849542	849604	849665
708	850033	850095	850156	850217	850279
709	850646	850707	850769	850830	850891
710	851258	851319	851381	851442	851503
711	851870	851931	851992	852053	852114
712	2.852480	2.852541	2.852602	2.852663	2.852724
713	853089	853150	853211	853272	853333
714	853698	853759	853820	853881	853941
715	854306	854367	854427	854488	854549
716	854913	854974	855034	855095	855156
717	855519	855580	855640	855701	855761
718	856124	856185	856245	856306	856366
719	856729	856789	856850	856910	856970
720	857332	857393	857453	857513	857574
721	857935	857995	858056	858116	858176
722	858537	858597	858657	858718	858778
723	2.859138	2.859198	2.859258	2.859318	2.859378
724	859739	859798	859858	859918	859978
725	860338	860398	860458	860518	860578
726	860937	860996	861056	861116	861176
727	861534	861594	861654	861714	861773
728	862131	862191	862251	862310	862370
729	862727	862787	862847	862906	862966
730	863323	863382	863442	863501	863561
731	863917	863977	864036	864096	864155
732	864511	864570	864630	864689	864748
733	865104	865163	865222	865282	865341
734	2.865696	2.865755	2.865814	2.865873	2.865933
735	866287	866346	866405	866465	866524
736	866878	866937	866996	867055	867114
737	867467	867526	867585	867644	867703
738	868056	868115	868174	868233	868292
739	868644	868703	868762	868821	868879
740	869232	869290	869349	869408	869466
741	869818	869877	869935	869994	870053
742	870404	870462	870521	870579	870638
743	870989	871047	871106	871164	871223
744	871573	871631	871690	871748	871806

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849112	849174	849235	849296	849358	61
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854610	854670	854731	854792	854852	61
855216	855277	855337	855398	855459	61
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864808	864867	864926	864985	865045	59
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866583	866642	866701	866760	866819	59
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870696	870755	870813	870872	870930	58
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871865	871923	871981	872040	872098	58

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747	873321	873379	873437	873495	873553
748	873902	873960	874018	874076	874134
749	874482	874540	874598	874656	874714
750	875061	875119	875177	875235	875293
751	875640	875698	875756	875813	875871
752	876218	876276	876333	876391	876449
753	876795	876853	876910	876968	877026
754	877371	877429	877486	877544	877602
755	2.877947	2.878004	2.878062	2.878119	2.878177
756	878522	878579	878637	878694	878751
757	879096	879153	879211	879268	879325
758	879669	879726	879784	879841	879898
759	880242	880299	880356	880413	880471
760	880814	880871	880928	880985	881042
761	881385	881442	881499	881556	881613
762	881955	882012	882069	882126	882183
763	882524	882581	882638	882695	882752
764	883093	883150	883207	883264	883321
765	883661	883718	883775	883832	883889
766	2.884229	2.884285	2.884342	2.884399	2.884455
767	884795	884852	884909	884965	885022
768	885361	885418	885474	885531	885587
769	885926	885983	886039	886096	886152
770	886491	886547	886603	886660	886716
771	887054	887111	887167	887223	887280
772	887617	887673	887730	887786	887842
773	888179	888236	888292	888348	888404
774	888741	888797	888853	888909	888965
775	889302	889358	889414	889470	889526
776	889862	889918	889974	890030	890086
777	2.890421	2.890477	2.890533	2.890588	2.890644
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779	891537	891593	891649	891705	891760
780	892095	892150	892206	892262	892317
781	892651	892707	892762	892818	892873
782	893207	893262	893318	893373	893429
783	893762	893817	893873	893928	893984
784	894316	894371	894427	894482	894538
785	894870	894925	894980	895036	895091
786	895422	895478	895533	895588	895643
787	895975	896030	896085	896140	896195

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874192	874250	874308	874366	874424	58
874772	874830	874887	874945	875003	58
875351	875409	875466	875524	875582	58
875929	875987	876045	876102	876160	58
876506	876564	876622	876680	876737	58
877083	877141	877198	877256	877314	58
877659	877717	877774	877832	877889	58
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879383	879440	879497	879555	879612	57
879956	880013	880070	880127	880185	57
880528	880585	880642	880699	880756	57
881099	881156	881213	881270	881328	57
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882240	882297	882354	882411	882408	57
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883945	884002	884059	884115	884172	57
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886209	886265	886321	886378	886434	56
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887898	887955	888011	888067	888123	56
888460	888516	888573	888629	888685	56
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889582	889638	889694	889750	889806	56
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891259	891314	891370	891426	891482	56
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892373	892428	892484	892540	892595	56
892929	892985	893040	893096	893151	56
893484	893540	893595	893651	893706	56
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791	898176	898231	898286	898341	898396
792	898725	898780	898835	898890	898944
793	899273	899328	899383	899437	899492
794	899820	899875	899930	899985	900039
795	900367	900422	900476	900531	900586
796	900913	900968	901022	901077	901131
797	901458	901513	901567	901622	901676
798	2.902003	2.902057	2.902112	2.902166	2.902220
799	902547	902601	902655	902710	902764
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801	903632	903687	903741	903795	903849
802	904174	904228	904283	904337	904391
803	904715	904770	904824	904878	904932
804	905256	905310	905364	905418	905472
805	905796	905850	905904	905958	906012
806	906335	906389	906443	906497	906550
807	906873	906927	906981	907035	907089
808	907411	907465	907519	907573	907626
809	2.907948	2.908002	2.908056	2.908109	2.908163
810	908485	908539	908592	908646	908699
811	909021	909074	909128	909181	909235
812	909556	909609	909663	909716	909770
813	910090	910144	910197	910251	910304
814	910624	910678	910731	910784	910838
815	911158	911211	911264	911317	911371
816	911690	911743	911797	911850	911903
817	912222	912275	912328	912381	912435
818	912753	912806	912859	912912	912966
819	913284	913337	913390	913443	913496
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823	915400	915453	915505	915558	915611
824	915927	915980	916033	916085	916138
825	916454	916507	916559	916612	916664
826	916980	917033	917085	917138	917190
827	917505	917558	917610	917663	917715
828	918030	918083	918135	918188	918240
829	918555	918607	918659	918712	918764
830	919078	919130	919183	919235	919287

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898451	898506	898561	898615	898670	55
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899547	899602	899656	899711	899766	55
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900640	900695	900749	900804	900858	55
901186	901240	901295	901349	901404	55
901731	901785	901840	901894	901948	54
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903361	903416	903470	903524	903578	54
903903	903958	904012	904066	904120	54
904445	904499	904553	904607	904661	54
904986	905040	905094	905148	905202	54
905526	905580	905634	905688	905742	54
906065	906119	906173	906227	906281	54
906604	906658	906712	906766	906820	54
907142	907196	907250	907304	907358	54
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910358	910411	910464	910518	910571	53
910891	910944	910998	911051	911104	53
911424	911477	911530	911584	911637	53
911956	912009	912063	912116	912169	53
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918292	918345	918397	918450	918502	52
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831	2.919601	2.919653	2.919705	2.919756	2.9198[illegible]
832	920123	920175	920228	920280	920332
833	920645	920697	920749	920801	920853
834	921166	921218	921270	921322	921374
835	921686	921738	921790	921842	921894
836	922206	922258	922310	922362	922414
837	922725	922777	922829	922881	922933
838	923244	923296	923348	923399	923451
839	923762	923814	923865	923917	923969
840	924279	924331	924383	924434	924486
841	2.924796	2.924848	2.924899	2.924951	2.925002
842	925312	925364	925415	925467	925518
843	925828	925879	925931	925982	926034
844	926342	926394	926445	926497	926548
845	926857	926908	926959	927011	927062
846	927370	927422	927473	927524	927576
847	927883	927935	927986	928037	928088
848	928396	928447	928498	928549	928601
849	928908	928959	929010	929061	929112
850	929419	929470	929521	929572	929623
851	929930	929981	930032	930083	930134
852	2.930440	2.930491	2.930541	2.930592	2.930643
853	930949	931000	931051	931102	931153
854	931458	931509	931560	931610	931661
855	931966	932017	932068	932118	932169
856	932474	932524	932575	932626	932677
857	932981	933031	933082	933133	933183
858	933487	933538	933588	933639	933690
859	933993	934044	934094	934145	934195
860	934498	934549	934599	934650	934700
861	935003	935054	935104	935154	935205
862	935507	935558	935608	935658	935709
863	2.936011	2.936061	2.936111	2.936162	2.936212
864	936514	936564	936614	936664	936715
865	937016	937066	937116	937167	937217
866	937518	937568	937618	937668	937718
867	938019	938069	938119	938169	938219
868	938520	938570	938620	938670	938720
869	939020	939070	939120	939170	939220
870	939519	939569	939619	939669	939719
871	940018	940068	940118	940168	940218
872	940516	940566	940616	940666	940716
873	941014	941064	941114	941163	941213

5	6	7	8	9	Diff.
2.919862	2.919914	2.919967	2.920019	2.920071	52
920384	920436	920489	920541	920593	52
920906	920958	921010	921062	921114	52
921426	921478	921530	921582	921634	52
921946	921298	922050	922102	922154	52
922466	922518	922570	922622	922674	52
922985	923037	923088	923140	923192	52
923503	923555	923607	923658	923710	52
924021	924072	924124	924176	924228	52
924538	924589	924641	924693	924744	52
2.925054	2.925106	2.925157	2.925209	2.925260	52
925570	925621	925673	925724	925776	52
926085	926137	926188	926239	926291	52
926600	926651	926702	926754	926805	51
927114	927165	927216	927268	927319	51
927627	927678	927730	927781	927832	51
928140	928191	928242	928293	928345	51
928652	928703	928754	928805	928856	51
929163	929214	929266	929317	929368	51
929674	929725	929776	929827	929878	51
930185	930236	930287	930338	930389	51
2.930694	2.930745	2.930796	2.930847	2.930898	51
931203	931254	931305	931356	931407	51
931712	931763	931814	931864	931915	51
932220	932271	932321	932372	932423	51
932727	932778	932829	932879	932930	51
933234	933285	933335	933386	933437	51
933740	933791	933841	933892	933943	51
934246	934296	934347	934397	934448	51
934751	934801	934852	934902	934953	50
935255	935306	935356	935406	935457	50
935759	935809	935860	935910	935960	50
2.936263	2.936313	2.936363	2.936413	2.936463	50
936765	936815	936865	936916	936966	50
937267	937317	937367	937418	937468	50
937769	937819	937869	937919	937969	50
938269	938319	938370	938420	938470	50
938770	938820	938870	938920	938970	50
939270	939319	939369	939419	939469	50
939769	939819	939868	939918	939968	50
940267	940317	940367	940417	940467	50
940765	940815	940865	940915	940965	50
941263	941313	941362	941412	941462	50

Num.	0	1	2	3	4
874	2.941511	2.941561	2.941611	2.941660	2.941710
875	942008	942058	942107	942157	942206
876	942504	942554	942604	942653	942702
877	943000	943049	943099	943148	943198
878	943494	943544	943593	943643	943692
879	943989	944038	944088	944137	944186
880	944483	944532	944581	944631	944680
881	944976	945025	945074	945124	945173
882	945469	945518	945567	945616	945665
883	945961	946010	946059	946108	946157
884	2.946452	2.946501	2.946550	2.946600	2.946649
885	946943	946992	947041	947090	947139
886	947434	947483	947532	947581	947630
887	947924	947973	948021	948070	948119
888	948413	948462	948511	948560	948608
889	948902	948951	948999	949048	949097
890	949390	949439	949488	949536	949585
891	949878	949926	949975	950024	950073
892	950365	950413	950462	950511	950560
893	950851	950900	950949	950997	951046
894	951337	951386	951435	951483	951532
895	2.951823	2.951872	2.951920	2.951969	2.952017
896	952308	952356	952405	952453	952502
897	952792	952841	952889	952938	952986
898	953276	953325	953373	953421	953470
899	953760	953808	953856	953905	953953
900	954242	954291	954339	954387	954435
901	954725	954773	954821	954869	954918
902	955206	955255	955303	955351	955399
903	955688	955736	955784	955832	955880
904	956168	956216	956264	956312	956360
905	956649	956697	956745	956792	956840
906	2.957128	2.957176	2.957224	2.957272	2.957320
907	957607	957655	957703	957751	957799
908	958086	958134	958181	958229	958277
909	958564	958612	958659	958707	958755
910	959041	959089	959137	959184	959232
911	959518	959566	959614	959661	959709
912	959995	960042	960090	960138	960185
913	960471	960518	960566	960613	960661
914	960946	960994	961041	961089	961136
915	961421	961468	961516	961563	961611
916	961895	961943	961990	962038	962085

5	6	7	8	9	Diff.
2.941760	2.941809	2.941859	2.941909	2.941958	49
942256	942306	942355	942405	942454	49
942752	942801	942851	942900	942950	49
943247	943297	943346	943396	943445	49
943742	943791	943841	943890	943939	49
944236	944285	944335	944384	944433	49
944729	944779	944828	944877	944927	49
945222	945272	945321	945370	945419	49
945715	945764	945813	945862	945911	49
946207	946256	946305	946354	946403	49
2.946698	2.946747	2.946796	2.946845	2.946894	49
947189	947238	947287	947336	947385	49
947679	947728	947777	947826	947875	49
948168	948217	948266	948315	948364	49
948657	948706	948755	948804	948853	49
949146	949195	949244	949292	949341	49
949633	949683	949731	949780	949829	49
950121	950170	950219	950267	950316	49
950608	950657	950705	950754	950803	49
951095	951143	951192	951240	951289	49
951580	951629	951677	951726	951774	49
2.952066	2.952114	2.952163	2.952211	2.952259	48
952550	952599	952647	952696	952744	48
953034	953083	953131	953180	953228	48
953518	953566	953615	953663	953711	48
954001	954049	954098	954146	954194	48
954484	954532	954580	954628	954677	48
954966	955014	955062	955110	955158	48
955447	955495	955543	955591	955640	48
955928	955976	956024	956072	956120	48
956409	956457	956505	956553	956601	48
956888	956936	956984	957032	957080	48
2.957368	2.957416	2.957464	2.957511	2.957559	48
957847	957894	957942	957990	958038	48
958325	958373	958420	958468	958516	48
958803	958850	958898	958946	958994	48
959280	959328	959375	959423	959471	48
959757	959804	959852	959900	959947	48
960233	960280	960328	960376	960423	48
960708	960756	960804	960851	960899	48
961184	961231	961279	961326	961374	47
961658	961706	961753	961801	961848	47
962132	962180	962227	962275	962322	47

Num.	0	1	2	3	4
917	2.962369	2.962417	2.962464	2.962511	2.962559
918	962843	962890	962937	962985	963032
919	963315	963363	963410	963457	963504
920	963788	963835	963882	963929	963977
921	964260	964307	964354	964401	964448
922	964731	964778	964825	964872	964919
923	965202	965249	965296	965343	965390
924	965672	965719	965766	965813	965860
925	966142	966189	966236	966283	966329
926	966611	966658	966705	966752	966798
927	2.967080	2.967127	2.967173	2.967220	2.967267
928	967548	967595	967642	967688	967735
929	968016	968062	968109	968156	968203
930	968483	968530	968576	968623	968670
931	968950	968996	969043	969090	969136
932	969416	969462	969509	969556	969602
933	969882	969928	969975	970021	970068
934	970347	970393	970440	970486	970533
935	970812	970858	970904	970951	970997
936	971276	971322	971369	971415	971461
937	971740	971786	971832	971879	971925
938	2.972203	2.972249	2.972295	2.972342	2.972388
939	972666	972712	972758	972804	972851
940	973128	973174	973220	973266	973313
941	973590	973636	973682	973728	973774
942	974051	974097	974143	974189	974235
943	974512	974558	974604	974650	974696
944	974972	975018	975064	975110	975156
945	975432	975478	975524	975570	975616
946	975891	975937	975983	976029	976075
947	976350	976396	976442	976487	976533
948	976808	976854	976900	976946	976991
949	2.977266	2.977312	2.977358	2.977403	2.977449
950	977724	977769	977815	977861	977906
951	978180	978226	978272	978317	978363
952	978637	978683	978728	978774	978819
953	979093	979138	979184	979230	979275
954	979548	979594	979639	979685	979730
955	980003	980049	980094	980140	980185
956	980458	980503	980549	980594	980640
957	980912	980957	981003	981048	981093
958	981365	981411	981456	981501	981547
959	981819	981864	981909	981954	982000

5	6	7	8	9	Diff.
2.962606	2.962653	2.962701	2.962748	2.962795	47
963079	963126	963174	963221	963268	47
963552	963599	963646	963693	963741	47
964024	964071	964118	964165	964212	47
964495	964542	964590	964637	964684	47
964966	965013	965060	965108	965155	47
965437	965484	965531	965578	965625	47
965907	965954	966001	966048	966095	47
966376	966423	966470	966517	966564	47
966845	966892	966939	966986	967033	47
2.967314	2.967361	2.967408	2.967454	2.967501	47
967782	967829	967875	967922	967969	47
968249	968296	968343	968389	968436	47
968716	968763	968810	968856	968903	47
969183	969229	969276	969323	969369	47
969649	969695	969742	969788	969835	47
970114	970161	970207	970254	970300	47
970579	970626	970672	970719	970765	47
971044	971090	971137	971183	971229	46
971508	971554	971600	971647	971693	46
971971	972018	972064	972110	972156	46
2.972434	2.972480	2.972527	2.972573	2.972619	46
972897	972943	972989	973035	973082	46
973359	973405	973451	973497	973543	46
973820	973866	973913	973959	974005	46
974281	974327	974373	974420	974466	46
974742	974788	974834	974880	974926	46
975202	975248	975294	975340	975386	46
975661	975707	975753	975799	975845	46
976121	976166	976212	976258	976304	46
976579	976625	976671	976717	976762	46
977037	977083	977129	977175	977220	46
2.977495	2.977541	2.977586	2.977632	2.977678	46
977952	977998	978043	978089	978135	46
978409	978454	978500	978546	978591	46
978865	978911	978956	979002	979047	46
979321	979366	979412	979457	979503	46
979776	979821	979867	979212	979958	46
980231	980276	980322	980367	980412	45
980685	980730	980776	980821	980867	45
981139	981184	981229	981275	981320	45
981592	981637	981683	981728	981773	45
982045	982090	982135	982181	982226	45

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960	2.982271	2.982316	2.982362	2.982407	2.982452
961	982723	982769	982814	982859	982904
962	983175	983220	983265	983310	983356
963	983626	983671	983716	983762	983807
964	984077	984122	984167	984212	984257
965	984527	984572	984617	984662	984707
966	984977	985022	985067	985112	985157
967	985426	985471	985516	985561	985606
968	985875	985920	985965	986010	986055
969	986324	986369	986413	986458	986503
970	2.986772	2.986816	2.986861	2.986906	2.986951
971	987219	987264	987309	987353	987398
972	987666	987711	987756	987800	987845
973	988113	988157	988202	988247	988291
974	988559	988603	988648	988693	988737
975	989005	989049	989094	989138	989183
976	989450	989494	989539	989583	989628
977	989895	989939	989983	990028	990072
978	990339	990383	990428	990472	990516
979	990783	990827	990871	990916	990960
980	991226	991270	991315	991359	991403
981	2.991669	2.991713	2.991757	2.991802	2.991846
982	992111	992156	992200	992244	992288
983	992553	992598	992642	992686	992730
984	992995	993039	993083	993127	993172
985	993436	993480	993524	993568	993613
986	993877	993921	993965	994009	994053
987	994317	994361	994405	994449	994493
988	994757	994801	994845	994889	994933
989	995196	995240	995284	995328	995372
990	995635	995679	995723	995767	995811
991	996074	996117	996161	996205	996249
992	2.996512	2.996555	2.996599	2.996643	2.996687
993	996949	996993	997037	997080	997124
994	997386	997430	997474	997517	997561
995	997823	997867	997910	997954	997998
996	998259	998303	998346	998390	998434
997	998695	998739	998782	998826	998869
998	999130	999174	999218	999261	999305
999	999565	999609	999652	999696	999739

5	6	7	8	9	Diff.
2.982497	2.982543	2.982588	2.982633	2.982678	45
982949	982994	983040	983085	983130	45
983401	983446	983491	983536	983581	45
983852	983897	983942	983987	984032	45
984302	984347	984392	984437	984482	45
984752	984797	984842	984887	984932	45
985202	985247	985292	985337	985382	45
985651	985696	985741	985786	985830	45
986100	986144	986189	986234	986279	45
986548	986593	986637	986682	986727	45
2.986995	2.987040	2.987085	2.987130	2.987174	45
987443	987487	987532	987577	987622	45
987890	987934	987979	988024	988068	45
988336	988381	988425	988470	988514	45
988782	988826	988871	988915	988960	45
989227	989272	989316	989361	989405	45
989672	989717	989761	989806	989850	44
990117	990161	990206	990250	990294	44
990561	990605	990650	990694	990738	44
991004	991049	991093	991137	991182	44
991448	991492	991536	991580	991625	44
2.991890	2.991934	2.991979	2.992023	2.992067	44
992333	992377	992421	992465	992509	44
992774	992818	992863	992907	992951	44
993216	993260	993304	993348	993392	44
993657	993701	993745	993789	993833	44
994097	994141	994185	994229	994273	44
994537	994581	994625	994669	994713	44
994977	995021	995064	995108	995152	44
995416	995460	995504	995547	995591	44
995854	995898	995942	995986	996030	44
996293	996336	996380	996424	996468	44
2.996730	2.996774	2.996818	2.996862	2.996905	44
997168	997212	997255	997299	997343	44
997605	997648	997692	997736	997779	44
998041	998085	998128	998172	998216	44
998477	998521	998564	998608	998652	44
998913	998956	999000	999043	999087	44
999348	999392	999435	999478	999522	44
999783	999826	999870	999913	999957	43

ANSWERS TO THE EXERCISES,

IN THE FOREGOING WORK.

NUMERATION.

IN FIGURES.

1. 354	6. 200209
2. 506	7. 47800000
3. 3086	8. 45607564300
4. 9070	9. 624,371,264,635,995,442
5. 16785	

IN WORDS.

1. Sixty-two.
2. Ninety.
3. Six hundred and thirty.
4. Seven hundred and four.
5. Six thousand, seven hundred, and eighty-nine.
6. Fifty thousand, four hundred, and seventy-one.
7. Three hundred and seven thousand, six hundred and five.
8. Four millions, seven hundred and sixty-eight thousand, and seventy-three.
9. Twenty-five millions, twenty-four thousand, seven hundred and thirty-eight.
10. Eight hundred and sixty millions, seven hundred and four thousand, and twenty-three.

ADDITION.

2. 14252.	9. 374574.
3. 17468.	10. 511541.
4. 19806.	11. 152385.
5. 443509.	12. 427574.
6. 4023322.	13. 517751.
7. 24229854.	14. 271254.
8. 152385.	

SUBTRACTION.

2. 2667585.	6. 11183.
3. 15676087.	7. 7388177.
4. 621361.	8. 5333665.
5. 2106745.	9. 2529596.

MISCELLANEOUS EXERCISES.

1. 36 years.	3. £401.
2. 367 years.	4. 1780.

MULTIPLICATION.

RULE I.

7. 88246944.	11. 1059648564.
8. 138933048.	12. 1977335789.
9. 638348300.	13. 3652127142.
10. 651297105.	14. 2719416339.

RULE II.

1. 395424.	4. 20461356.
2. 770504.	5. 32811219.
3. 3506304.	6. 8124048.

RULE III.

1. 487567444198.	3. 38060745866.
2. 24998400625.	4. 2837244458048.

MISCELLANEOUS EXERCISES.

1. 360 degrees.	5. 660 difference.
2. 6570 balls.	6. 41975 miles.
3. 1st. 90100 l.; 2d. 29750 l.	7. £32000.
4. 5080208 product.	8. 2810304 letters.

DIVISION.

RULE I.

7. 131070966.	10. 60189850—2.
8. 286206413—3.	11. 32005834—11.
9. 84775340—4.	12. 68764383—1.

RULE II.

1. 94838—23.	4. 53361—10.
2. 11301—41.	5. 7476—86.
3. 7688—34.	6. 2587—21.

RULE III.

1. 9700—15.	7. 729.
2. 8583.	8. 121168—2398.
3. 1576.	9. 60—6396.
4. 133.	10. 95—4912.
5. 524—162.	11. 12737—90069.
6. 943—143.	12. 10539—69275.

RULE IV.

1. 680—83.	3. 6755—133.
2. 23—778.	4. 572—8948.

RULE V.

1. 2377—12.	4. 1763—90.
2. 509—103.	5. 3485—77.
3. 2837.	6. 7863—369.

RULE VI.

1. 742—123.	4. 517—113.
2. 3164.	5. 7495—201.
3. 30919—13.	6. 4831—104.

MISCELLANEOUS EXERCISES.

1. £426$\frac{1}{11}$.	5. 21 days.
2. 251 trees.	6. 1415—211699 m.
3. 17 feet.	7. 69—$\frac{1}{14}$.
4. 23$\frac{1}{4}$ yards.	

SIMPLE PROPORTION. p. 38.

1. 32 days.	11. 210 crowns.
2. 12 men.	12. ~~94$\frac{1}{11}$ boys, 96$\frac{2}{11}$ girls.~~ 864 girls 936 boys.
3. 9 days.	13. 28 yards.
4. 63 barrels.	14. 150 yards.
5. 20 men.	15. 4 hours, 21$\frac{2}{11}$ minutes.
6. 32.	16. 48 lb.
7. 12 miles.	17. 5$\frac{1}{4}$ days.
8. £12500.	18. 6$\frac{6}{7}$ days.
9. 105 bushels.	19. 4 hours.
10. 49 hours.	20. 15 oxen.

COMPOUND PROPORTION.

1. 200 men.
2. 40 acres
3. $11\frac{1}{4}$ men.
4. £$2\frac{13}{16}$.
5. 1200 bushels
6. £$9\frac{3}{8}$. or 7 6
7. £$613\frac{1}{3}$.
8. £500.
9. $403\frac{1}{5}$ sheep.
10. Worked 30 days, and idle 10 days.
11. 60 ells at Frankfort
12. 18657 bolls and 53 lasts
13. 15 men.
14. 42 oxen.

VULGAR FRACTIONS.

CASE I.

1. 19
2. 188
3. 48
4. 4
5. 36
6. 47

CASE II.

1. 24
2. 120
3. 24
4. 60
5. 90
6. 672

CASE III.

1. $\frac{3}{4}$
2. $\frac{3}{35}$
3. $\frac{7}{9}$
4. $\frac{37}{105}$
5. $\frac{21}{31}$
6. $\frac{17}{19}$
7. $\frac{67}{109}$
8. $\frac{17}{45}$
9. $\frac{271}{459}$
10. $\frac{8}{25}$
11. $\frac{183}{617}$
12. $\frac{103}{207}$

CASE IV.

1. $\frac{11}{1}$
2. $\frac{18}{1}$
3. $\frac{42}{7}$
4. $\frac{247}{13}$
5. $\frac{37}{7}$
6. $\frac{106}{9}$
7. $\frac{167}{12}$
8. $\frac{256}{11}$
9. $\frac{7215}{99}$
10. $\frac{406}{9}$

CASE V.

1. $4\frac{5}{8}$
2. $5\frac{6}{7}$
3. $8\frac{9}{10}$
4. $19\frac{1}{2}$
5. $20\frac{1}{105}$
6. $48\frac{5}{37}$

CASE VI.

1. $\frac{10}{27}$
2. $\frac{1}{3}$
3. $\frac{14}{65}$
4. $\frac{616}{75}$
5. $\frac{60}{9}$
6. $\frac{9}{5}$

CASE VII.

1. $\frac{12}{24}, \frac{16}{24}, \frac{18}{24}$.
2. $\frac{144}{216}, \frac{135}{216}, \frac{168}{216}$.
3. $\frac{60}{210}, \frac{168}{210}, \frac{35}{210}$.
4. $\frac{140}{210}, \frac{120}{210}, \frac{126}{210}, \frac{105}{210}$.
5. $\frac{168}{448}, \frac{336}{448}, \frac{224}{448}, \frac{3776}{448}$.
6. $\frac{360}{1050}, \frac{525}{1050}, \frac{7420}{1050}$.

CASE VIII.

1. $\frac{45}{66}, \frac{42}{66}, \frac{55}{66}$.
2. $\frac{63}{126}, \frac{84}{126}, \frac{90}{126}, \frac{112}{126}$.
3. $\frac{12}{24}, \frac{21}{24}, \frac{16}{24}, \frac{22}{24}$.
4. $\frac{27}{36}, \frac{20}{36}, \frac{15}{36}, \frac{19}{36}$.
5. $\frac{308}{672}, \frac{360}{672}, \frac{600}{672}, \frac{272}{672}$.
6. $\frac{105}{120}, \frac{110}{120}, \frac{248}{120}, \frac{96}{120}$.

CASE IX.

3. $\frac{5}{12}$
4. $\frac{23}{48}$
5. $\frac{653}{1286}$
6. $\frac{109}{480}$
7. $\frac{3049}{9680}$
8. $\frac{7}{32}$

CASE X*.

3. 5s. 8d.
4. 2s. $0\frac{12}{67}$d.
5. 4 oz. 10 dwt.
6. 3 r. 8 p.
7. 0 qr. 3 lb. $9\frac{25}{31}$ oz.
8. 0 qr. 21 lb.
9. 10 dwt. 16 gr.
10. 3 b. 3 p.

ADDITION OF VULGAR FRACTIONS.

5. $1\frac{1}{8}$
6. $2\frac{2}{9}$
7. $2\frac{17}{36}$
8. $2\frac{83}{120}$
9. $15\frac{239}{280}$
10. $133\frac{111}{140}$
11. £1. 1s. $6\frac{1}{4}\frac{37}{77}$d.
12. 5 oz. 8 dwt. $11\frac{109}{120}$ gr.
13. 1 qr. 13 lb. 9 oz. 16 dwt. $7\frac{3}{11}$ gr.
14. 35 qr. 7 b. 1 p. $4\frac{4}{7}$ q.

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* This Case and the Exercises in Addition, Subtraction, Multiplication, and Division, as well as the exercises in Decimals, which relate to Money, Weight, or Measure, may be neglected till the Student has proceeded as far as Proportion in Commercial Arithmetic.

SUBTRACTION OF VULGAR FRACTIONS.

3. $\frac{1}{24}$
4. $\frac{2}{81}$
5. $\frac{23}{77}$
6. $1\frac{4}{5}$
7. $3\frac{4}{5}$
8. $20\frac{5}{48}$
9. $\frac{7}{24}$
10. $9\frac{623}{1560}$
11. $\frac{181}{360}$
12. $\frac{167}{672}$
13. $\frac{1}{14}$
14. $8\frac{31}{42}$
15. 7s. 3d. $\frac{1}{4}$ $\frac{1}{3}$
16. 2s. 3d.
17. 6 oz. 13 dwt. $16\frac{16}{21}$ gr.
18. 6 cwt. 3 qr. 16 lb. 12 oz. $12\frac{4}{5}$ dr.

MULTIPLICATION OF VULGAR FRACTIONS.

3. $\frac{9}{14}$
4. $\frac{35}{256}$
5. $\frac{7}{30}$
6. $\frac{8}{45}$
7. $\frac{7}{36}$
8. $3\frac{2}{3}$
9. $221\frac{1}{7}$
10. $6\frac{9}{16}$
11. $793\frac{13}{24}$
12. £23. 12s.
13. $25\frac{4}{5}$ guineas.
14. £91. 0s. $3\frac{3}{4}$ $\frac{4}{31}$d.
15. £16. 19s. $0\frac{3}{4}$d.

DIVISION OF VULGAR FRACTIONS.

3. $\frac{3}{4}$
4. $1\frac{1}{4}$
5. $1\frac{3}{7}$
6. $1\frac{8}{55}$
7. $4\frac{1}{2}$
8. $13\frac{1}{2}$
9. $6\frac{1}{2}$
10. $\frac{9}{13}$
11. $\frac{31}{60}$
12. $17\frac{16}{39}$
13. $75\frac{1}{7}$
14. $2\frac{523}{1716}$
15. $6\frac{39}{49}$
16. $27\frac{131}{168}$
17. $1\frac{136}{185}$
18. $21\frac{11}{12}$
19. D $44\frac{73}{103}$, E $44\frac{73}{103}$, F $44\frac{73}{103}$, G $26\frac{29}{36}$.

PROPORTION.

3. $457\frac{1}{7}$ lb.
4. £$7\frac{1}{2}$
5. $\frac{1}{5600}$ of a guinea.
6. £$6\frac{21}{64}$
7. $8\frac{17}{33}$ days.
8. £$11\frac{1}{16}$
9. £$160\frac{5}{7}$
10. $6\frac{9}{76}$ miles.

REDUCTION OF DECIMALS.

CASE I.

3. .5
4. .75
5. .25
6. .875
7. .692307
8. .0025
9. .592
10. .136
11. .7265625
12. .013
13. .1011904761
14. .01265822784810

CASE II.

3. .016
4. .9125
5. .6427083
6. .7125
7. .8052083
8. .366071428571
9. 3.15733119419642857I
10. .4325
11. .103515625
12. .33267361

MENTALLY.

2. .138
3. .432
4. .284
5. .931
6. .119
7. .098
8. .712
9. .571

CASE III.

3. £0. 17s. 6d.
4. £0. 5s. 6d.
5. £0. 0s. 9d.
6. £0. 0s. $7\frac{1}{2}$d.
7. £0. 6s. $0\frac{3}{4}$d.
8. 2 q. 3 n.
9. 2 qr. 14 lb.
10. 9 oz. 15 dwt.
11. 4 b. 2 p. $5\frac{63}{125}$ q.
12. 16 b. 2 p.
13. 1 r. 14 p.
14. 1 w. 4 d. 23 h. 59′ 56″ 30‴

CASE IV.

2. £0. 7s. $3\frac{3}{4}$d.
3. £0. 19s. 10d.
4. £0. 19s. 2d.
5. £0. 18s. 8d.
6. 5 oz.
7. 8 cwt. 1 qr. 9 lb. 5 oz. $5\frac{1}{3}$ dr.
8. 15 cwt. 3 qr. 18 lb. 0 oz. $11\frac{17}{24}$ dwt.
9. 10 h. 56′ $1\frac{1}{5}$″

CASE V.

2. £0. 4s. $3\frac{1}{4}\frac{21}{37}$d.
3. £0. 12s. $8\frac{1}{2}\frac{10}{11}$d.
4. £0. 17s. $5\frac{1}{4}\frac{9}{11}$d.
5. 1 r. 30 p. $7\frac{4678}{111111}$y.
6. £0. 6s. 9d.
7. 5 b. 1 p. $6\frac{6}{11}$q.
8. 7 f. 60 y.
9. 5 d. 23 h. 6′ 40″

MENTALLY.

1. £0. 19s.
2. £0. 16s. 6d.
3. £0. 10s. $3\frac{1}{4}$d.
4. £0. 14s. $9\frac{1}{2}$d.
5. £0. 15s. 11d.
6. £0. 0s. $1\frac{3}{4}$d.
7. £0. 0s. $10\frac{3}{4}$d.
8. £0. 11s. $6\frac{3}{4}$d.

CASE VI.

2. $\frac{3}{5}$
3. $\frac{3}{4}$
4. $\frac{1}{4}$
5. $\frac{1}{8}$
6. $\frac{7}{8}$
7. $\frac{13}{16}$
8. $\frac{643}{800}$
9. $\frac{1}{400}$
10. $\frac{8777}{8000}$
11. $\frac{13}{1000}$
12. $\frac{81}{400}$

CASE VII.

2. $\frac{1}{3}$
3. $\frac{4}{27}$
4. $\frac{7}{1111}$
5. $\frac{6}{41}$
6. $\frac{8}{33}$
7. $\frac{1}{333}$
8. $\frac{4}{7}$
9. $\frac{8}{13}$

CASE VIII.

2. $\frac{1}{15}$
3. $\frac{1}{150}$
4. $\frac{7}{55}$
5. $\frac{5}{8}$
6. $\frac{16}{75}$
7. $\frac{5}{34}$
8. $\frac{125}{164}$

CASE IX.

2. .416666
.636363
.396396

3. .23148148
.90909090
.10416666

4. .2727272727
.0975609756
.6666666666

5. .95274390243902
.38109756097560
.56818181818181

ADDITION OF DECIMALS.

CASE I.

2. 1285.61826
3. 1506.2325
4. 981.2673
5. 14670.7845

CASE II.

2. $4824.8\dot{3}$
3. $27583.5\dot{6}$
4. $154.63\dot{8}$
5. £$74.9208\dot{5}$
6. £$55.04791\dot{6}$
7. cwt. 17.09027
8. oz. 29.8624

CASE III.

2. $210.364\dot{1}7748\dot{9}$
3. $95.2828\dot{5}$
4. $47.62\dot{5}$
5. $42.0\dot{4}931516\dot{5}$

SUBTRACTION OF DECIMALS.

CASE I.

3. .4455
4. 11.56465
5. 36.875
6. .002845
7. 44.16875
8. .875

CASE II.

3. $29.37860\dot{1}8\dot{5}$
4. $21.5\dot{3}0949\dot{1}$
5. $.295\dot{2}$
6. $44.041\dot{6}$
7. $43.52\dot{7}$
8. 46.648, &c. to 30 places
9. $36.657291\dot{7}$
10. $1.5089\dot{1}8571\dot{4}$

MULTIPLICATION OF DECIMALS.

CASE I.

2. 2.466
3. .018864
4. 43.5802151
5. 15.278254
6. .00515625
7. .413532
8. 115.5465
9. 43.125
10. 7550
11. 75.5

CASE II.

3. $2699.58\dot{3}$
4. 75833333
5. $6.29963\dot{5}$
6. 659.875
7. $6896.1\dot{2}3076\dot{9}$
8. $1.187815\dot{9}\dot{0}$

CASE III.

5. .36
6. .[illegible]
7. .58406
8. 9.[illegible]
9. 312.8
10. 3063.45
11. .273402
12. 90.334236
cont. 675700090

CASE IV.

2. 572.08
3. 13.921
4. 19.6105
5. [illegible].9
6. .1812
7. .00316391.

DIVISION OF DECIMALS

CASE I.

3. 144
4. 1.44
5. 14.4
6. 14400
7. 14.4
8. .0144
9. .13232682+
10. 68.96551724+
11. .001772+
12. .076095[illegible]
13. .18919+
14. .589680
15. 98.75
16. .009875.

CASE II.

2. [illegible]
3. 27.36633
4. 4.242375
5. 1.58878+
6. .27
7. 176
8. 1508.91
9. 24.20673+

CASE III.

2. 853.3682
3. 1,54
4. 7.2191
5. 13.33371
6. .085265.

POSITION.

2. £144.
3. £250.
4. A's stock, £705; and B's, £545.
5. A's share, 218 guineas; C's 324 guineas; and D's 600 guineas.
6. £6⅓ for the harness; £13⅓ for the horse; and £40 for the chaise.
7. A's share 195 guineas; B's 135 guineas; and C's 105 guineas.

DOUBLE POSITION.

2. 84.
3. 1st horse, £30; 2d horse, £40.
4. $23\frac{1}{4}$ days.
5. 27 g. of the 1st sort, and 36 g. of the 2d sort.
6. £1480.
7. 36.

INVOLUTION.

CASE I.

2. 729.
3. 54756.
4. 157464.
5. 1500625.
6. 166375.

TO INVOLVE A FRACTION.

2. $\frac{16}{81}$.
3. $\frac{27}{64}$.
4. $\frac{81}{4096}$.
5. $32\frac{16}{125}$.
6. $2592\frac{4}{81}$.

EVOLUTION.

2. 782.
3. 2345.
4. 672.
5. 1108.041515+.
6. 1.4142135+.
7. 12.536745+.
8. .035.
9. 20.297783+.
10. .8660254+.
11. 1.52752523+.
12. $3\frac{1}{7}$.

MISCELLANEOUS EXERCISES.

1. 3969 yards.
2. £32. 18s.
3. 294.
4. 3615 feet.
5. $78\frac{2}{9}$ lb.
6. 12.1067+.
7. £20+.
8. 6 hours.
9. 42.4265 yards.
10. 21.563858 m.
11. 141.4213 feet.

CUBE ROOT.

2. 72.
3. 38.
4. 99.
5. 402.

6. 1.259921+.
7. 2.08+.
8. 4.0615478+.
9. 7.937+.
10. .333.
11. 32.947075+.
12. .82207069+.
13. .7211247+.
14. $\frac{3}{5}$.
15. $3\frac{1}{8}$.
16. 2.376894+.

MISCELLANEOUS EXERCISES.

1. 14 inches.
2. 6 and 18.
3. 36 and 216.
4. 784 feet.
5. 72 lb.
6. 6.6843 inches.
7. 1.44224 cubits.
8. $37\frac{13}{16}$ grains.
9. 25 inches.
10. 90.714 ft. long, 31.498 ft. broad, 16.378 ft. deep } to carry twice as much.
    57.146 ft. long, 19.842 ft. broad, 10.318 ft. deep } to carry half as much.
11. 356 tons.—
12. 4332 d. 14 h. 27′ 10″.
13. 30451 d. 21 h. 0′ 36″.
14. Ceres' mean distance, 263,208,328 miles; Saturn's mean distance, 903,690,197 miles.
15. 12.649568+ diameters, or 1127962 miles.

DUODECIMALS.

2. 13 feet, 8′, 4″.
3. 27 feet, 1′, 6″.
4. 81 feet, 1′, 6″.
5. 134 feet, 8′, 5″.
6. 273 feet, 6′, 6″.
7. 94 yards, 5 feet.
8. 42 yards, 7 feet, 36 inches
9. 75 yards, 8 feet, 51 inches
10. 851920 feet.
11. 22 feet, 6′, 0″, 7‴, 6⁗.
12. 1330560 yards.

ARITHMETICAL PROGRESSION.

1. 21000.
2. 17325.
3. $711\frac{3}{4}$.
4. 100 and d. 3.
5. 25755 and 101.
6. £$68\frac{9}{10}$ and 52.
7. 300 strokes.
8. 26 years.
9. £36 and £3.

GEOMETRICAL PROGRESSION.

1. 11998.
2. 8748 ounces.
3. 32768 and 65535.
4. 1048575.
5. 524288.
6. 2.
7. End of 7 years
8. 3-fold.
9. 201326592.

PERMUTATION AND COMBINATION.

2. 252.
3. 16777211.
4. 65780.
5. 70.
6. 255.
7. 479001600.
8. 27720.

COMMERCIAL ARITHMETIC.

COMPOUND ADDITION.

1. £69. 13s. 1d.
2. £67. 1s. $5\frac{1}{4}$d.
3. £691. 13s. 1d.
4. £688. 0s. 10d.
5. £657. 5s. $3\frac{1}{4}$d.
6. £6331. 19s. $8\frac{1}{4}$d.

TROY WEIGHT.

7. 660 lb. 1 oz. 8 dwt. 19 gr.
8. 673 lb. 11 oz. 12 dwt. 11 gr.

AVOIRDUPOIS WEIGHT.

9. 712 tons, 13 cwt. 1 qr. 8 lb. 10 oz. 5 dr.
10. 735 tons, 11 cwt. 3 qrs. 12 lb. 12 oz. 7 dr.

CLOTH MEASURE.

11. 6180 yds. 3 qrs. 0 nails.
12. 7370 yds. 2 qrs. 1 nail

WINE MEASURE.

13. 730 tuns, 1 hhd. 28 gal. 6 pints.
14. 668 tuns, 3 hogsheads 14 gallons.

DRY MEASURE.

15. 774 qr. 6 bushels, 0 pecks, 13 pints.
16. 860 qr. 6 bushels, 1 peck, 2 pints.

COMPOUND SUBTRACTION.

1. £29. 4s. 0¼d.
2. £13. 9s. 10d.
3. £489. 5s. 7½d.
4. £476. 17s. 11½d.
5. £74. 3s. 2¼d.
6. £226. 16s. 7½d.
7. £643. 4s. 2¼d.
8. £65. 16s. 2½d.
9. £48. 8s. 8¼d.

TROY WEIGHT.

10. 34 lb. 10 oz. 18 dwt. 14 gr.
11. 24 lb. 5 oz. 17 dwt. 15 gr.
12. 8 lb. 9 oz. 17 dwt. 13 gr.

AVOIRDUPOIS WEIGHT.

13. 102 lb. 15 oz. 13 drams.
14. 174 cwt. 2 qr. 17 lb.
15. 46 tons, 9 cwt. 3 qr. 25 lb.

CLOTH MEASURE.

16. 16 yds. 0 qr. 3 nails.
17. 15 yds. 3 qr. 2 n. 1¼ inch.
18. 4 yds. 1 qr. 2 nails.

WINE MEASURE.

19. 12 hhds. 59 gal. 5 pints.
20. 31 tuns, 3 hhds. 51 gal.
21. 50 tuns, 2 hhds. 51 gal.

DRY MEASURE.

22. 8 qrs. 3 bushels, 7 gal.
23. 101 qrs. 1 bushel, 7 gal.
24. 531 qrs. 6 bushels, 0 pecks, 14 pints.

MISCELLANEOUS EXERCISES.

1. £2. 2s. 10d.
2. £17. 19s. 4d.
3. 12s. 10d.
4. 32 cwt. 2 qr. 24 lb.
5. 91 cwt. 0 qr. 5 lb.
6. 9 yards, 2 qr.
7. £2243. 9s. 0¼d.
8. £267. 1s. 0½d.
9. £2252. 9s. 11d.
10. £157. 4s.
11. £1065. 12s. 11d.

COMPOUND MULTIPLICATION.

CASE I.

1. £77. 16s. 7½d.
2. £47. 2s. 11d.
3. £19. 10s. 2¼d.
4. £65. 13s. 4½d.
5. £41. 1s. 9½d.
6. £6. 3s. 11½d.
7. £34. 17s. 11d.
8. 50 lb. 11 oz. 12 dwt. 22 gr.
9. 16 tons, 17 cwt. 0 qr. 12 lb. 11 oz.
10. 87 yds. 3 qr. 1 nail.
11. 454 qr. 5 bus. 1 pk. 2 pints

CASE II.

1. £8. 14s. 8d¼.
2. £8. 14s. 4¼d.
3. £66. 11s. 2d.
4. £141. 12s. 9d.
5. £3946. 10s.
6. £328. 18s.
7. £628. 10s.
8. £192. 17s.
9. 665 cwt. 2 qr. 8 lb.
10. 87 lb. 11 oz. 7 dwt. 6 gr.
11. 22549 acres, 2 roods, 8 poles, 22¼ yards.
12. 628 tuns, 3 hhds, 11 gal.
13. 983 qrs. 2 bushels, 1 peck, 8 pints.
14. 93 years, 288 d. 3 h. 19′.

CASE III.

1. £82. 6s. 8d.
2. £24903. 17s. 4d.
3. £61. 4s. 0½d.
4. £8. 4s. 10d.
5. £4454. 15s. 3¼d.
6. £397. 3s. 9¼d.
7. 87 tons, 7 cwt.
8. 6456 tons, 8 cwt. 1 qr. 12 lb.
9. 1055 lb. 8 oz. 9 dwt. 10 gr.
10. 1420 acres, 2 r. 19½ poles

CASE IV.

1. £494. 5s. 11¾d.
2. £52. 16s. 8¼d.
3. £1994. 8s.
4. £626. 5s. 11d.
5. 39998 t. 19 cwt. 0 qr. 14 lb.
6. 74 lb. 8 oz. 3 dwt. 7 gr.
7. 1285507 qr. 0 bush. 1 pk.
8. 27 lb. 4 oz. 2 dwt. 12 gr.

MISCELLANEOUS EXERCISES.

1. £47. 1s. 4d.
2. £40. 0s. 0d.
3. £1276. 16s. 0d.
4. £337. 14s. 8d.
5. £350. 9s. 3d.
6. £887. 9s. 10d.
7. £31. 10s. 0d.
8. £69. 12s. 0d.
9. £253. 18s. 9d.
10. £2260. 8s. 4d.
11. £626. 5s. 11d.
12. 1 ton, 5 cwt. 1 qr.
13. 9 tons, 0 cwt. 1 qr. 20 lb.
14. 18 tons, 1 cwt. 2 qr. 27 lb.
15. 21 tons, 12 cwt. 1 qr. 21 lb.
16. £0. 0s. 2¼ $\frac{47}{64}$d.
17. £50. 1s.
18. £387. 16s. 10½d.
19. He runs into debt, £13. 16s. a-year.
20. £97. 2s. 6d.
21. £333. 6s. 8d.

22. 950 y. 1 qr. 3 n.
23. 2 lb. 2 oz. 19 dwt. 14 gr.
24. 12 tons, 4 cwt. 0 qr. 19 lb.
25. £249950.
26. £25442773. 12s. $0\frac{3}{4}$d — reckoning the year at 365d. 5 h. 48′ $48\frac{1}{2}$″
27. 132706800 yards.

COMPOUND DIVISION.

CASE I.

1. £56. 11s. $10\frac{3}{4}\,\frac{1}{3}$d.
2. £131. 17s. $7\frac{1}{7}$d.
3. £117. 13s. $1\frac{1}{2}\,\frac{6}{7}$d.
4. £11. 3s. 4d.
5. £0. 14s. $8\frac{1}{7}\,\frac{2}{3}$d.
6. £14. 18s. $0\frac{3}{4}\,\frac{7}{17}$d.
7. £138. 5s. 2d.
8. £6. 16s. $11\frac{1}{2}$
9. £6. 8s. $9\frac{3}{4}\,\frac{4}{13}$d.
10. £0. 8s. $11\frac{3}{4}$d.
11. £31. 11s. $8\frac{1}{4}\,\frac{7}{37}$d.
12. £12. 0s. $4\frac{3}{4}\,\frac{29}{48}$d.
13. £4. 7s. $4\frac{3}{4}\,\frac{5}{12}$d.
14. 19 cwt. 0 qr. 7 lb. 12 oz. $7\frac{1}{9}$ dr.
15. 1 lb. 10 oz. 8 dwt. $16\frac{1}{4}$ gr.
16. 68 q. 3 b. 1 p. $5\frac{1}{4}\,\frac{1}{9}$p.
17. 1 acre, 3 r. 3 p. $11\frac{4}{7}\,\frac{5}{8}$ yds.
18. 13 cwt. 2 qr. 22 lb. 11 oz. $7\frac{2}{8}\,\frac{4}{7}$ dr.
19. 4 years, 12 days
20. 32 m. 2 r. 24 p. 4 y. 1 f. $1\frac{2}{4}\,\frac{8}{9}$ in.

CASE II.

1. 379
2. 42
3. $60\frac{134}{886}$
4. $53\frac{9823}{88668}$
5. $11\frac{644}{1055}$
6. $72\frac{432}{49956}$

MISCELLANEOUS EXERCISES.

1. £26. 12s. $8\frac{3}{4}$d.
2. £0. 0s. $9\frac{1}{4}\,\frac{1}{14}$d.
3. £0. 0s. $4\frac{1}{4}\,\frac{1}{7}$d.
4. £0. 0s. $3\frac{1}{4}\,\frac{13}{24}$d.
5. £50. 1s.
6. £38. 0s. 10d.
7. £0. 2s. $10\frac{1}{2}\,\frac{8}{9}$d.
8. $20\frac{1}{2}$ yards
9. $581\frac{1}{5}$ miles, nearly.
10. £0. 10s. 6d. per gallon.
11. £0. 13s. loss.
12. £11. 5s. each boy.
    £22. 10s. each woman.
    £33. 15s. each man.
13. A $416\frac{2}{3}$ gs.
    B $260\frac{2}{3}$ gs.
    C $322\frac{2}{3}$ gs.
14. £1. 4s. per boll.
15. £63. 10s. 6d.
16. £3. 7s. 6d.
17. £734. 11s. 8d. captain
    £214. 5s. $0\frac{5}{6}$d. each officer
    £124. 19s. $7\frac{3}{7}\,\frac{5}{8}$d. each man
18. 1536 lbs.
19. 11 cwt. 1 qr. 19 lb. value £47. 19s. 3d.
    13 cwt. 0 qr. 25 lb. value £55. 10s. 9d.
20. 4 gallons.

REDUCTION.

CASE I.

1. 396461 farthings.
2. 33417 halfpence.
3. 14364 pence.
4. 1159 twopences.
5. 2546 threepences.
6. 4619 fourpences.
7. 36545 sixpences.
8. 104220 pence.
9. 55907 grains.
10. 1931 pennyweights.
11. 236760 grains.
12. 19152 pounds.
13. 1751 pounds.
14. 758 nails.
15. 459972 inches.
16. 10080 pints.
17. 368 pints.
18. 7220 sheets.
19. 315569280 seconds.
20. 31556880 seconds.

CASE II.

1. £106. 16s. 4½d.
2. £1234. 10s.
3. £2101. 10s.
4. £1358. 2s. 6d.
5. £4166. 13s. 4d.
6. 9 lb. 8 oz. 15 dwt. 10 gr.
7. 187 lb. 6 oz.
8. 97 lb. 8 oz.
9. 8260 lb.
10. 98 cwt. 1 qr. 21 lb.
11. 331 y. 0 q. 2 n.
12. 1870 ells, 1 qr. 1 n.
13. 5010 yards.
14. 7 acres, 2 r. 9 p. 17$\frac{1}{4}$ y.
15. 3205 gallons.
16. 44808 qrs. 11202 g.
17. 140160 hours, 5840 d.

CASE III.

1. 35480 gs.
2. £793. 16s.
3. 5964 moidores.
4. £10638.
5. 21399 crowns.
6. 5760 lbs. Troy.
7. 3456 English ells.
8. 12273$\frac{3}{7}$ guineas.
9. £60075.

MISCELLANEOUS EXERCISES.

1. £10. 18s. 8d.
2. £167. 8s. 7½d.
3. £0. 0s. 10$\frac{44}{374}$
4. £24. 2s. 2¼d.
5. 6 m. 7 f. 81 y. 1 foot.
6. 56 weeks.
7. 2295 souls.
8. 364661$\frac{131}{2800}$
9. 195071$\frac{488}{747}$ miles.
10. 14 tons, 2 cwt. 0 qr. 6 lb.
11. 6 d. 14 h. 8$\frac{4}{7}$ min.
12. 5 dwt. 9$\frac{38}{89}$ gr.
13. 4 dwt. 22$\frac{58}{89}$ gr.
14. 3 dwt. 20$\frac{28}{31}$ gr.
15. 3 dwt. 13$\frac{20}{31}$ gr.
16. 4s. 3¼d. $\frac{27}{74}$
17. 244
18. 215
19. 47$\frac{41}{63}$
20. £425. 12s. 10d.
21. 434$\frac{3}{8}$
22. 380 y. 96 d. $\frac{38}{45}$ h.
23. 123$\frac{14}{47}$.
24. 5 g. 4 p.
25. 1466$\frac{2}{7}$

SIMPLE PROPORTION.

1. £1. 4s. 6d.
2. £1. 0s. 0d. $\frac{3}{17}$
3. £103. 1s. 6d.
4. £15. 18s. 6d.
5. £120. 4s. $8\frac{1}{4}$d.
6. £211. 6s. $5\frac{1}{2}$d. $\frac{1}{2}$
7. 26 yards.
8. 27 yards, 1 qr.
9. £0. 10s. $10\frac{1}{4}$d. $\frac{17}{74}$
10. £591. 14s. $11\frac{1}{4}$d. $\frac{1}{11}$
11. £0. 1s. 6d.
12. £0. 3s. $1\frac{1}{2}$d. $\frac{1}{4}$
13. £100.
14. £700.
15. £4. 10s. $3\frac{1}{4}$d. $\frac{1}{3}$
16. £15. 16s. $6\frac{1}{4}$d. $\frac{1}{7}$
17. £337. 10s.
18. £35. 0s. $9\frac{1}{2}$d. $\frac{146}{159}$
19. 7 cwt. 1 qr. 25 lb. 10 oz. $3\frac{13}{17}$ dr.
20. $11\frac{1}{2}$d.
21. £0. 18s. $7\frac{1}{4}$d. $\frac{49}{61}$
22. 228 yards.
23. £135. 2s. $2\frac{1}{4}$d. $\frac{1}{3}$
24. £12. 6s. $3\frac{1}{4}$d. $\frac{3}{5}$
25. $103\frac{3}{59}$ yards.
26. hhds. 57. 9 gal. £857. 2s. 10d. $\frac{3}{7}$
27. £17. 7s. $0\frac{1}{4}$d. $\frac{1}{7}$

COMPOUND PROPORTION.

1. £1. 4s.
2. £6. 1s. $0\frac{1}{4}$d.
3. £500.
4. 8s. $11\frac{1}{4}$d. $\frac{43}{55}$
5. 1s. $1\frac{1}{4}$d. $\frac{41}{45}$
6. $4\frac{1}{4}$d.
7. £1403. 3s. $7\frac{1}{4}$d. $\frac{6}{11}$
8. £2. 10s. 5d. $\frac{71}{165}$
9. £250.
10. 18 weeks.

PRACTICE.

CASE I.

	£	s.	d.		£	s.	d.
1.	2336	10	0	12.	105	5	0
2.	2611	6	8	13.	57	1	9
3.	1141	5	0	14.	68	13	2
4.	1033	16	0	15.	3278	8	$10\frac{1}{4}$ $\frac{2}{3}$
5.	597	10	0	16.	1268	12	1
6.	956	0	0	17.	284	18	$0\frac{1}{2}$ $\frac{2}{3}$
7.	765	16	0	18.	230	10	$11\frac{1}{4}$ $\frac{1}{3}$
8.	86	5	0	19.	63	12	$8\frac{1}{4}$ $\frac{1}{2}$
9.	182	17	0	20.	59	12	$6\frac{1}{4}$
10.	79	16	0	21.	39	12	$8\frac{3}{4}$ $\frac{1}{5}$
11.	121	16	6				

CASE II.

	£	s.	d.		£	s.	d.
1.	1274	8	0	12.	1873	11	0
2.	1800	1	0	13.	1257	6	0
3.	2290	4	0	14.	4611	15	0
4.	2317	18	0	15.	65	8	0
5.	3590	5	0	16.	1152	16	0
6.	5013	6	0	17.	2920	10	0
7.	14158	11	0	18.	3386	8	0
8.	4512	18	0	19.	2812	16	0
9.	8797	10	0	20.	3281	12	0
10.	1312	13	0	21.	2025	12	0
11.	2330	10	0	22.	10839	8	0

CASE III.

	£	s.	d.		£	s.	d.
1.	40	7	8	7.	115	0	0
2.	93	11	0	8.	67	3	3
3.	149	1	0	9.	346	3	7
4.	165	4	7	10.	154	10	0
5.	198	16	0	11.	78	15	5
6.	217	7	7	12.	226	16	7

NOTE TO CASE III.

	£	s.	d.		£	s.	d.
1.	5973	0	0	6.	7453	12	0
2.	3497	12	0	7.	4391	6	8
3.	3449	5	0	8.	5862	7	6
4.	4428	18	0	9.	3485	8	7 1/5
5.	3726	11	3	10.	3105	19	8 3/4

CASE IV.

	£	s.	d.		£	s.	d.
1.	66	13	6	6.	4	3	8 1/2
2.	168	5	3	7.	108	16	8 1/2
3.	14	6	10 1/2	8.	60	13	6
4.	9	9	7 1/4	9.	298	18	4 1/2
5.	91	3	9	10.	131	1	3

CASE V.

	£	s.	d.		£	s.	d.
1.	120	7	8	6.	1546	13	9
2.	453	7	11	7.	1241	17	1 1/2
3.	711	11	3	8.	2348	1	9 1/2
4.	797	6	10 1/2	9.	1340	14	8
5.	1327	16	3	10.	1852	1	1 1/2

	£	s.	d.		£	s.	d.
11.	4505	12	6	16.	9060	0	0
12.	3677	2	0	17.	5060	6	8
13.	2724	3	4	18.	6091	17	6
14.	2199	7	6	19.	18784	10	$6\frac{3}{4}$
15.	6917	16	1				

CASE VI.

	£	s.	d.		£	s.	d.
1.	12	6	$1\frac{1}{8}$	6.	101	13	4
2.	76	15	$1\frac{1}{4}\frac{3}{8}$	7.	72	0	$5\frac{1}{4}\frac{1}{3}$
3.	76	11	$2\frac{3}{4}\frac{3}{5}$	8.	58	6	$1\frac{1}{5}$
4.	86	8	$4\frac{3}{4}\frac{1}{5}$	9.	95	10	$2\frac{1}{4}\frac{1}{7}$
5.	209	1	$8\frac{1}{8}$	10.	79	6	6

NOTE TO CASE VI.

	£	s.	d.		£	s.	d.
1.	1	15	0	7.	31	8	$4\frac{1}{4}\frac{1}{4}$
2.	2	4	$11\frac{3}{4}$	8.	24	8	7
3.	3	1	$10\frac{3}{16}$	9.	43	14	6
4.	1	7	$9\frac{1}{2}$	10.	38	17	$3\frac{1}{2}$
5.	3	8	$11\frac{1}{4}\frac{1}{3}$	11.	20	2	$5\frac{1}{2}\frac{1}{2}$
6.	4	7	$1\frac{3}{4}$	12.	53	11	$2\frac{1}{4}$

CASE VII.

	£	s.	d.		£	s.	d.
1.	4	0	$5\frac{3}{4}$	10.	628	15	$0\frac{3}{4}$
2.	12	16	3	11.	1728	15	$8\frac{1}{2}+$
3.	17	3	9	12.	220	2	$9\frac{1}{7}$
4.	37	13	$10\frac{43}{248}$	13.	103	17	$8\frac{1}{4}\frac{1}{16}$
5.	1167	1	3	14.	89	11	$5\frac{1}{2}\frac{1}{2}$
6.	966	8	$5\frac{1}{4}\frac{3}{14}$	15.	476	6	$9\frac{3}{4}\frac{19}{24}$
7.	9521	6	$0\frac{1}{4}\frac{1}{2}$	16.	15147	3	$1\frac{1}{3}+$
8.	239	15	$4\frac{3}{8}$	17.	450637	13	$11\frac{1}{3}+$
9.	39	8	$8\frac{1}{2}\frac{1}{2}$	18.	4401599	12	$2\frac{1}{3}+$

DEDUCTION FROM WEIGHTS.

CASE I.

1. 38 cwt. 3 qr. $0\frac{1}{4}$ lb. +
2. 20 cwt. 0 qr. 3 lb.
3. 2 cwt. 2 qr. $27\frac{13}{96}$ lb.
4. 7 cwt. 0 qr. $0\frac{1}{3}\frac{8}{9}$ lb.
5. £77. 14s. $2\frac{3}{4}$d. $\frac{27}{176}$
6. £325. 7s. $11\frac{3}{4}$d. $\frac{1}{2}$
7. 49 cwt. 0 qr. $5\frac{4}{7}$ lb.
8. £182. 6s. 6d.

CASE II.

1. 41 cwt. 2 qr. 17 lb.
2. 171 cwt. 1 qr. 20 lb. +
3. £360. 12s. 8¾d. $\frac{32}{45}$
4. 113 cwt. 0 qr. 13 lb.
5. £186. 14s. 9¾d.
6. £198. 5s. 9d.
7. 712 lb.
8. 58 cwt. 1 qr. 26 lb.
9. £196. 5s. 2d.
10. £152. 4s.
11. £222. 11s. 3d.

COMMISSION AND BROKERAGE.

	£	s.	d.		£	s.	d.
1.	16	9	0	16.	20	16	0
2.	13	7	8½ $\frac{1}{9}$	17.	189	2	0
3.	49	13	8	18.	9	3	0½
4.	9	16	8½	19.	3	11	9½
5.	14	2	10¾	20.	5	9	6½
6.	277	0	10¼	21.	0	14	3
7.	23	19	11½	22.	15	18	6½
8.	59	18	1¾	23.	3	9	4½
9.	1	8	9	24.	29	4	0½
10.	2	8	3¼	25.	93	16	7
11.	2	14	9½	26.	1733	0	8¼
12.	4	18	8	27.	367	9	0¾
13.	1	9	3¼	28.	528	0	3
14.	0	5	3	29.	584	16	3¼
15.	1	16	5				

SIMPLE INTEREST.

CASE I.

	£	s.	d.		£	s.	d.
1.	126	10	0	6.	156	8	0
2.	124	6	6	7.	31	14	11¾
3.	197	6	3	8.	150	4	11¼ $\frac{1}{3}$
4.	332	15	3	9.	36	17	6
5.	159	5	0	10.	146	12	6

CASE II.

	£	s.	d.		£	s.	d.
1.	9	8	9	6.	9	18	0¾
2.	1	0	0	7.	9	8	3
3.	1	0	0	8.	32	3	2½
4.	4	1	2½	9.	235	9	3½
5.	39	6	5½	10.	1030	15	5¼

CASE III.

	£	s.	d.
1.	2	19	2
2.	21	3	$1\frac{1}{4}$
3.	8	1	$5\frac{1}{4}$
4.	1	11	$0\frac{3}{4}$
5.	106	0	2
6.	8	17	$9\frac{3}{4}$

CASE IV.

1. £4. 3s. 6d. due to Coutts and Co.
2. £2. 15s. $9\frac{1}{4}$d. balance due to D. C.
3. £0. 14s. 7d. interest due to Street.
4. £33. 5s. $4\frac{1}{4}$d. balance due to Jay and Vanderpot.
5. £195. 7s. 6d. balance due to John Jay.

CASE V.

	£	s.	d.
1.	362	5	11
2.	422	7	$5\frac{1}{4}$
3.	1880	10	7
4.	150	15	$6\frac{1}{4}$

DISCOUNT.

CASE I.

1. £16. 13s. 4d.
2. £99. 0s. $5\frac{1}{4}$d.+
3. £66. 13s. 4d.
4. £1. 1s. $8\frac{1}{4}$d.
5. $9\frac{11}{17}$ years
6. 4 per cent.

CASE II.

	£	s.	d.	
1.	247	4	$6\frac{1}{4}$	
2.	247	2	4	
3.	4	16	$9\frac{1}{4}$	
4.	196	4	$4\frac{1}{4}$	
5.	45	0	$6\frac{1}{4}$	
6.	131	6	$8\frac{1}{4}$	
7.	294	19	6	
8.	12	14	5	discount
	$15\frac{1}{4}$	0	0	net proceeds
9.	142	2	$5\frac{1}{4}$	balance due to house in Hamburgh.

EQUATION OF PAYMENTS.

1. 8 months
2. $7\frac{1}{2}$ months
3. $10\frac{3}{4}$ months
4. $8\frac{1}{11}$ months
5. $74\frac{7}{17}$ days
6. 23rd May.

PROFIT AND LOSS.

CASE I.

	£	s.	d.			£	s.	d.	
1.	16	13	4		3.	20	6	$6\frac{1}{7}$	
2.	10	0	0		4.	20	10	3	nearly.

CASE II.

1. 4s.
2. 90s.
3. £56. 14s.
4. 3s. $1\frac{1}{4}$d. $\frac{9}{40}$
5. £1. 2s. $3\frac{1}{4}$d. $\frac{9}{25}$
6. £3. 16s. $5\frac{1}{4}$d. $\frac{5}{61}$

CASE III.

1. 4s. $8\frac{1}{4}$d. $\frac{7}{13}$
2. £14
3. $26\frac{3}{4}$ yards
   12s. prime cost p. yd.

CASE IV.

	£	s.	d.			£	s.	d.	
1.	14	13	9	per cent	3.	4	10	0	gain per cent
2.	3	1	$10\frac{1}{4}$	loss per cent	4.	2	10	0	loss per cent

CASE V.

	£	s.	d.		£	s.	d.
1.	3	15	0	4.	0	19	$8\frac{1}{4}$
2.	0	18	9	5.	0	2	$10\frac{349}{1150}$
3.	0	4	0	6.	0	18	$10\frac{5}{73}$

BARTER.

1. 17 lb. of tea
2. $2133\frac{1}{3}$ yards Irish linen
3. 2s. $6\frac{1}{2}$d.
4. 84s. and 21 cwt.
5. D has the better bargain by £1. 13s. 4d., and A receives 140 spindles
6. 88s. per cwt.

PARTNERSHIP.

		£	s.	d.
1.	A paid...	£553	16	$11\frac{1}{13}$
	B.........	369	4	$7\frac{6}{13}$
	C.........	276	18	$5\frac{7}{13}$
2.	A's share	£4023	11	4
	B's	3520	12	5
	C's	2011	15	8
	D's	1005	17	8

3. A's share £726. 16s. 8d. & the two others are each, £420. 7s. 11d.

		£	s.	d.
4.	A, ...	£105	4	10
	B, ...	122	15	$7\frac{1}{2}\frac{2}{3}$
	C, ...	52	12	5
	D, ...	35	1	$7\frac{1}{4}\frac{1}{5}$

5. A rec. £82 19 $1\frac{1}{11}$
B......... 176 19 $4\frac{8}{11}$
C......... 193 11 $2\frac{6}{11}$
D......... 276 10 $3\frac{7}{11}$

6. A 81 5 0
B 46 17 6
C 22 10 0
D 17 10 0
E 30 12 6
Proprietors, 51 5 0

7. A's share £638 9 $5\frac{1}{3}$
B's ...... 888 9 $5\frac{1}{3}$
C's ...... 1353 9 $5\frac{1}{3}$

8. A's share of the profits is, £128 0 $7\frac{4}{5}$, and B's, 87 7 $1\frac{1}{5}$.

9. A ... £3244 15 $6\frac{24}{65}$
B .... 2163 3 $8\frac{16}{65}$
C ...... 1271 14 $1\frac{5}{13}$

10. A gets £95 1 $8\frac{1}{4}\frac{20}{59}$
B ...... 71 6 $3\frac{3}{4}\frac{15}{59}$
C ...... £57 1 $0\frac{3}{4}\frac{52}{59}$
D ...... 47 10 $10\frac{3}{4}\frac{10}{59}$
E ...... 40 15 $0\frac{1}{4}\frac{58}{59}$

11. A 281 a. 1 r. 35 p.+
B 187 2 23+
C 159 2 12+
D 110 2 10—

12. A ... £74 7 $7\frac{1}{4}\frac{85}{103}$
B ... 105 15 $8\frac{1}{4}\frac{98}{103}$
C ... 330 11 $7\frac{3}{4}\frac{23}{103}$

13. A ... £20 18 $7\frac{1}{4}\frac{1}{25}$
B ... 10 19 $9\frac{3}{4}\frac{6}{25}$
C ... 13 1 $7\frac{1}{2}\frac{6}{25}$

14. 10s. 6d. in the pound.

15. 7s. 6d. in the pound.

16. £2060.

17. The creditors will gain 7d. per pound, by accepting 20s., without interest, at 5 years hence.

ALLIGATION.

CASE I.

1. 17s. $9\frac{1}{4}$d. $\frac{3}{5}$
2. 5s. $4\frac{1}{4}$d. $\frac{7}{9}$
3. 77s. 3d. $\frac{7}{48}$ per oz.
4. 64s. $11\frac{1}{3}$d. $\frac{74}{901}$

CASE II.

1. 20 lb. at 8s.
8 lb. at 7s. 6d.
8 lb. at 7s.
2 lb. at 6s. 8d.
8 lb. at 6s.

2. 3 gal. at 8s. 6d.
1 gal. at 9s.
1 gal. at 9s. 6d.
3 gal. at 10s.

3. 2 oz.

4. 2 lb. at 7d.
2 lb. at 8d.
3 lb. at 11d.

CASE III.

1. 140 lb. at 10d.
28 lb. at 9d.
28 lb. at 8d.
28 lb. at 7d.

2. 50 oz. 16 dwt. $18\frac{18}{31}$ gr.

3. $8\frac{11}{13}$ gallons of water, and 8 gallons of each.

CASE IV.

1. $93\frac{3}{4}$ oz. of gold
   $56\frac{1}{4}$ oz. of silver.
2. 8 oz. of 19
   8 oz. of 21
   32 oz. of 23.
3. $46\frac{26}{69}$ gal. of brandy
   $17\frac{43}{69}$ gal. of water.

STOCKS.

CASE I.

	£	s.	d.
1.	424	18	9
2.	390	6	2
3.	570	12	3
4.	375	18	$1\frac{1}{2}$
5.	35	6	$8\frac{1}{4}$
6.	360	6	8

CASE II.

	£	s.	d.
1.	1131	6	3
2.	824	9	3
3.	7269	5	$10\frac{1}{4}$
4.	1598	0	$2\frac{1}{4}$
5.	6794	7	7

CASE III.

	£	s.	d.	
1.	4	8	$8\frac{1}{2}$	
2.	4	0	0	
3.	5	16	8	
4.	131	5	0	
5.	$4\frac{1}{4}$	0	0	
6.	5	4	8,	5 per cent.
	4	18	6,	3 per cent cons.
	4	19	3,	3 per cent red.

CASE IV.

	£	s.	d.
1.	8466	13	4
2.	4091	13	4
3.	10645	16	8
4.	3307	10	0

CASE V.

1. £1. 7s. $9\frac{1}{5}$d.
2. £11. 9s. $5\frac{3}{4}$d.
3. £0. 9s. 11d. diff.

TERMINABLE ANNUITIES.

CASE I.

1. £2327. 18s. $1\frac{1}{2}$d.
2. £3741. 11s. $4\frac{1}{5}$d.
3. £2740. 18s. $5\frac{1}{4}$d.

CASE II.

1. £196. 4s. 11d.
2. £27. 6s. 4d.

EXCHEQUER BILLS.

CASE I.

1. £16. 4s. 4d.
2. £45. 10s.
3. £5. 16s. 2d.

CASE II.

	£	s.	d.
1.	4070	13	4
2.	3029	16	6
3.	685	8	4
4.	2022	10	0
5.	408	11	9
6.	810	0	9

OMNIUM AND SCRIP.

CASE I.

1. £263. 15s.
2. £178. 15s.
3. £2772.
4. £409. 10s.

CASE II.

1. £21. 18s. 4d.
2. £9. 2s. 7¼d.
3. £9. 2s. 5½d.
4. £6. 18s. 0½d.

MARINE INSURANCE.

CASE I.

1. £36. 12s. 6d.
2. £43. 2s. 6d.
3. £70. 13s. 9d.
4. £260. 17s. 6d.

CASE II.

1. £16. 8s. 7d.
2. £70. 0s. 11d.
3. £24. 14s. 8½d.

CASE III.

1. £87. 6s.
2. £57. 3s.

CASE IV.

1. £85. 10s.
2. £15. 18s. 9d.

CASE V.

	£	s.	d.	
1.	36	19	2¼	whole return for short interest,
	1	13	0	return per cent,
	121	2	2¼	net cost of insurance.
2.	7	10	0	return for short interest,
	0	12	0	return per cent,
	34	16	3	net cost of insurance.
3.	21	0	0	return for short interest,
	0	9	6¼ 1/7	return per cent,
	151	4	4	net amount of insurance.

CASE VI.

	£	s.	d.	
1.	65	0	0	amount for short interest,
	135	0	0	amount for convoy and arrival,
	5	0	0	return per cent,
	124	0	0	net cost of the insurance.
2.	41	5	0	amount of return for short interest,
	90	0	0	return for convoy and arrival,
	133	5	0	net cost of the insurance.

CASE VII.

1. £ 640
2. £1124
3. £1259
4. £3358
5. £2317

CASE VIII.

1. £1769
2. £1299

CASE IX.

1. £4. 5s. 6d. amount of return for short interest.

CASE X.

	£	s.	d.	
1.	78	0	0	whole return of premium,
	91	9	9¼	net cost of the insurance,
	2	5	7¼	return per cent.
2.	2	0	8¾	return for short interest,
	21	10	9¾	return for convoy and arrival,
	35	8	8	net cost of the insurance,
	2	16	6	return per cent.

CASE XI.

	£	s.	d.	
1.	1315	13	5	whole amount to be recovered from the underwriters.
2.	664	19	11	whole sum for which the underwriters are liable.

CASE XII.

	£	s.	d.	
1.	549	18	0	whole cost of the insurance,
	2535	0	0	net proceeds of the loss,
	2416	14	0	amount received credit for,
	57	4	0	return for short interest,
	124	16	0	return for convoy and arrival,
	2	6	8	return per cent.

AVERAGES.

		£	s.	d.
1*.	The ship must contribute	59	5	$2\frac{2}{9}$
	Freight	8	5	$11\frac{1}{9}$
	A, ...	35	11	$1\frac{1}{9}$
	B, ...	47	8	$1\frac{7}{9}$
	C, ...	9	9	$7\frac{4}{9}$
2.	£4 per cent general average loss,			
	Owners of the ship must pay...	£140	0	0
	Proprietors of the Goods	100	0	0
	Underwriters	260	0	0
3.	They recovered in all............	£343	9	$4\frac{1}{2}$
	and £14. 6s. 2¾d. per cent.			

FOREIGN EXCHANGES.

AMSTERDAM.

1. 6842 f. 13 st. 12 pen. cur.
2. 4351 f. 2 st. $14\frac{14}{131}$ banco
3. 7233 f. 2 st. 10 pen. cur.
4. £962.
5. 6099 f. 12 st.
6. £408. 10s. 4d.
7. 17575 f. 1 st. 5 pen.
8. £3. 0s. 1¼d. per cwt.
9. 8075 f. 1 st. 8 pen. current
10. 15724 f. 6 st. 5 pen. cur.

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* In this exercise the value of the ship ought to have been stated at £1000, and the freight at £140.

ROTTERDAM.

1. 5750 florins
2. 5915 flor. 0 st. 4 pen.
3. £715. 11s. 11d.
4. £605. 8s. 11d.
5. £739. 0s. 1d.

HAMBURGH.

1. £372. 1s. 10½d. sterling
2. £676. 12s. 7d.
3. 2307 m. 11s. 1 p. banco
4. 2742 flor. 12 stiv. 12 pen. current
5. £537. 10s. gained by the transaction
6. £17. 15s. 4d. difference in favour of Hambro', or 17s. 9d. per cent
7. 5379 m. 10 s. 9 phen.
8. £343. 10s. 5d.
9. £692. 1s. 7d.
10. £351. 4s. 10d.
11. First draft .... M. 7179 0 9

Second ditto				£529	6	7
Commis. on both sums, ...	£5	6	6			
Brokerage on ditto.........	1	6	7			
Letters	0	11	8			
				7	4	9
House in London has to pay				£536	11	4
First draft				535	10	0
House in Loudon pays for 4 mo. acccom.				£1	1	4
						3
Ditto .... in a year, ...				£3	4	0

Rate per cent per annum, ...£0. 12s. 0d.

FRANCE.

1. 5226 francs, 68 cents.
2. £81. 3s. $\frac{364}{307}$ sterling
3. £305. 9s. 8d. sterling
4. 1080 florins banco
5. 2157 francs, 57 cents.
6. 8802 m. 14 s. 4 p. banco
7. £137. 14s. 8½d. sterling.

SPAIN.

1. 3617 pias. 3 reals, 1 mar.
2. £846. 19s. 10d. sterling
3. £19. 6s. sterling
4. 473 m. 9s. 11 phen. banco
5. 9509 f. 2 st. 6⅖ ph. banco.
6. 348 f. 12 st. 6 pen. cur.
7. 32814 reals, 12 mar. vel.
8. £128. 10s. 7d. gain.

LISBON AND OPORTO.

1. £362. 14s. sterling
2. £850. 10s. sterling
3. 1553.924 milrees
4. 4920 milrees, 446 rees; or £1148. 2s. 1d.
5. 11628 f. 15 st. 10 p. cur.
6. He gains £180 sterling.

LEGHORN.

1. £355. 18s. 4½d. sterling
2. £180. 17s. 2d. sterling
3. 1564 piastres, 8 soldi
4. 11421 florins banco
5. 2910 p. 1 r. 8¼ m. of Spain
6. 5s. 7$\frac{1}{16}$d. nearly, per bar.

GENOA.

1. 1149 lire, 18 soldi
2. £944. 4s. 7⅛d. sterling
3. 3289 lire
4. 210 pezze, 16 soldi, 8 den.
5. £280. 13s. 2⅓d. sterling
6. 1382 lire, 3 soldi, 1$\frac{5}{7}$ den. fuori di banco.

VENICE.

1. 18291 francs, 63 cents
2. £121. 0s. 9d. sterling
3. 3372 francs, 30 cents, French money
4. 7587 lire, 13 soldi, 6 den. Venetian money
5. 12137 lire, 0 soldi, 3 den. picooli
6. 28901 lire, 1 soldi, 1¼ den. piccoli

NAPLES.

1. £257. 13s. 4d. sterling
2. £249. 8s. 8½d. sterling
3. 4644 ducats, 88 grains
4. 9372f. 14st. 6p. of Holland

MALTA.

1. £35. 18s. 11¾d. sterling
2. 5410 scudi, 6 tari, 6 grains
3. £60. 9s. 8¾d. sterling.

PALERMO.

1. 730 ounces. 0 tari, 10 grs.
2. £348. 4s. 1⅕d. sterling
3. 1576 ounces, 20 tari
4. 4764 pias. 2r. 20m. of plate

VIENNA.

1. £201. 6s. 11½d. sterling
2. £228. 10s. 4d. sterling
3. 6962 g. 22 creut. 2⅘ pfe.
4. 2236 rixdol. of account, 43 creut. 2 pfenings.

TURKEY.

1. £106. 2s, 8½d. sterling
2. 3574 piastres, 32½ paras.

RUSSIA.

1. £423. 12s. 11d. sterling
2. 6053 rubles, 39 copecks
3. £1. 4s. 5½d. per pood
4. 7 tons, 9 cwt. 1 qr. 4 lb. value £374. 8s. 7½d.

SWEDEN.

1. £2009. 3s. 4d. sterling
2. 1479 rixdol. 45 skillings.
3. 215 rixd. 26 skil. 7 fenings
4. 1544 marcs banco.

COPENHAGEN.

1. 3834 rixd. 3 skil. 1 fening
2. £823. 6s. 6d. sterling
3. 6408 rixd. 15 skil. 7 fenings
4. 3616 rixd. 4 skil. 7 fenings.

DANTZIC, KONIGSBERG, AND MEMEL.

1. £275. 16s. 4d. sterling
2. 4964 guldens, 21 groschen
3. £2. 12s. $9\frac{1}{2}$d. per cwt.
4. £4. 9s. 2d. nearly, per qr.

SCOTLAND.

1. £5. 17s. $0\frac{1}{4}$d.
2. £227. 11s. $6\frac{1}{4}$d.

AMERICAN STOCKS.

1. £325. 19s. $4\frac{3}{4}$d.
2. 3127 dollars, 31 cents.

EXCHANGE WITH THE UNITED STATES.

1. £200. 13s. 1d.
2. £987. 13s. $0\frac{1}{4}$d.
3. 2366 dollars
4. £1703. 9s. $8\frac{1}{4}$d.
5. £1186. 17s. 6d.
6. 6895 flor. 6 stiv. $10\frac{2}{3}$ pen.
7. 757 dollars, 90 cents
8. 1009 dollars, 12 cents.

WEST INDIES.

1. £1382. 16s. 3d. Jamaica currency
2. £5408. 11s. $6\frac{1}{2}$d. sterling
3. £421. 14s. 9d. sterling
4. £1627. 19s. $4\frac{1}{2}$d. sterling
5. 2316 dollars, 60 cents.

SIMPLE ARBITRATION.

1. $51\frac{3}{16}$d. nearly.
2. 37s. $3\frac{3}{10}$ gr.
3. 22 f. 3 cents, nearly.
4. 34s. 2 gr.
5. 37s. $7\frac{1}{4}$ gr.
6. $42\frac{1}{12}$d.
7. 41 f. 36 cents.

COMPOUND ARBITRATION.

1. $39\frac{5}{47}$d. per dollar. The direct exchange is therefore more advantageous.
2. $70\frac{5}{7}$d. per milree. The circular operation is therefore more advantageous.
3. £4. 9s. 4d. gain, by the circular operation.
4. 1 combin. $41\frac{3}{4}$d. nearly ⎫
   2 ditto $38\frac{1}{2}$d. ⎪
   3 ditto $39\frac{1}{4}$d. ⎬ per dol.
   4 ditto $38\frac{1}{3}$d. ⎪
   5 ditto $38\frac{1}{2}\frac{1}{3}$d. ⎪
   6 ditto $42\frac{9}{16}$d. ⎭

WHEN CHARGES ARE INCLUDED.

1. 34$\frac{1}{2}$d. per dollar. 2. £454. 18s. 2$\frac{1}{4}$d.

STANDARDING GOLD.

1. 30 oz. 1 dwt. 12 gr. 2. 107 guineas.

STANDARDING SILVER.

1. 261 oz. 15$\frac{1}{7}$ dwt. 2. 243 oz. 11$\frac{3}{4}$ dwt.

TO FIND THE VALUE OF GOLD AND SILVER, &c.

1. £4. 2s. 1$\frac{7}{8}$d.
2. 24 livres, 5$\frac{1}{3}$s.
3. 5s. 1$\frac{13}{16}$d.
4. 62$\frac{1}{2}$d.
5. 59$\frac{1}{3}\frac{1}{4}$d.
6. 38$\frac{5}{12}$d.

SIMPLE INTEREST.

1. £294. 7s. 6.
2. 4 years.
3. 5 per cent
4. £174. 10s. 4d.
5. £432. 5s.
6. 4 years.
7. 3$\frac{3}{4}$ per cent.

COMPOUND INTEREST.

1. £1480. 4s. 9$\frac{3}{5}$d.
2. £379. 19s. 2$\frac{4}{25}$d.
3. 4 per cent.
4. 14 years, 75 days.
5. £6442905*, &c.
6. £13362. 15s. 3d.

ANNUITIES IN ARREARS.

1. £2157. 16s. 5d.
2. £46. 6s. 10$\frac{1}{3}$d.
3. 24 years.
4. 4$\frac{1}{2}$ per cent.
5. £98. 14s. 2$\frac{1}{4}$d.
6. †£848. 8s. 3d.

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* The answer to this exercise consists of 19 places of figures, of which the above are the *first* seven places.

† This exercise should have been placed among those of Reversionary Annuities.

PRESENT WORTH OF ANNUITIES.

1. £60. 0s. $4\frac{4}{5}$d.
2. £1359. 0s. $7\frac{1}{5}$d.
3. £863. 6s. $3\frac{1}{4}$d.
4. £20.
5. 28 years.
6. £5000.
7. £64. 7s. $11\frac{1}{3}$d.

REVERSIONARY ANNUITIES.

1. Same as Ex. 6. Page 464.
2. £60.
3. 12 years.
4. 4 years hence.
5. £41. 9s. $11\frac{1}{4}$d.

MISCELLANEOUS EXERCISES.

1. £37. 6s.
2. Lease better by £1308. 7s. $9\frac{3}{4}$d.
3. £876. 15s. $0\frac{1}{4}$d.
4. £33. 1s. $11\frac{1}{2}$d. add. rent. £83. 1s. 11d. whole rent.
5. 53 years, 99 days.
6. £238,635,494 debt. £16,931,774 fund.

ANNUITIES ON LIVES.

PROBLEM I.

1. $\frac{1541}{3666}$ Probability.
2. $\frac{2701}{4448}$ Prob.* $\frac{1747}{4448}$ Prob. shall die.
3. $\frac{481}{4748}$ Prob.

PROBLEM II.

1. $\frac{2901}{5158}\times\frac{3347}{5666}=.3324$ Prob.
2. $\frac{3286}{3911}\times\frac{4441}{4903}=.761$ Prob.
3. $\frac{336}{941}\times\frac{1709}{2579}=.2366$ Prob.

PROBLEM III.

1. 25.09 Exp.
2. 39.15 Exp.
3. 33.2 Exp.

PROBLEM IV†.

1. 3.331
2. 8.789
3. 5.783
4. 14.034

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* This is supposed to be a male life.

† Although the words male and female occur in the three first Exercises, they are to be disregarded, as Table VIII. from which the answers are taken, only contains lives in general.

PROBLEM V.

1. 13.296
2. 10.884
3. 8.402

PROBLEM VI.

1. 18.184
2. 18.215
3. 20.182

PROBLEM VII.

1. £12. 16s. 10⅘d.
2. £156. 1s. 7d.

PROBLEM VIII.

1. £42. 9s. 1d.
2. £0. 0s. 2¾d.

PROBLEM IX.

1. £172. 19s. 3d.
An. P. £10. 3s. 4d.
2. £924. 16s. 9½d.
An. P. £55. 4s. 2d.

PROBLEM X.

1. £16. 12s. 10½d.
An. P. £3. 12s. 10½d.
2. £183. 5s. 7¼d.
An. P. £17. 2s. 11d.
3. £19. 1s. 0d.
An. P. £2. 5s. 0d.

THE END.

J. Swan, Printer, 76. Fleet Street, London.